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Performance of MIMO RADAR Using Two-way MUSIC

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Abstract — Multiple Input Multiple Output (MIMO) concept have been a prevalent technology in the wireless communication system (IEEE 802.11n and 4G), as it helps to increase transmit power and data throughput by array gain and diversity gain, while reducing multipath fading. In recent years, the concept of MIMO technology has been exploited for outdoor RADAR system for direction finding of targets besides using the conventional mono-static and bi-static radar system. Direction finding subspace algorithms such as MUSIC, ESPRIT have been often used to estimate the angle of arrival (AOA) of signal path that has been transmitted from the single element transmitters.

In this paper, we explore using the MUSIC algorithm for direction finding of the target in the space for RADAR system equipped with antenna array at both transmitter and receiver. This algorithm is known as Two-way MUSIC for MIMO RADAR system. We will assess the performance of the Two-way MUSIC algorithm for MIMO RADAR such as the advantages and limitations in terms of maximum number of target elements that can be detected for given number of transmitters and receivers. We will also explore the optimum performance of the system for different practical scenarios.

1. INTRODUCTION

Due to the inefficiency of the current RADAR systems to detect multiple targets, MIMO RADAR concept has been explored by scientists and engineers using numerous methods and algorithms. Two-way MUSIC comes handy out of other algorithms because of its ability to perform well in the presence of noise and target Radar Cross Section (RCS), which is a nuisance in other methods. Brief description and a derivation for MIMO concept and Two-way MUSIC algorithm are presented in the section below.

2. MIMO RADAR CONCEPT AND TWO-WAY MUSIC

MIMO RADAR concept was introduced to the world through Multistatic RADAR systems. The integration of MIMO concept used in communications systems with Multistatic RADAR system gave rise to the MIMO RADAR system. One reason behind the success achieved by MIMO RADAR Systems is the usage of multiple antennas for both transmitting and receiving purpose [1, 2].

When MIMO concept is being approached through Two-way MUSIC system, a key factor to be taken care of is the ability to detect different received signals with respect to different transmitters. In order to achieve that, orthogonality of the transmitted signals is adapted. In other words, antennae will transmit signals with orthogonality among them [1], so that through match filters at the receiver end, it is capable of identifying which transmitter transmit particular signal. Different transmitted signals \( S \), from different transmitters are shown in Figure 1. This is a key requirement when Two-way MUSIC algorithm is implemented to the MIMO RADAR system.

In order to derive the Two-way MUSIC algorithm, it’s required to model the MIMO RADAR concept along with transmitters, receiver and targets, as given in Figure 2. For the purpose of identifying the transmitted signals emitted by different transmitters, orthogonal signals will be tagged with the transmitted steering vectors, given in Equation (1), with \( \theta_t \) as Angle Of Departures (AOD), and \( \theta_r \) the Angle Of Arrivals (AOA). \( M_t \) is the number of transmitters and \( M_r \) is the number of receivers. And \( \lambda \) is the wavelength of the transmitted signals [1, 5].

\[
T(\theta_t) = \begin{bmatrix}
1 & e^{j2\pi \sin(\theta_t) \ast dt/\lambda} & e^{j2\pi \sin(\theta_t) \ast 2dt/\lambda} & \ldots & e^{j2\pi \sin(\theta_t) \ast (M_t-1)dt/\lambda}
\end{bmatrix}^T
\]

\[
T = [T(\theta_{t1}), T(\theta_{t2}), \ldots, T(\theta_{tQ})]
\]

(1)

Same way the receiving steering vector can be defined as explained in the Equation (2).

\[
R(\theta_r) = \begin{bmatrix}
1 & e^{j2\pi \sin(\theta_r) \ast dr/\lambda} & e^{j2\pi \sin(\theta_r) \ast 2dr/\lambda} & \ldots & e^{j2\pi \sin(\theta_r) \ast (M_r-1)dr/\lambda}
\end{bmatrix}^T
\]

\[
R = [R(\theta_{r1}), R(\theta_{r2}), \ldots, R(\theta_{rQ})]
\]

(2)
Figure 1: Transmitted signals from respective transmitters.

Figure 2: MIMO RADAR concept in Mathematical descriptive way.

And the target can be modeled by its reflection coefficient or RCS $\zeta$, which is a complex random variable as shown in Equation (3). And $Q$ is the number of targets $[1,2,5]$.

$$\Sigma = \left(\frac{1}{\sqrt{2}}\right) \text{diag}(\zeta_1, \ldots, \zeta_Q)$$  \hspace{1cm} (3)

By combining transmitting, receiving steering vectors and target model, the received signal can be modeled with the addition of Additive White Gaussian Noise, $W$, which is also known as AWGN.

$$H = R \Sigma (T^T) S + W$$  \hspace{1cm} (4)

After passing through the matching filters of the receivers, the transmitted signal $S$, where $S = [s_0, s_1, \ldots, s_{M_1-1}]$, and $s_i(t) = e^{j\omega_it}$, will be cancelled off from the received signals. And the received signals can be rearranged in matrix format as in Equation (5), where $N$ is the number of samples and $C^T$ is vectorised target RCS, and $Z$ is vectorised AWGN.

$$X_v = \frac{1}{N^2} (T \odot R) C^T + Z$$  \hspace{1cm} (5)

where $\odot$ means Khatri-Rao product and $C^T = [\Sigma v_1^T, \Sigma v_2^T, \ldots, \Sigma v_{M_1}^T]$, where $M_1$ is the number of samples, the covariance matrix will be created as given in the Equation (6) [5].

$$R_{xx} = (X_v X_v^H)$$  \hspace{1cm} (6)

Then the Eigen decomposition will be performed on the covariance matrix as given by the Equation (7), in order to separate signal Eigenvalues ($v$) and noise Eigenvalues ($\sigma^2$) and $D$ will be respective Eigenvectors [5].

$$\begin{bmatrix}
  v_1 + \sigma^2 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
  0 & v_2 + \sigma^2 & \cdots & 0 & 0 & \cdots & 0 \\
  \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
  0 & 0 & \cdots & v_Q + \sigma^2 & 0 & \cdots & 0 \\
  0 & 0 & \cdots & 0 & \sigma^2 & \cdots & \vdots \\
  \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
  0 & 0 & \cdots & 0 & 0 & \cdots & \sigma^2 \\
\end{bmatrix} = D^H$$  \hspace{1cm} (7)

So at the end, the spectrum for Two-way MUSIC can be defined as the reciprocal of the product of steering vectors and noise Eigenvectors $D_N$, as given in the Equation (8) where $T \cdot R = T \odot R$ $[1,5]$. As the Eigenvalues of $D_N$ are equal or close to zero, a spike can be observed in the pseudo spectrum where $\theta_t = \text{AOA}$, and $\theta_r = \text{AOD}$. In other words, the spike points to the location of targets.

$$P_{\text{2way MUSIC}}(\theta_t; \theta_r) = \frac{1}{T \cdot R(\theta_t; \theta_r)^H D_N D_N^H T \cdot R(\theta_t; \theta_r)}$$  \hspace{1cm} (8)
3. RESULTS AND DISCUSSIONS

The simulation was carried out under different environments in order to find out the performance as well as limitation of Two-way MUSIC when implemented in MIMO RADAR. All the results were simulated with 3 transmitters and 3 receivers. And the system can perform at a reasonable transmit power of 50–100 kW [8], and can detect targets located at a distance of 6–8 km. The operating frequency, or in other words carrier frequency is 2 GHz. The 3 different environments that the simulation results presented are scattered targets, linear targets (when the magnitude of AOA and AOD are the same), and either the AOA or AOD repeats more than 2 times.

3.1. Maximum Number of Targets

The maximum number of targets can be detected depends on the Eigen structure created. In the absence of noise, for $3 \times 3$ system, it can be detected up to 8 targets or ($Mt \times Mr - 1$) as only one noise Eigenvector is sufficient to generate the pseudo spectrum, as shown in the Figure 3. But in the presence of noise and path loss factor, as shown in the Figure 4, it is required at least 2 noise Eigenvectors to generate the pseudo spectrum, so that only 7 targets or ($Mt \times Mr - 2$) can be detected.

![Figure 3](image1.png) Maximum number of targets = 8 in the absence of noise and path loss.  

![Figure 4](image2.png) Maximum number of targets = 7 in the presence of noise and path loss.  

Figure 5: Physical location of the targets.

3.2. Linear Target (Magnitude of AOA and AOD are the Same)

When the magnitude of the AOA and AOD are the same, it can be detected only 4 targets, which is ($Mr + Mr - 2$), as shown in the Figure 6. This is due to the unique behavior when combined steering vector is generated using Kronecker product. And it ultimately results in decreasing the rank of the matrix and therefore required more the 2 noise Eigenvectors for the detection of the targets and to generate the pseudo spectrum [5, 7].

![Figure 6](image3.png) Maximum number of targets when the magnitude of AOA and AOD the same.  

![Figure 7](image4.png) Physical location of the targets.

3.3. When AOA or AOD Repeats

When AOA or AOD repeats more than two times, the rank for the transmitting and or receiving steering matrix will decrease and fail to detect the all the 7 targets. Instead, only 4 peaks can be observed, as shown in the Figure 8 [5, 7]. This is due to the drop in the rank of steering vectors. In other words, it follows the Kruskal’s uniqueness [5, 7].

![Figure 8](image5.png)
4. CONCLUSION

So it can be concluded that Two-way MUSIC algorithm is a viable solution to be implemented in MIMO RADAR system, which can be used in the systems such as Military RADAR and airport surveillance RADAR systems, with some limitations to be explored further.

REFERENCES