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<td><strong>Author(s)</strong></td>
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4f amplified in-line compressive holography

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Abstract: Compressive holography is a combination of compressive sensing and holography. In this paper, an approach to expand the amplification ratio and enhance the axial resolution in in-line compressive holography is proposed. Firstly the basic principle of 4f amplified in-line compressive holography is described. Next the feasibility of reconstructing object and analysis of reconstruction quality is verified. Finally, both simulated and real experiments on multilayer objects with non-overlapping and overlapping patterns are demonstrated to validate the approach.

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References and links

1. Introduction

Digital Holography is a two-step process: recording a hologram on a CCD and recovering the object wavefront by back-propagating the numerically reconstructed hologram to the object plane. While digital holography has significant advantages and widespread applications in acquiring and storing object information [1, 2], its application is limited by the need for huge storage and bandwidth requirements. The relatively new framework of compressive sensing can reduce the requirement on storage and hence transmitting of information. It has drawn extensive attention and has been widely applied as it can realize accurate N-dimensional information reconstruction from $M \leq N$ dimensional measured value using the optimization algorithms [3, 4].

Compressive sensing technique is revolutionary in many areas, and digital holography is no exception. Brady et al [5] suggested the reconstruction of holograms using compressive sensing method and called it compressive holography. Rivenson et al [6] introduced the theory of compressive sensing in a review paper and presented examples of applications of compressive sensing to digital holography [7]. Some applications of compressive holography include: recovering object information with compressive holography from single exposure in-line [8] or off-axis hologram [9], applying compressive holography in millimeter wave band testing [10], retrieving phase of lens by compressive phase-shifting holography with single-pixel detector [11], recovering the phase based on an phase-modulated optical system with sparse representation [12], wire reconstruction using multiple view projection compressive holography [13] and reconstructing two live water cyclopes with microscopic compressive holographic tomography [14]. Reference [15] verifies that in-line compressive holography has higher axial resolution than back-propagation method. Reference [16] indicates that combining of lens results in an image with a greater depth of focus, and the depth of focus refers to the resolvable axial resolution of imaging systems. The reconstruction of a small object magnified by microscopy or a diverging beam will introduce a spherical phase term due to mismatch of the reference and test wavefronts. In this paper, tomographic reconstruction of small objects using a 4f amplified in-line compressive holography technique is proposed. Some advantages of this approach is

- The effect of the spherical phase factor need not be considered
- It maintains maximal space-bandwidth product for the in-line hologram
- It enables tomographic reconstruction of the small objects with high axial resolution.

In this paper, the principle and applications of a 4f amplified in-line compressive holography (4FICH) is described. This approach combines the Gabor in-line setup with 4f recording system to capture the diffraction field of 3D object with amplification. The outline of this paper is as follows: in Section 2, an introduction is given to the basic principle of 4f amplified in-line compressive holography. In Section 3, the feasibility of object reconstruction based on 4f amplified in-line compressive holography is verified through simulation and the quality of object reconstruction of the 4f amplifying in-line compressive holography compared to in-line compressive holography is analyzed. Finally in Section 4, real experiments on reconstructions of small multilayer objects with non-overlapping and overlapping patterns are demonstrated.
2. The principle of 4f amplified in-line compressive holography

2.1 Compressive sensing

Compressive sensing is a novel technology, which can implement accurate reconstructing from fewer samples than required by conventional methods. In this paper, compressive sensing is applied to a digital hologram captured using a 4f amplified in-line hologram recording system.

To make accurate reconstruction using compressive sensing, two conditions have to be satisfied. The first one is signal has the sparsity, which means signal can be exposed to a sparse signal under some transform, such as the Fourier transform. For example, the Fourier transform realizes the sparse representation of signal and can be denoted by $\psi$. If $f$ is a $(N \times 1)$ vector, then $x = \psi^T f$ is the sparse signal of original signal $f$. Here $\psi^T$ is the Hermitian transpose of $\psi$. The second condition is that when designing an appropriate sensing matrix $H$, the sparse representation matrix and the sensing matrix must satisfy restricted isometric property:

$$
(1 - \delta_s) \|x\|^2 \leq \|H\psi x\|^2 \leq (1 + \delta_s) \|x\|^2
$$

where $\|\cdot\|^2$ denotes the Euclidean norm and $\delta_s$ is much smaller than one.

The projection of the sensing matrix $H \in \mathbb{R}^{M \times N}$ ($M \ll N$),

$$g = Hf$$

will compress the signal $f \in \mathbb{R}^N$ to $g \in \mathbb{R}^M$.

For accurate reconstruction from the sensing data $g \in \mathbb{R}^M$, the Fourier-identity or Fourier-wavelet matrix pairs satisfy the above-mentioned criterion and can be appropriately chosen for compressive holography [17, 18]. Thus a highly accurate reconstruction can be obtained by solving Eq. (3),

$$\hat{x} = \arg \min \|x\| \text{ s.t. } g = H\psi x$$

where $\|x\| = \sum_i |x_i|$.

The connection between compressive sensing and 4f amplified in-line compressive holography and the reconstruction process are explained in detail, in section 2.2 and 2.3.

2.2 The principle of 4f amplified in-line compressive holography (4FICH)

The structure of 4FICH is very simple compared with traditional objective lens or tube lens system used in microscopy. It consists of only two lenses, and does not introduce a spherical phase factor in the reconstruction results. Figure 1 illustrates the schematic diagram of a 4FICH recording system, and two lenses in the dashed line box form the 4f amplification system. Let the focal length of two lenses $L_1$ and $L_2$ be $f_1$ and $f_2$ respectively, and the distance between two lenses be $d = f_1 + f_2$. 

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On the basis of the 4f system, a three dimensional object point \(O(x',y',z')\) can be imaged into a three dimensional image point \(\text{Im}(x'',y'',z'')\). The transverse magnification \(\beta_{x,y}\) and axial magnification \(\beta_z\) on the image plane of the 4f system are described in Eq. (4), where the negative sign indicates image inversion. The transverse and axial magnifications are independent of the transverse and axial coordinates of the object.

\[
\beta_{x,y} = -\frac{f_2}{f_1}, \beta_z = \left(\frac{f_2}{f_1}\right)^2 \tag{4}
\]

According to the transverse and axial magnifications of a 4f system, the relationship between pixel sizes on object plane and image plane can be described by Eq. (5), where \(\Delta_x \times \Delta_y \times \Delta_z\) and \(\Delta_x' \times \Delta_y' \times \Delta_z'\) is pixel size on object plane and image plane respectively.

\[
\Delta_x = -\frac{f_2}{f_1} \Delta_x', \Delta_y = -\frac{f_2}{f_1} \Delta_y', \Delta_z = \beta_z \Delta_z' = \left(\frac{f_2}{f_1}\right)^2 \Delta_z' \tag{5}
\]

The image diffraction field \(E(x,y)\) and a plane reference beam \(R(x,y)\) generate a hologram \(H(x,y)\) on the recording plane. The irradiance distribution of in-line hologram is given by Eq. (6), where the plane reference beam is set to unity.

\[
I_H(x,y) = |R(x,y) + E(x,y)|^2 = 1 + |E(x,y)|^2 + 2 \text{Re}(E(x,y)) \tag{6}
\]

where \(I_H(x,y)\) is the irradiance distribution of the hologram on the recording plane and \(|E(x,y)|^2\) contributes to the system error term \(n(x,y)\). Thus Eq. (6) can be simplified to Eq. (7) after eliminating DC term [5].

\[
\overline{I_H(x,y)} = 2 \text{Re}(E(x,y)) + n(x,y) \tag{7}
\]

Equation (7) indicates that irradiance distribution on hologram plane and image diffraction field have a linear mapping relation with system error. So a 4FICH recording system is considered as a linear space-invariant system. According to the optical propagation theory, the image diffraction field can be defined as the convolution of image diffraction wave and the point spread function \(h\) as:

\[
\text{Im}(x'',y'',z'') = \int \int \int h(x-z,y-w,z-u) |E(x,y,z)|^2 \, dz \, dw \, du \tag{8}
\]
\[ E(x, y) = \text{Im} \{ g(x', y', z') \ast h(x', y', z') \} = F_{N_{x}, N_{y}}^{-1} \left\{ \sum_{k, l} F_{k, l} \left[ \text{Im} \{ g(x', y'; z') \ast h(x', y'; z') \} H(k_{x}, k_{y}; z') \} \right\} \]

(8)

The angular spectrum method is used as the distance between image plane and recording plane is small. The point spread function of the angular spectrum method can be considered as the inverse Fourier transform of the transfer function \( H = \exp \left( jz \sqrt{k_{x}^{2} - k_{y}^{2}} \right) \).

Assuming the sample interval is \( \Delta_{x} = \Delta_{y} \) on hologram plane and \( N_{x}, N_{y} \) are the number of pixels along respectively then \( \text{Im} \{ g(x', y'; z') \} \) can be divided into \( N_{x} \times N_{y} \times N_{z} \) pixels of size \( \Delta_{x} \times \Delta_{y} \times \Delta_{z} \). The image diffraction field can then be discretized as:

\[ E(k_{x}, l_{y}; z) = F_{N_{x}, N_{y}}^{-1} \left\{ \sum_{m, n} F_{m, n} \left[ \text{Im} \{ m_{x}, n_{y} \ast q_{z} \} H(m_{x}, n_{y}; q_{z}) \} \right\} \exp \left( jq_{z} \sqrt{k_{x}^{2} - k_{y}^{2}} \right) \]

(9)

Defining \( Y_{(m-1)N_{y}, (n-1)N_{x}} = E_{m, n} \), \( X_{m, n} \) is a block diagonal matrix, \( F_{2D} \) is the matrix representing the 2D DFT with size of \( N_{x} \times N_{y} \) with size \( N_{x} \times N_{y} \), \( Q = [P_{0}, P_{1}, \ldots, P_{N_{y}-1}] \) with \( P_{q} = [P_{q, m}]_{N_{x}} \) representing the transfer function \( \exp \left( jq_{z} \sqrt{k_{x}^{2} - k_{y}^{2}} \right) \), which is Fourier transformed from point spread function at \( q_{z} \) plane and \( T_{2D} \) represents the 2D inverse DFT matrix and \( H_{i, j} = (T_{2D} Q)_{i, j} \).

2.3 The reconstruction process of 4f amplified in-line compressive holography (4FICH)

To realize compressive holographic reconstruction of the digital hologram, sampling is applied to the Fourier transform results of hologram. Most of the information is concentrated at the center of the frequency domain and the information density decays increasingly away from the origin [19]. Thus more data should be sampled around the origin and less data on the periphery. To generate the special undersampling pattern, which satisfies characteristic of high density at the center of the frequency domain and low density around the periphery, here suppose the probability of random sampling obeys the distribution as shown in Eq. (11).

\[ p(x, y) = \exp \left( -\frac{\sqrt{x^{2} + y^{2}}}{\beta_{r}^{2}} \right) \]

(11)

Where \( p(x, y) \) indicates the probability of sampling the point \( (x, y) \), \( \alpha_{r} \) and \( \beta_{r} \) denote respectively the sampling ratio and the sampling number.
The special sampling pattern is shown in Fig. 2, where the white points represent the position of sampled data and the black points denote the position of un-sampled data. The sampling ratio can be defined as the number of sampling data divided the overall data. Figure 2 show that the sampling pattern in the frequency domain has a sampling ratio of 25%.

![Image](image_url)

Fig. 2. The sampling pattern in the frequency domain with sampling ratio of 25%.

According to compressive sensing theory, the pair of Fourier and canonical basis is incoherent [17], which guarantees a reconstruction of the compressed hologram by Total Variation (TV) minimization. Hence the reconstruction problem can be solved by TV minimization of Eq. (12).

$$\hat{f} = \arg \min \|f\|_{TV}, \text{s.t. } g = Hf$$

Here $$\|f\|_{TV} = \sum_{m} \sum_{n} \sum_{q} \sqrt{(f_{m+1,n,q} - f_{m,n,q})^2 + (f_{m,n,q+1} - f_{m,n,q})^2 + (f_{m,n,q+1} - f_{m,n,q})^2}$$

In other words, the whole processes of (4FICH) can be divided into four steps.

1. Hologram G is recorded by 4f amplified in-line hologram recording system.
2. A variable density sampling mask is used to sample the FT data from the hologram.
3. Information on the image plane can be reconstructed from the compressed hologram by optimization algorithm based on the constraint of TV minimization.
4. According to the relationship between object wave and image wave, the information of object plane can be reconstructed by the information on the image plane.

The detailed flow chart for illustrating the reconstruction processes is shown in Fig. 3.
3. Simulation analysis

In this section, the feasibility of object reconstruction based on 4FICH for a small object is studied. Two holograms are simulated: the first is in-line recorded without amplification and the second is in-line recorded with 4f amplification. For reconstruction, two methods are tested: one is the back-propagation reconstruction algorithm used in traditional digital holography and the other is compressive holography reconstruction algorithm described in Section 2 of this paper. Thus, four groups of simulated results are compared: back-propagating reconstruction for in-line hologram (BP-H), compressive holography reconstruction for in-line hologram (CH-H), back-propagating reconstruction for 4FICH (BP-4FH) and compressive holography reconstruction for 4FICH (CH-4FH).

The simulated experimental setup is similar to the one in Fig. 1, and the simulated object with the amplification system is shown Fig. 4. A simulated object with two planes (155 μm × 155 μm × 2 layers) is used as the test sample while the distance between two focusing plane is 0.02 mm. There are three vertical lines on the front plane and three horizontal lines on the back plane. The width of all lines is δw = 13.95 μm. The front plane is at the focus of lens $L_1$ and its distance from the detector plane is $z'_1 = 402 \text{ mm}$. The pixel size of hologram plane is $\Delta_x = \Delta_y = 4.65 \mu \text{m}$ and there are 100 × 100 pixels. The wavelength is 632.8 nm and the focal lengths of two lenses $L_1, L_2$ in the amplification system are $f_1 = 50 \text{ mm}$, $f_2 = 150 \text{ mm}$ respectively.
Fig. 4. Simulated experimental setup.

Firstly, the experimental feasibility of object reconstruction based on 4f amplified in-line compressive holography is demonstrated. The object and its images obtained by the 4f system are shown in Fig. 5, where the image plane array of measured object is transposed. According to Eq. (4), the transverse magnification and the axial magnification of image plane are $\beta_{x,y} = 3$ and $\beta_z = 9$ respectively. In the experimental setup, the distance from corresponding image planes to detector are $z_1, z_2$ respectively, the line width on image plane of the measured object is $\delta_{ld}$ and the pixel size of measured object is $\Delta = \Delta_z$. According to the principle of 4f amplification, the distance of image planes and detector can be defined by Eq. (13), the line width $\delta_{ld}$ on image plane of the measured object can be shown as Eq. (14).

$$z_1 = z'_1 - 2(f_1 + f_2), \quad z_2 = z_1 + \beta_x(z'_1 - z'_1) \quad (13)$$

$$\delta_{ld} = \beta_{x,y} \delta_{ld} \quad (14)$$

According to these Eqs, $z_1 = 2\text{mm}, z_2 = 2.18\text{mm}, \delta_{ld} = 41.85\mu\text{m}, \Delta_x = \Delta_y = 1.55\mu\text{m}$, can be deduced.

![Fig. 5. Simulated object and its image: (a) two slices with line width $\delta_y = 13.95\mu\text{m}$ placed at $z'_1 = 402\text{mm}, z'_2 = 402.02\mu\text{m}$, (b) the images of two slices with line width $\delta_y = 41.85\mu\text{m}$ imaged at $z_1 = 2\text{mm}, z_2 = 2.18\text{mm}$ from the detector respectively.](image)

The reconstruction results are shown in Fig. 6. Figures 6(a)-6(d) show the reconstruction results of BP-H, CH-H, BP-4FH and CH-4FH, respectively. From Fig. 6(d), it can be seen that two images of two slices from 4f amplifying in-line hologram can be distinguished clearly by compressive holography, and the axial resolution can be improved with 4f amplifying system. This means that the proposed system can overcome the shortcoming of axial resolution limit of the compressive holography.
To further evaluate the reconstruction results, an image quality index [20] defined in Eq. (15) is adopted to compare the reconstruction results.

\[
\text{Im}Q = \frac{4\sigma_\phi \overline{\sigma}_\phi}{\left(\sigma_\phi^2 + \sigma_\omega^2\right) \left(\overline{\phi}^2 + \overline{\omega}^2\right)}
\]

where \(\overline{\phi}\) denotes the average amplitude value from the original image, where \(\overline{\omega}\) denotes that from the reconstructed image, \(\sigma_\phi^2\) denotes the variance of amplitude value from the original image and \(\sigma_\omega^2\) that from the reconstructed image, \(\sigma_{\phi\omega}\) denotes the cross-correlation of the original image and the reconstructed image. The dynamic range of \(\text{Im}Q\) is from \([-1, 1]\), where the highest quality index (\(\text{Im}Q = 1\)) is obtained in the case \(\omega = \phi\).

The quality indexes of the reconstructed results by back-propagating reconstruction algorithm and compressive holography for in-line hologram and 4f amplified in-line hologram are shown in Table 1. Table 1 indicates that reconstruction by compressive holography has higher quality indexes than that by back-propagating reconstruction algorithm and the quality index of compressive holography reconstruction increases while adding 4f amplifying system into in-line hologram recording setup. Here there is a difference between the \(\text{Im}Qs\) of the horizontal (\(\text{Im}Q = 0.635\)) and vertical (\(\text{Im}Q = 0.8826\)) lines for compressive holography reconstruction of in-line hologram. The main reason is that the reconstruction process firstly satisfies the reconstruction quality of last layer (horizontal line in this example which is far away from detector).

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<td>Vertical line by BP</td>
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<td>Horizontal line by BP</td>
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4. Experimental analysis

The recording setup for the 4FICH system is shown in Fig. 7. A 632.8 nm He-Ne laser is used as the light source to record a 900×900 pixel hologram with a pixel size of 4.65µm. The 4f amplification system consists of two lenses, with the focal lengths of \(f_1 = 50mm\) and
$f_2 = 150\text{mm}$ respectively. There are two test objects. One is a $3.18\text{mm}$ thick optical element with the triangle and circular pattern engraved on the front and back surfaces respectively and the two patterns are non-overlapping. The other one is a similar optical element but the two patterns are overlapping. The test objects are shown in Fig. 8. In both samples, the relative parameters of the patterns are the same. The side length of triangle pattern is $1\text{mm}$, and the outside diameter of the circular pattern $0.5\text{mm}$. The groove width of the two patterns is $40\mu\text{m}$.

![Experimental setup of recording 4f amplifying in-line hologram](image)

Fig. 7. Experimental setup of recording 4f amplifying in-line hologram: (1) laser light source, (2) microscopic objective, (3) pin hole, (4) collimating lens, (5) mirror, (6) tested object, (7) two lenses with focal length $f_1$ and $f_2$, (8) CCD.

![Two tested objects](image)

Fig. 8. The two tested objects: (a) Object with two no-overlapping patterns, (b) Object with two overlapping patterns.

In the experimental setup, the front surface of the test optic is placed at the front focus of lens $L_1$, and then triangle is imaged at the back focus of lens $L_2$. Hence, according to Eq. (13), the distance between the image plane of circular pattern and that of triangle pattern is $28.62\text{mm}$. Two experiments are performed with the experimental system shown in Fig. 7. In each experiment, two in-line holograms are recorded: one with 4f amplification and the other is without amplification. The reconstructions are performed on a computer with Intel Core5 CPU running at $2.6\text{GHz}$ with $4\text{GB}$ of RAM.

1) Experiment of the object with non-overlapping patterns

The in-line holograms without and with 4f amplification are shown in Figs. 9(a) and 9(b). Figure 10 shows their reconstruction results at two positions $z_1 = 428.62\text{mm}$ and $z_2 = 431.80\text{mm}$, by back-propagating reconstruction algorithm and compressive holography. The data processing takes about one hour by compressive holography reconstruction and several seconds by back-propagating reconstruction. Figures 10(a)-10(c) show the reconstruction results by CH of the in-line hologram with normal system, BP of the in-line hologram with 4f amplifying system and CH of the in-line hologram with 4f amplifying system, at $z_1 = 428.62\text{mm}$ and $z_2 = 431.80\text{mm}$ respectively. Figures 10(d) and...
10(e) show the corresponding highlighted cross line of Figs. 10(a)-10(c) at $z_1 = 428.62\, \text{mm}$ and $z_2 = 431.80\, \text{mm}$ respectively. It is very obvious that compressive holography reconstruction has a better reconstruction quality than back-propagating reconstruction algorithm does. 4f amplifying compressive holography can make further improvement on the axial resolution of reconstruction via compressive holography. Furthermore, it shows that tomographic reconstruction of triangle and circular patterns can be realized from the cross line with blue color.

Fig. 9. Recorded in-line holograms of the objects: (a) Hologram of two no-overlapping patterns without 4f amplification. (b) Hologram of two no-overlapping patterns with 4f amplification. (c) Hologram of two overlapping patterns without 4f amplification. (d) Hologram of two overlapping patterns with 4f amplification.

Fig. 10. Reconstruction results of the holograms in Figs. 9(a) and 9(b): (a) Reconstruction by CH of the in-line hologram with normal system at two positions. (b) Reconstruction by BP of the in-line hologram with 4f amplification at two positions. (c) Reconstruction by CH of the in-line hologram with 4f amplification at two positions. (d) Highlighted cross line of Figs. 10(a)-10(c) at $z_1 = 428.62\, \text{mm}$. (e) Highlighted cross line of Figs. 10(a)-10(c) at $z_2 = 431.80\, \text{mm}$. 

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2) Experiment of the object with two overlapping patterns

To further verify the applicability of the method described in the paper, reconstruction of the test optic with two overlapping patterns is tested using the setup shown in Fig. 7.

The amplified and not amplified in-line holograms of the test optic are shown in Figs. 9(c) and 9(d). The reconstruction results of optical element with two overlapping patterns are shown in Figs. 11(a)-11(e). The data processing takes about one hour by compressive holography reconstruction and several seconds by back-propagating reconstruction. Figures 11(a)-11(c) show the reconstruction results by CH of the in-line hologram with normal system, BP of the in-line hologram with 4f amplification and CH of the in-line hologram with 4f amplification, at \( z_1 = 428.62\text{mm} \) and \( z_2 = 431.80\text{mm} \) respectively. Figures 11(d) and 11(e) show the corresponding highlighted cross line of Figs. 11(a)-11(c), at \( z_1 = 428.62\text{mm} \) and \( z_2 = 431.80\text{mm} \) respectively. From the reconstruction results, we can see that 4f amplified in-line compressive holography has a higher axial resolution than non-amplified in-line compressive holography and tomographic reconstruction of overlapping triangle and circular patterns can also be clearly realized from the cross line with blue color.

![Fig. 11. Reconstruction results of the holograms shown in Figs. 9(c) and 9(d): (a) Reconstruction results by CH of the in-line hologram with normal system at two positions. (b) Reconstruction results by BP of the in-line hologram with 4f amplifying system at two positions. (c) Reconstruction results by CH of the in-line hologram with 4f amplifying system at two positions. (d) Corresponding highlighted cross line of Figs. 11(a)-11(c) at \( z_1 = 428.62\text{mm} \). (e) Corresponding highlighted cross line of Figs. 11(a)-11(c) at \( z_2 = 431.80\text{mm} \).](image-url)
5. Discussion on the axial resolution for testing experiment

In in-line compressive holography recording setup without 4f system (not putting \( L_1 \) and \( L_1 \) in Fig. 4), we define \( w_{\text{obj}} \) indicates the feature size of object, \( H_{\text{obj}} \) the distance between two slice planes and \( \Delta \) is the pixel pitch on the hologram.

The numerical aperture \( NA_1 \) of this in-line hologram recording system is \( NA_1 = N_1 \Delta / 2z_1 \). According to the classical resolution equations, the transverse and axial resolution are respectively

\[
\Delta_{\text{tr}} = \frac{\lambda}{NA_1} \quad \Delta_{\text{ax}} = \frac{\lambda}{NA_1^2} \quad \text{[10]}
\]

So the theoretical axial resolution of object is

\[
\Delta_{z_{\text{in}}} = \frac{4\lambda z_1^2}{N_1^2 \Delta^2}.
\]

In 4f amplified in-line compressive holography recording setup as shown in Fig. 4, the object is presented to an amplified image. The distance between two slices’ imaging planes is

\[
H_{\text{img}} = \left( \beta_{x,y} \right)^2 H_{\text{obj}}
\]

and the feature size of images are

\[
w_{\text{img}} = \beta_{x,y} w_{\text{obj}}.
\]

The numerical aperture \( NA_2 \) of the optical system behind image plane is \( NA_2 = N_2 \Delta / 2z_1 \), so the theoretical axial resolution of 4f amplifying system is

\[
\Delta_{z_{\text{ax}}} = \frac{4\lambda z_1^2}{\left( \beta_{x,y} N_2 \Delta \right)^2}.
\]

So, we can obtain the ratio value \( M \) of axial resolutions for these two recording setups, as shown in Eq. (16):

\[
M = \frac{\Delta_{z_{\text{in}}}}{\Delta_{z_{\text{ax}}}} = \left( \frac{z_1}{\beta_{x,y} z_1} \right)^2.
\]

If the ratio value \( M < 1 \), namely \( z_i < \beta_{x,y} z_1 \), the axial resolution can be improved by 4f amplifying in-line compressive holography compared with in-line compressive holography. The condition \( z_i < \beta_{x,y} z_1 \) can be easily implemented by 4f amplifying system.

In testing experiment, the theoretical axial resolutions for in-line compressive holography and 4f amplified in-line compressive holography are respectively \( \Delta_{z_{\text{in}}} = 26.6\, \text{mm} \) and \( \Delta_{z_{\text{ax}}} = 0.013\, \text{mm} \). So for the two slice planes with distance 3.18\,mm, it can be separated by 4f amplified in-line compressive holography, not by in-line compressive holography. In fact, the actual resolution is lower than the theoretical that because of noise and effective aperture.

6. Conclusions

In this paper, compressive sensing is applied to a 4f amplified in-line hologram recording system. Although the present method needs more computational effort compared to the back-propagating reconstruction algorithm, it has some special advantages. The present method enables accurate tomographic reconstruction for dual focusing planes. Compared with in-line compressive holography, it can distinguish dual focusing planes with higher axial-resolution and the reconstruction quality of all focusing planes can be improved highly. The efficiency is same whether the test samples at the two planes are overlapping or non-overlapping. This method can also resolve tomographic reconstruction of multiple layers with sparse, only partially overlapping objects.

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