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Research Article

Discrete-Time $H_\infty$ Preview Control Problem in Finite Horizon

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This paper considers the $H_\infty$ preview control problem for discrete-time systems. It investigates the problem via game theory and dynamic programming. Different from the existing results, on one hand, the proposed approach is suitable for dealing with the corresponding multiple preview channels problem; on the other hand, the approach provides a possibility in explaining how the preview controller improves the $H_\infty$ performance and why the performance will be saturated.

1. Introduction

Preview control is a means of using the future information of the reference or disturbance input for control. In the context of tracking, it amounts to tracking a delayed reference. As for the preview disturbance, it is a signal which can be obtained before it disturbs the system. It thus can be redefined as a delayed disturbance. Accordingly, the preview information often appears in the form of the delayed exogenous disturbance or reference signal. The preview control problem thus falls into the category of the delayed control problem. The delayed magnitude of the reference or disturbance is usually referred to as the width of the preview window and it reflects the ability of the system to obtain information in advance.

Numerous facts have shown that the preview control can improve the system performance effectively. For instance, semiactive and active suspension systems can always provide the better closed-loop performance including ride comfort and the handling performance than passive suspension because the former exploit the preview information for control [1–3]. Hence, preview control has extensive applications such as path following [4], bottom following for autonomous underwater vehicles [5], navigation [6], and robot trajectory control [7], which thus attracts lots of attention with respect to how to utilize the preview information [8–11]. So far, the optimal preview control theory has been relatively mature [2] and the $H_\infty$ preview control theory has also been developed greatly. The result of Tadmor and Mirkin [10, 11] is the most beautiful one to resolve the $H_\infty$ preview control problems. Unfortunately, it seems not to attack the multiple-preview cases in that there does not exist a pair of state and costate of the Hamilton Jacobi system that the former typically captures the contribution of past inputs/initial data, whereas the costate captures the contributions of future inputs. Or rather, there is not a suitable instant to separate the past inputs/initial data and future inputs completely. An alternative comprehensive result for the continuous-time system is proposed by Kojima and Ishijima [9]. It can settle a generalized class of delay system covering the multiple input delay and preview $H_\infty$ control problem, yet the infinite-dimensional system theory and operator manipulation employed there are actually very abstract and complicated to understand. Besides the pure state-space approaches mentioned previously, Meinsma and Mirkin [12] also give another complete solution to the multiple I/O delays control problem, which is converted into a series connection of Adobe problem to solve. The solution can provide an insight into the effect of the lag on the controller and has the remarkable physical sense. Adobe input problem, as its special case, covers the preview problem. Yet, given that the transfer function/matrix is a critical intermediate to solve the problem, we know that the approach is actually confined to settle the problem for the time-invariant system. There seem few studies dealing with the finite horizon $H_\infty$ preview control problem [8]. Note that
[8] provides no necessity of the problem. In addition, there seem few works mentioning how the $H_{\infty}$ preview control performance is improved and the explicit evidence that the $H_{\infty}$ performance will be saturated as the width of the preview window increases.

Compared with the existed literatures [8–12], the property of the proposed solution procedure is still very attractive. Our approach borrows the idea of dynamic programming. It can attack the multiple-preview case, where the preview may be from a channel or multichannel, so it can resolve the varying preview problem. What is more interesting, the structure of our solution is more similar to that of the standard $H_{\infty}$ controller. In detail, the state gain of the proposed preview controller is in the form of $-B_{2}' P(i) (B_{1}' P(i + 1) B_{2} + I)^{-1} B_{2}' P(i + 1) A$, where the matrix $P(i + 1)$ admits an $H_{\infty}$-like Riccati equation with the same dimension as the state.

We will organize the paper as follows. In Section 2, we propose the $H_{\infty}$ preview control problem. Sections 3 and 4 are devoted to the analysis and solution of the problem, respectively. A numerical example is provided in Section 5. Some conclusions are achieved in Section 6.

**Notation.** In the paper, the superscript $'$ means matrix transpose. $\|\cdot\|_{l_{2}(a,b)}$ is the 2-norm in $l_{2}(a,b)$. A quadratic form $x' P x$ will be denoted as $|x|_{P}^{2}$ and $|x|_{I}^{2}$ with $P = I$, and $I$ is the identity matrix. $\tilde{x}_{i}$ represents the column vector stacked by $x(k), w(k-1), \ldots, w(k-d)$. $L(x)$ and $Q(x)$ only imply the linear and quadratic function over $x$, respectively.

## 2. Discrete-Time $H_{\infty}$ Preview Control Problem

Consider the system
\begin{equation}
\begin{aligned}
    x(i + 1) &= A x(i) + B_{1} w(i - d) + B_{2} u(i), \\
    z(i) &= C x(i) + D u(i),
\end{aligned}
\end{equation}
where $A, B_{1}, B_{2}, C, D$ are matrices with compatible dimensions, $d > 0$ is the width of the preview window, $x(i)$ is the state, $u(i)$ is the control input, $w(i)$ is the previewable information, and $z(i)$ is the regulated signal.

The finite-horizon $H_{\infty}$ preview control problem for the system (1) can be stated as follows.

For a prescribed $\gamma > 0$, find a control-law
\begin{equation}
    u(i)^{*} = F(x(i), w(s) \mid s = i - d, \ldots, i)
\end{equation}
such that
\begin{equation}
    \sup_{w} \frac{\|z(i)\|_{l_{2}[0,N]}^{2} + |x(N + 1)|_{P_{N+1}}^{2}}{\|w(i)\|_{l_{2}[0,N-d]}^{2}} < \gamma^{2},
\end{equation}
where $N$ is the control terminal time, and $P_{N+1}$ is the positive semidefinite weight matrix over the terminal state.

Equation (3) shows a remarkable difference from a general $H_{\infty}$ performance index that the involved $z(\cdot)$ and $w(\cdot)$ are in the different time horizon, which originates from the fact that $w(i), i = N - d + 1, \ldots, N$ actually does not affect the system. The difference will be actually shared with the infinite-horizon $H_{\infty}$ preview control problem. However, due to $[0, \infty) = [0, \infty - d)$, the infinite-horizon $H_{\infty}$ preview control problem for the system (1) is as follows.

For a prescribed $\gamma > 0$, find a control-law $u(i)^{*} = F(x(i), w(s) \mid s = i - d, \ldots, i)$ such that
\begin{equation}
    \sup_{w} \frac{\|z(i)^{\ast}\|_{l_{2}[0,\infty)}^{2}}{\|w(i)^{\ast}\|_{l_{2}[0,\infty)}^{2}} < \gamma^{2}.
\end{equation}

Equation (2) means that the control-law we pursued is full-information. The $H_{\infty}$ full-information controller $u(i)$ achieves (3) or (4) implies that the controller regulates the state so as to minimize the energy of the regulated signal when the system encounters the worst-case preview information.

In order to assure that the underlying problem is well defined, we have to make the standard assumptions.

(i) $(A, B_{2})$ is stabilizable.

(ii) $[A, B_{2}, C, D]$ has no invariant zeros on the unit circle and is left invertible.

In addition, to simplify the derivation, we make an orthogonal assumption without loss of generality as

(i) $D'[C \quad D] = [0 \quad l]$.

## 3. Analyze the Problem via Zero-Sum Game Approach

The $H_{\infty}$ preview control problem is so involved that the game theory will be introduced. It will provide a preliminary analysis for solving the problem.

It should be noted that the standard $H_{\infty}$ control problem has been connected with a game problem, where the cost functional of two players is
\begin{equation}
    J(u, w) = \sum_{i=0}^{N} \left( |z(i)|^{2} - \gamma^{2} |w(i)|^{2} \right) + |x(N + 1)|_{P_{N+1}}^{2},
\end{equation}
subject to (1) with $d = 0$, $u$ is the minimizing player, and $w$ is the maximizing player [13, 14].

Analysis shows that the $H_{\infty}$ finite preview control problem reserves the essence of the game above except that the information pattern is restricted. We thus associate the problem with the game which has cost functional of two players
\begin{equation}
    J(u, w) = \sum_{i=0}^{N} |z(i)|^{2} - \gamma^{2} \sum_{i=0}^{N-d} |w(i)|^{2} + |x(N + 1)|_{P_{N+1}}^{2},
\end{equation}
subject to (1), where the minimizing player is $u$ and the maximizing player is $w$. Different from the standard $H_{\infty}$ control problem, $w$ is previewable in finite time interval $d$. 
In other words, the action of the player \( w \) at each time always takes effect after \( d \) units of time. Strategies of the players are confined to have the information patterns as

\[
\begin{align*}
\psi(i) &= F_u(x(i), w(i-d), \ldots, w(i)), \\
\psi(i) &= F_w(x(i), w(i-d), \ldots, w(i-1))
\end{align*}
\]  

(7) (8)

which means that, at each time \( i \), the two players have access to \( x(i) \) as well as \( w(i-1), \ldots, w(i-d) \) and the strategy of \( w \) is available for the player \( u \) while the player \( u \) knows nothing about the strategy of \( w \). For ease of use, the aforementioned information pattern will be also called the state feedback hereafter in the paper.

It is a fact that whatever the strategies before \( t \) stage are, decisions at \( t \) stage can never influence the states, the strategies, and the part before \( t \) stage in the game. We thus borrow the idea of dynamic programming to investigate the game problem (6) related to the preview problem.

At \( N \) stage, collect terms influenced by two players \( u(N) \), \( w(N) \) in cost function (6) and write them together as \( J_N \)

\[
J_N = |z(N)|^2 + |x(N+1)|^2_{P(N+1)},
\]

(9)

that \( w(N) \) is not involved and \( J_N \) is quadratic over \( u(N), w(N-d), x(N) \) follows from (1) and (9). Rewrite \( J_N \) as

\[
\begin{align*}
|\psi(N) - \psi^*(N)|^2_{Y(N)} + Q(w(N-d)) + 2|x(N)|L(w(N-d)) + Q(x(N)),
\end{align*}
\]

(10)

where \( Y(N) = I + B_1^2P(N+1)B_2 \) is a positive matrix from \( P(N+1) \geq 0 \). \( L(a) \) means the linear homogeneous term over a variable \( a \), \( Q(a) \) denotes a quadratic homogeneous term over \( a \), and \( \psi^*(N) \) is a linear function with respect to \( x(N) \), \( w(N-d) \). They are explicit via completion of square over \( u(N) \). In view of \( Y(N) > 0 \), it is clear that the optimal solution with the state feedback structure as (7) to \( J_N \) is \( (\psi^*(N), w^*(N)) \) with any \( w^*(N) \) since \( w(N) \) never disturbs the system before \( N-d \).

At \( N-1 \) stage, combine with (10) together the terms in \( J(u, w) \) impacted by \( u(N-1), w(N-1) \) as

\[
\begin{align*}
|z(N-1)|^2 + 2|x(N)|L(w(N-d)) + Q(x(N)).
\end{align*}
\]

(11)

With a similar argument to \( i = N \), there will hold

\[
\begin{align*}
|\psi(N-1) - \psi^*(N-1)|^2_{Y(N-1)} + 2|x(N-1)|L(w(N-d), w(N-d-1)) + Q(w(N-d), w(N-d-1)) + Q(x(N-1)),
\end{align*}
\]

(12)

where \( L(a,b) \) and \( Q(a,b) \) are used to represent a linear function and a quadratic one in variables \( a, b \), respectively, and \( \psi^*(N-1) \) is a homogeneous linear with respect to \( x(N-1), w(N-1-d), w(N-d) \). \( L(a,b), Q(a,b), \) and \( \psi^*(N-1) \) can be also explicit. \( Y(N-1) \) is determined by terms \( Q(x(N)) \) and \( |z(N-1)|^2 \). Likewise, if \( Y(N-1) > 0 \), then \( (\psi^*(N-1), w^*(N-1)) \) where any \( w^*(N-1) \) is still the optimal solution to (11).

Repeat the above process until \( N-d \), and we will find that the terms in (6) involving \( u(N-d), w(N-d) \) are a quadratic form as

\[
\begin{align*}
|z(N-d)|^2 - y^2|w(N-d)|^2 + Q(w(N-d), \ldots, w(N-2d+1)) + x(N-d)|L(w(N-d-1), \ldots, w(N-2d)) + Q(x(N-d-1)).
\end{align*}
\]

(13)

With reminiscence that \( \psi(i) \) is available for \( u(i) \), we have

\[
\begin{align*}
|u(N-d) - u^*(N-d)|^2_{Y(N-d)} + |w(N-d) - w^*(N-d)|^2_{M(N-d)} + 2x(N-d)L(w(N-d-1), \ldots, w(N-2d)) + Q(w(N-d-1), \ldots, w(N-2d)) + Q(x(N-1)),
\end{align*}
\]

(14)

where \( L(\ldots, \ldots), Q(\ldots, \ldots) \) stand for “linear” and “quadratic”, respectively. Note that \( u^*(N-d) \) is a homogeneous linear with respect to \( x(N-d), w(N-d), \ldots, w(N-2d) \), \( w^*(N-d) \) is a homogeneous linear with respect to \( x(N-d), w(N-d-1), \ldots, w(N-2d) \). \( L(\ldots, \ldots), Q(\ldots, \ldots) \), and \( u^*(N-1) \) can be still explicit. Different from instant before \( N-d \), from \( N-d \) instant, there needs to be assured that the player \( u \) has an adverse impact on \( J(u, w) \) so that the noncooperative game is kept. Hence, \( \Lambda(N-d) < 0 \) should hold simultaneously beside \( Y(N-d) > 0 \).

The case at \( i, i = 0, \ldots, N-d-1 \), bears the analogous analysis to the stage \( N-d \).

Based on the preceding analysis, we can achieve such a conclusion which will play an important role in solving the game problem as well as the original \( H_\infty \) preview control problem.

**Proposition 1.** Consider the game problem

\[
J_i(u, w) = \sum_{j=i}^{N} \left| z(j) \right|^2 - y^2 \sum_{j=i}^{N-d} \left| w(j) \right|^2 + \left| x(N+1) \right|^2_{P(N+1)},
\]

(15)

where \( i = 0, \ldots, N \), subject to (1). If the information pattern of strategies \( u^*(j), w^*(j) \) is restricted to be full-information, then \( J_i(u, w) \) can be expressed as

\[
\sum_{j=i}^{N} \left| u(j) - u^*(j) \right|^2_{Y(j)} + \sum_{j=i}^{N-d} \left| w(j) - w^*(j) \right|^2_{M(j)} + f(\bar{x}_i),
\]

(16)

where \( f(\bar{x}_i) \) is a quadratic form over \( \bar{x}_i \)

\[
f(\bar{x}_i) = \sum_{j=1}^{d} \sum_{k=1}^{d} w(i-j)^{\prime} \beta_{j,k}(i) w(i-k)
\]

(17)

\[+ 2 \sum_{j=1}^{d} w(i-j)^{\prime} \alpha_{j}(i) x(i) + x(i)^{\prime} P(i) x(i).
\]
In Proposition 1, it is not important what \( u^*(j), w^*(j), Y(j), \Lambda(j), \beta(j), \alpha(j), P(i) \) exactly are. Here, we just want to say that \( I_j(u, w) \) can be expressed by such a quadratic form. In the sequel, we will characterize those parameters as well as the saddle point.

From (15), it is obvious that
\[
I_k(u, w) = I_{k+1}(u, w) + |z(k)|^2 - y^2|w(k)|^2.
\] (18)

Applying Proposition 1 to \( I_{k+1}(u, w) \) leads to
\[
I_{k+1}(u, w) = \sum_{j=k}^{N} |u(j) - u^*(j)|^2_{\gamma(j)} + \sum_{j=1}^{N-d} |w(j) - w^*(j)|^2_{\Lambda(j)} + f(\tilde{x}_{k+1}).
\] (19)

Plugging (19) into (18) and completing square over \( u(k), w(k) \), we will have the following relations:
\[
\mathcal{A}_{j-1}(k) = \alpha_j(k+1) - S'_j(k) \Lambda(k)^{-1} \alpha_j(k+1), \quad j = 2, \ldots, d,
\]
\[
\mathcal{B}_j(k) = B'_jP(k+1) - B_n(k)\alpha'_j(k+1) \Lambda(k)^{-1} \alpha_j(k+1),
\] (20)
\[
\alpha_j(k) = \mathcal{A}_j(k) \alpha_x(k), \quad j = 1, \ldots, d,
\]
\[
S_j(k) = \beta_{1,j}(k+1) - \alpha_1(k+1) G(k) \alpha_j(k+1),
\]
\[
\beta_{1,j}(k) = \mathcal{B}_j(k) B_x(k),
\]
\[
\beta_{j-1,j}(k) = \beta_{j,j}(k+1) - \alpha_1(k+1) G(k) \alpha'_j(k+1) - S'_j(k) \Lambda(k)^{-1} S_j(k),
\]
\[
\Lambda(k) = -y^2 I + \beta_{1,1}(k) - \alpha'_1(k+1) G(k) \alpha_1(k+1).
\] (24)

In the above,
\[
Y(k) = I + B'_jP(k+1)B_2,
\]
\[
G(k) = B_2 Y(k)^{-1} B'_2,
\]
\[
[A_x(k) B_x(k)] = [I - G(k) P(k+1)] [A B_1].
\] (27)

Observing (20)–(24), it is not hard to find that the left-hand side of (20)–(24) will be calculated step by step as we have initial values \( S_j(N) = 0 \) (i = 1, \ldots, d - 2), \( \Lambda(i) = -y^2 I \) (i = \( N - d + 1, \ldots, N \)), \( \alpha_j(N+1) = 0 \) (j = 1, \ldots, d), \( \beta_{j,j}(N+1) = 0 \) (k = 1, \ldots, d - 2), \( j = 1, \ldots, k - 1 \).

As \( Y(k) \), \( \Lambda(k) \) are nonsingular, it is straightforward to get that
\[
I_k(u, w) = \sum_{j=0}^{N} |u(j) - u^*(j)|^2_{\gamma(j)} + |w(j) - w^*(j)|^2_{\Lambda(j)} + f(\tilde{x}_k).
\] (28)

\( P(k) \) in \( f(x_k) \) is related to \( x(k) \) and updated as
\[
P(k) = H_2 (P(k+1)) - A_x(k) \alpha'_1 \Lambda(k)^{-1} \alpha_1 A_x(k),
\] (29)

where
\[
H_2 (P(k+1)) = C'C + A^T P(k+1) A
\]
\[
- A^T P(k+1) B_2 Y(k)^{-1} B'_2 P(k+1) A.
\] (30)

Moreover, we have \( u^*(k), w^*(k) \) as
\[
u^*(k) = -Y(k)^{-1} B'_2 \left\{ P(k+1) [A_x(k) x(k) + B_x(k) w(k-d)] + \sum_{j=1}^{d} \alpha_j(k+1) (k+1) \right\} + \sum_{j=1}^{d} S_j(k) w(k-j)
\] (31)
\[
w^*(k) = -\Lambda(k)^{-1} \left\{ \alpha_1(k+1) [A_x(k) x(k) + B_x(k) w(k-d)] + \sum_{j=1}^{d} S_j(k) w(k-j) \right\}.
\] (32)

Note that we can take any \( k \in [0, N] \) in the derivation above, so a similar conclusion holds for any \( k \in [0, N] \). Of course, the \( N \) stage is only a special case including one decision.

For all \( \Lambda(j), Y(j) \) j = 0, 1, \ldots, N, are nonsingular, we can achieve
\[
J_0(u, w) = \sum_{j=0}^{N} |u(j) - u^*(j)|^2_{\gamma(j)} + \sum_{j=0}^{N-d} |w(j) - w^*(j)|^2_{\Lambda(j)} + f(\tilde{x}_0),
\] (33)

where \( f(\tilde{x}_0) \) is as shown in (17) and is a quadratic form totally determined by the initial data.

**Theorem 2.** Consider the game problem (6) subject to (1). If (29) has a solution such that \( Y(j) > 0, \Lambda(j) < 0 \), then \((u^*(j), w^*(j)), j = 0, \ldots, N \) as in (32) are saddle point of \( f(u, w) \) in (6).

**Proof.** As (29) has a solution such that \( Y(j) > 0, \Lambda(j) < 0 \), it is easy to get that
\[
J(u^*, w) \leq J(u^*, w^*) \leq J(u, w^*),
\] (34)

where \( u^* \) means that for all \( j = 0, \ldots, N \), \( u(j) \) is replaced by \( u^*(j) \) in the cost function \( f(u, w) \), so does \( w^* \). The proof is complete. \(\square\)
4. Finite Horizon $H_\infty$ Preview Controller

Now we start with giving the solution to the finite-horizon $H_\infty$ preview control problem for the discrete-time system (1) and the sufficient and necessary condition thereof.

**Theorem 3.** Consider the $H_\infty$ preview control problem with (3) for the state-space model (1) with zero initial data. The problem is solvable if and only if

(C1) recursive equation (29) has a positive semidefinite solution,

(C2) $\Lambda(i) < 0$.

Moreover, if the problem is solvable, then (31) provides the preview controller satisfying (3).

**Proof.** For any $i, 0 \leq i \leq N$. As (29) has a positive semidefinite solution $P(i) \geq 0$, there holds $Y(i) > 0$. As $\Lambda(i) < 0$, that $u(i), i = 0, \ldots, N$ are the controllers such that (3) holds for the zero-initial data which follows from Theorem 2. The sufficient part is complete.

As for the necessity of Theorem 3, we can apply the existing $H_\infty$ full-information control results [15] for the augmented system corresponding to (1).

We have the following algorithm to realize the $H_\infty$ preview controller provided by Theorem 3 in finite horizon.

1. Set $y = y_0$;
2. Set $i = N, P(N + 1) = P_{N+1}, \alpha_j(N + 1) = 0, \beta_{k,j}(N + 1) = 0, S_j(N) = 0, \Lambda(N) = -\gamma^2 I$;
3. Use (29) to compute $P(i)$ and check its positive semidefiniteness. If it holds, go on; otherwise, go to step 1 and reset a larger value than $y_0$ for $y$;
4. Use (21) and (20) to compute $\alpha(i)$ and $\beta_{k,j}(i)$;
5. Use (24) to compute $\Lambda(i)$ and check its negativity. If it holds, go on; otherwise, go to step 1 and reset a larger value than $y_0$ for $y$;
6. Use (31) to compute the controller $u(i)$;
7. $i \leftarrow i - 1$, if $i - 1 \geq 0$, go to step 2; otherwise, end.

Why the preview control can improve the performance of the close-loop system? To answer the question, a similar idea will be utilized to solve the finite-horizon $H_\infty$ preview control problem for the system (1) with $d = 0$, namely, the standard full-information $H_\infty$ control problem in finite horizon. We will omit the detail of derivation and provide the control law as well as the worst-case disturbance.

**Corollary 4.** Consider the $H_\infty$ preview control problem with (3) for the state-space model (1) with $d = 0$ and zero-initial data. For a given $\gamma > 0$, the $H_\infty$ full-information control law $u(i) = F(x(i), w(i))$ achieving (3) with $d = 0$ exists if and only if

(i) recursive equation

\[
P(i) = H_2(P(i + 1)) - A_x(i)P(i + 1)B_1\Lambda(i)^{-1}B_1^TP(i + 1)A_x(i)
\]

has a positive semidefinite solution;

(ii) $\Lambda(i) < 0$

where

\[
\Lambda(i) = -\gamma^2 I + B_1^TP(i + 1)B_x(i).
\]  (36)

Moreover, there are the controller and the worst-case disturbance as follows:

\[u(i)^* = -Y(i)^{-1}B_1^TP(i + 1)[Ax(i) + B_1w(i)],\]  (37)

\[w(i)^* = -\Lambda(i)^{-1}B_1^TP(i + 1)A_x(i)x(i).\]  (38)

Corollary 4 is a standard result for the $H_\infty$ full-information control [15]. It is the special case of Theorem 3 with $d = 0$.

As for comparing the optimal control laws (37) with (31), the worst-case disturbances (38) with (32), and the Riccati equations (35) with (29), we will find that, for the preview case, the $d$-step delay enables the current control law to take not only the effect before $d$ units of time but also additional instant disturbance information into account.

Denote

\[
\Upsilon(i) = \sum_{j=1}^{d}\alpha_j(i + d + 1 - j)G(i + d - j)\alpha_j'(i + d + 1 - j),
\]

\[
\Lambda(j) = \sum_{j=1}^{d-1}\sum_{k=1}^{d-1}\alpha_j(i + d - j)\Lambda(i + d - j)^{-1}S_j(i + d - j)
\]

\[+ B_x(i + d)\alpha_d\Lambda(i + d)^{-1}\alpha_dB_x(i + d),\]  (39)

and then $\Lambda(i)$ has the form as

\[
\Lambda(i) = -\gamma^2 I + B_1^TP(i + d)B_x(i + d) - \Lambda(i) - \Upsilon(i).\]  (40)

Comparing (40) with (36), we will find that (29) is more complex than (35). In detail, the right side of (35) only involves information at $i + 1$ instant while the right side of (29) involves information from $i + 1$ instant to $i + d$ instant. Observing (40), we will see that $\Lambda(i)$ includes additional two terms, one is the term $\Upsilon(j)$ directly relating to $u(j)$, the other is the term $\Lambda(j)$ directly connecting with $w(j)$.

**Remark 5.** The formula (40) together with the formula (36) shows that using the previewable disturbance makes the performance improvement possible in finite horizon. However, it should be noted that the performance will achieve saturation for some preview length $d$, which stems from the structure of the control gain $\alpha_j(i)$.

Given that the proceeding analysis works on the case with any preview length $d$, yet it is not hard to find that the larger the preview length, the more complicated the problem. A fact should be noted that too much preview information may not help to achieve better performance. As a matter of fact, for the large enough preview window, the further the information away from the current position, the weaker the impact. In view of (26), $\|I - G(\cdot)P(\cdot)\|$ is less than 1, so $\alpha_j(\cdot) = *x\Pi_{k=0}^{i-1}(I-G(\cdot+k)P(\cdot+k))$, where * denotes a bounded entry whose
exact form is not relevant at the moment) has weaker weight as \( j \) increases. This analysis tells us that some tradeoffs may be reasonable and beneficial for the large preview problems, and it may also explain the saturation phenomenon mentioned by Tadmor and Mirkin [10] and by Mirkin and Meinsma [16] to a certain extent.

5. Numerical Example

To illustrate our results and their advantage based on Theorem 3, consider the \( H_{\infty} \) preview problem for the plant (1) with

\[
A = \begin{bmatrix} 0.9987 & 0.0512 \\ -0.0512 & 1.0500 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.0500 \\ 0.0013 \end{bmatrix}, \\
B_2 = \begin{bmatrix} 0.0013 \\ 0.0512 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & -0.2 \\ 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.
\] (41)

We are interested in the effect of the preview length on the achievable performance. Applying Theorem 3 and the standard \( \gamma \)-iteration to this system leads to the optimal achievable \( \gamma \) for each \( d \). The results are provided in Figure 1.

It shows that the achievable \( H_{\infty} \) performance is improved (from 1.755 in the preview-free problem to 0.899 with \( d \approx 30 \)) but any further increase of \( d \) has no effect on the achievable \( H_{\infty} \) performance after some finite \( d \); that is, the achievable \( H_{\infty} \) performance will be saturated as a function of the preview width \( d \).

6. Conclusion

The paper investigates the \( H_{\infty} \) preview control problem for the discrete-time systems. It settles the problem via completion of square and dynamic programming. The idea is suitable for the multiple preview channels and time-variant preview problems. It makes us realize another interesting problem: how to achieve the better performance via utilizing the suitable amount of preview information.

Disclosure

Parts of this work were presented at the 7th Asian Control Conference.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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