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<td>Author(s)</td>
<td>Zhao, Liya; Tang, Lihua; Yang, Yaowen</td>
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<td>Citation</td>
<td>Zhao, L., Tang, L., &amp; Yang, Y. (2013). Comparison of modeling methods and parametric study for a piezoelectric wind energy harvester. Smart materials and structures, 22(12), 125003-.</td>
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<tr>
<td>Date</td>
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Comparison of Modeling Methods and Parametric Study for Piezoelectric Wind Energy Harvester

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Abstract
Harvesting flow energy by exploiting transverse galloping of a bluff body attached to a piezoelectric cantilever is a prospective method to power the wireless sensing systems. In order to better understand the electroaeroelastic behavior and further improve the galloping piezoelectric energy harvester (GPEH), an effective analytical model is required, which needs to incorporate both the electromechanical coupling and aerodynamic force. Available electromechanical models for GPEH include lumped parameter single-degree-of-freedom (SDOF) model, approximated distributed parameter model based on Rayleigh-Ritz discretization, and distributed parameter model with Euler-Bernoulli beam representation. Each modelling method has its own advantages. The corresponding aerodynamic models are formulated using quasi-steady hypothesis (QSH). In this paper, SDOF model, Euler-Bernoulli distributed parameter model using single mode and Euler-Bernoulli distributed parameter model using multi-modes are compared and validated with experimental results. Based on the comparison and validation, the most effective model is employed for the subsequent parametric study. The effects of load resistance, wind exposure area of the bluff body, mass of the bluff body and length of the piezoelectric sheets on the power output are investigated. These simulations can be exploited for designing and optimizing GPEHs for better performance.

1. Introduction
Wireless sensing systems have been widely used in structural health monitoring. Due to the inconvenience and high cost of replacement of batteries, development of self-powered sensor nodes by harvesting ambient energy has been researched explosively in the past few years. Ambient mechanical vibrations can be converted to electricity via electrostatic [1], electromagnetic [2], piezoelectric [3] and magnetostrictive [4] mechanisms, among which piezoelectric transduction has attracted much interest because of its high power density and non-reliance on external magnetic field or initial voltage. Many studies have been conducted on energy harvesting using piezoelectric materials [5-11]. These studies mainly focus on converting the preexisting structural vibrations into electricity. However, piezoelectric energy harvesting from small wind has received only limited attention.

When structures are subjected to wind flows, aeroelastic instabilities such as vortex-induced vibrations, galloping, flutter and buffeting may occur given specific conditions. The energy of such wind-induced vibrations can be beneficially converted into electricity via the piezoelectric transduction mechanism. Many studies have been conducted on harnessing wind energy by exploiting these aeroelastic instabilities [12-39].
Some researchers proposed the energy harvesters with flapping wings to convert aerelastic flutter vibrations into electricity [12-22]. Bryant and Garcia [15] analyzed the flutter boundary and limit cycle behavior of a piezoelectric bimorph with a rigid flap connected to the tip. A peak power of 2.2mW was obtained at 7.9m/s. Erturk et al. [16] investigated the potential of harvesting energy from aerelastic vibrations with a piezoeelastic airfoil. Linear analysis of power generation at flutter boundary was performed. Different from the use of flapping wings, several other devices with alternative structural designs were proposed to extract power from aerelastic instabilities. Li et al. [23] developed a bio-inspired piezo-leaf architecture which generated electricity by cross-flow fluttering motion, gaining a peak power of 600μW. Kwon [24] experimentally investigated the power output of a T-shaped piezoelectric cantilever, and obtained the empirical formula to calculate the cut-in wind speed. Wind energy harvesters employing vortex-induced vibrations were also frequently investigated, which were usually designed as a linear mechanical resonator with a cylinder placed windward [25-29]. Maximum power output was achieved when the frequency of vortex shedding approaches the natural frequency of the harvester. Akaydin et al. [28] tested the efficiency of such a harvester and obtained a maximum power of 0.1mW at 1.192m/s. The main constraint of such harvesters is the narrow wind speed range for effective power generation. Jung and Lee [30] studied the performance of a harvester exploiting wake galloping with two paralleled cylinders. Electromagnetic induction was employed other than the piezoelectric transduction. Recent progress on wind energy harvesting also includes a damped cantilever pipe carrying flowing fluid proposed by Elvin and Elvin [31] and a harmonica-type aerelastic micropower generator developed by Bibo et al. [32].

Translational galloping is another aerelastic instability phenomenon giving rise to transverse oscillations normal to the wind flow direction in structures with weak damping, when the wind speed exceeds a critical value. It is a better choice to obtain structural vibrations for energy harvesting purpose compared to the vortex-induced vibrations and flutter, for its advantage of large oscillation amplitude and the ability of oscillating in infinite range of wind velocities [36,45]. Translational galloping was first analyzed by Den Hartog [40], and a criterion for galloping stability was introduced. Many researchers have studied the effect of various parameters on the galloping stability of different structures, including the angle of attack, cross-section geometries, flow turbulence, Reynolds number, etc [41-45]. Barrero-Gil et al. [33] theoretically analyzed the potential use of transverse galloping to harvest energy using an SDOF model. No specific design of harvester device was proposed. Abdelkefi et al. [34] considered a GPEH with a square prism as the bluff body, and investigated the effects of Reynolds number and load resistance on the threshold of galloping and the level of harvested power. They further theoretically compared the performances of different cross-section geometries of the bluff body for GPEHs with linear and nonlinear analysis [35]. An SDOF model was employed for these two studies, with the value of electromechanical coupling manually specified. Yang et al. [36] experimentally studied the influence of cross-section geometry on the performance of GPEHs, and validated the established SDOF model with the experimental results. Sirohi and Mahadik [37] studied a GPEH consisting of two piezoelectric cantilevers connected to a prism with equilateral triangular section. They also developed another GPEH using a composite piezoelectric cantilever connected in parallel to a prism with
D-shaped section [38]. For these two harvesters, an approximated distributed parameter model based on energy method (Rayleigh-Ritz type of discretization) was used. Abdelkefi et al. [39] employed the harvester design of Sirohi and Mahadik [37] and further studied the influence of load resistance on the threshold of galloping and harvested power level with an Euler-Bernoulli distributed parameter model. Single mode (fundamental mode) was considered in their analysis. In all the analytical models mentioned above, the aerodynamic forces were formulated based on QSH. Each model method has its own merits, such as the simple form and ease of application of the SDOF model, and the accuracy of the distributed parameter models. Although many studies have been conducted on the issues of the electromechanical models for vibration piezoelectric energy harvesters under base excitation [46, 47], more work need to done on the modeling and better designing of GPEH which incorporate the aerodynamic forces.

This article compares different modeling methods for GPEHs. SDOF model established in the preceding study [36], Euler-Bernoulli distributed parameter model using single mode and multi-modes are considered followed by experimental validation. The merits, disadvantages and applicability of these methods are discussed. Subsequently, employing the most effective model from the comparison, parametric study is performed to study the effects of load resistance, wind exposure area of the bluff body, mass of the bluff body and length of the piezoelectric sheets on the electroaeroelastic behavior (cut-in wind speed and power output level) of GPEHs. The results can be exploited for designing and optimizing GPEHs for better power output performance.

2. Mechanism of galloping

Translational galloping is a self-excited phenomenon giving rise to large amplitude oscillations of bluff body when subjected to wind flows. Assume that a bluff body is elastically mounted as shown in figure 1. The governing equation of the galloping motion can be written as

$$M\ddot{w}(t) + C\dot{w}(t) + Kw(t) = F_z(t)$$

(1)

where \(w\) is the vertical position of the bluff body in \(z\) direction; \(M\) is the mass of the bluff body; \(C\) is the damping coefficient; \(K\) is the stiffness of the system and \(F_z\) is the aerodynamic force. The overdot denotes differentiation with respect to time \(t\). \(F_z\) can be expressed as [45]

$$F_z = \frac{1}{2} \rho_w S_{wp} U^2 C_{Fz}$$

(2)
where $\rho_a$ is the air density; $S_{tip}$ is the exposure area facing the flow; $U$ is the wind speed and $C_{Fz}$ is the total aerodynamic force coefficient in $z$ direction. QSH is employed here, which considers that the aerodynamic force during galloping oscillation is equal to that when the bluff body is steady with the corresponding angle of attack. This hypothesis is applicable when the oscillation is slow enough, which requires that the characteristic timescale of the flow is much smaller than the characteristic timescale of the oscillation, corresponding to the character of galloping phenomenon. The larger the wind speed, the safer to use QSH. QSH has been confirmed to be able to successfully model galloping in many studies [40,41,45]. For a specific cross-section geometry, $C_{Fz}$ is a function of the angle of attack $\alpha$ (figure 1), and can be determined through experiments [40-45]. It is common to express $C_{Fz}$ as a polynomial expansion as

$$C_{Fz} = \sum_{r=1,2,...} A_r(\alpha)^r$$

where $A_r$ are empirical coefficients for the polynomial fitting [33,45]. If the bluff body undergoes only translational oscillation without rotation, $\alpha$ can be expressed as

$$\alpha = \frac{\dot{w}}{U}$$

Substituting Eqs. (2-4) into Eq. (1), and dividing both sides by $M$ yields

$$\ddot{w}(t) + \left[2\zeta\omega_n \frac{1}{2M} \rho_a S_{tip} U \sum_{r=1,2,...} A_r(\frac{\dot{w}}{U})^{r-1}\right] \dot{w} + \omega_n^2 w = 0$$

where $\zeta$ is the damping ratio and $\omega_n$ is the natural frequency of the system. The aerodynamic force can be considered as an effective nonlinear damping as shown in Eq.(5), rendering the galloping “self-excited”. The criterion for galloping instability, which is identified by Den Hartog [40], is expressed as

$$\frac{\partial C_{Fz}}{\partial \alpha} > 0$$

The criterion requires $A_1>0$. For arbitrarily small $\dot{w}$, the system is controlled by the linear damping $2\zeta\omega_n \frac{1}{2M} \rho_a S_{tip} U A_1$, which is positive, thus the oscillations will be damped to the zero equilibrium. When $U$ increases and exceeds a certain value, the linear damping becomes negative, giving rise to galloping oscillations of the bluff body (Hopf bifurcation). When $\dot{w}$ is large enough due to the increasing oscillation amplitude, the nonlinear damping $\frac{1}{2M} \rho_a S_{tip} U \sum_{r=2,...} A_r(\frac{\dot{w}}{U})^{r-1}$ should be taken into account which makes the overall damping non-negative. Limit cycle oscillation will occur when the damping reaches zero. Due to the self-excited and self-limiting characteristics of galloping, it is a prospective energy source for energy harvesting.

3. Comparison of modelling methods for galloping piezoelectric energy harvester

A typical GPEH is usually designed as a piezoelectric cantilever attached with a bluff body at the free end, as shown in Figure 2. The bluff body, which is with a specific cross section, oscillates in the direction normal to the incoming flow due to galloping. Two piezoelectric sheets are bonded to each side of the substrate beam, generating electricity from the
mechanical strain which is developed due to the bluff body oscillation. The analytical model of a GPEH should consider both the electromechanical coupling effect and the aerodynamic force acting on the bluff body. Commonly used models for GPEHs include the SDOF model, approximate distributed parameter model with Rayleigh-Ritz type of discretization, and Euler-Bernoulli distributed parameter model with exact analytical mode shapes. The main difference between these models lies in the representation of the electromechanical coupling term. This section will compare the merits, disadvantages and applicabilities of the lumped parameter and distributed parameter models. The aerodynamic forces are all formulated based on quasi-steady hypothesis, although their representation formulas are various due to the different mechanical parameters employed in the corresponding electromechanical equations. The power storage technique is not considered in this paper, so the simple electric circuit only consists of an external resistive load $R_L$.

![Figure 2. Schematic of a typical GPEH](image)

### 3.1. SDOF model

In the preceding study, a simple SDOF model was established to simulate the electroaeroelastic behavior of a GPEH [36]. In that model, the harvester was considered to oscillate close to the fundamental frequency, which was confirmed with visual observation during the experiment. The governing equations are given as

$$
\begin{align*}
&M_{\text{eff}} \ddot{w}(L,t) + C_{\text{eff}} \dot{w}(L,t) + K_{\text{eff}} w(L,t) + \Theta V(t) = F_z(t) = \frac{1}{2} \rho_w h l_{\text{tip}} U^2 C_{\text{p}} \\
&\frac{V(t)}{R_L} + C_p \dot{V}(t) - \Theta \dot{w}(L,t) = 0
\end{align*}
$$

(7)

where $w(L,t)$ is the displacement of the bluff body in the direction normal to the wind flow; $C_{\text{eff}}$ and $K_{\text{eff}}$ are the effective damping and stiffness of the harvester; $V(t)$ is the generated voltage across $C_p$; $C_p$ is the total capacitance of two piezoelectric sheets with parallel connection; $\Theta$ is the electromechanical coupling term; $h$ and $l_{\text{tip}}$ are the frontal dimension and length of the bluff body, the product of which equals to $S_{\text{tip}}$; and $M_{\text{eff}}$ is the effective mass approximated as $M_{\text{eff}} = \frac{33}{140} M_b + M_{\text{tip}}$, where $M_b$ and $M_{\text{tip}}$ are the mass of the cantilever and the bluff body. $\Theta$ is determined easily though experiments by

$$
\Theta = \sqrt{(\omega_{\text{osc}}^2 - \omega_{\text{exc}}^2) M_{\text{eff}} C_p}
$$

(8)
where $\omega_{noc}$ and $\omega_{nsc}$ are the open circuit and short circuit resonant frequencies of the harvester. $C_{Fz}$ is expressed as in Eq. (3), and the attack angle is modified to $\alpha = (L,t)/U + w'(L,t)$, where $w'(L,t)$ is the rotation angle at the free end due to the deflection of the beam, approximated as $w'(L,t) = 3w(L,t)/2L$.

By defining a state vector $X$:

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} w(L,t) \\ \dot{w}(L,t) \\ V(t) \end{bmatrix}$$  \hspace{1cm} (9)

the governing equations can be written in the state space form as

$$\begin{bmatrix} \dot{X}_2 \\ \dot{X}_3 \\ \dot{V}(t) \end{bmatrix} = \begin{bmatrix} -2\zeta\omega_n X_2 - \omega_n^2 X_1 - \Theta M_{ef} X_3 + \frac{\rho L h l_{eq} U^2}{2M_{ef}} \sum_{r=1,2,...} A_r \left( \frac{X_2}{U} + \frac{3X_1}{2L} \right) \\ \frac{\Theta C_p}{R_L C_p} X_2 - \frac{X_1}{R_L C_p} \end{bmatrix}$$  \hspace{1cm} (10)

where $2\zeta\omega_n = C_{ef}/M_{ef}$ and $\omega_n = \omega_{nsc}$. Eq. (10) can then be numerically solved in MATLAB using the solver like ode45 to determine the vibration response of the beam, cut-in wind speed, and generated voltage across $RL$. The average power $P_{ave}$ is related with the root mean square (RMS) voltage $V_{RMS}$ as $P_{ave} = V_{RMS}^2/R_L$.

3.2. Euler-Bernoulli distributed parameter model

![Figure 3. (a) Top view of the considered GPEH and (b) cross-section of the composite beam for $x_1 < x < x_2$.](image)
Distributed parameter electromechanical models based on Euler-Bernoulli beam theory have been studied a lot for vibration piezoelectric energy harvesters with base-excitations [5,46,47]. For a GPEH, the electromechanical equations are developed based on similar assumptions: (a) The Euler-Bernoulli beam assumptions are applied to the composite beam; (b) The perfectly conductive electrodes fully cover the top and bottom surfaces of the piezoelectric sheet inducing uniform electric field through the thickness; (c) Only the z-direction vibration (transverse vibration) is considered; (d) The damping mechanisms including the internal strain rate damping and external air damping satisfy the proportional damping criterion. The schematics of the GPEH showing the coordinate directions and the cross-section of the composite beam are presented in figure 3. We consider the parallel connected situation for the two piezoelectric sheets. The coupled governing equations for the GPEH can be written as [5, 39]

\[
\begin{aligned}
&\frac{\partial^2}{\partial x^2} \left[ Yl(x) \frac{\partial^2 w(x,t)}{\partial x^2} \right] + c_s(x) \frac{\partial w(x,t)}{\partial t} + c_a(x) \frac{\partial^2 w(x,t)}{\partial t^2} + m(x) \frac{\partial^2 w(x,t)}{\partial t^2}\right] + \theta(t) \left[ \frac{d\delta(x-x_1)}{dx} - \frac{d\delta(x-x_2)}{dx} \right] = F_i \delta(x-L) \\
&\frac{V(t)}{R_c} + C_p \frac{dV(t)}{dt} - \theta(t) \frac{\partial w(x,t)}{\partial x} = 0
\end{aligned}
\]

(11)

where \(w(x,t)\) is the transverse deflection of the beam in z direction; \(\delta(x)\) is the Dirac delta function; \(c_s\) and \(c_a\) are the coefficients of strain rate damping and air damping; \(m(x)\) is the distributed mass of the beam, being expressed as \(m(x) = \rho_s h_s b_s\) for \(0 < x < x_1\) and \(x_2 < x < L\), and \(m(x) = \rho_s h_s b_s + 2\rho_p h_p b_p\) for \(x_1 < x < x_2\), where \(\rho_s, h_s\) and \(b_s\) are the mass density, thickness and width of the substrate, respectively; \(\rho_p, h_p\) and \(b_p\) are the corresponding terms of the piezoelectric sheet, and \(x_1\) and \(x_2\) are respectively the starting and ending positions of the piezoelectric sheets along the beam; \(Yl(x)\) is the bending stiffness of the composite beam expressed as \(Yl(x) = Y_s b_s h_s^3 / 12\) for \(0 < x < x_1\) and \(x_2 < x < L\), and \(Yl(x) = Y_s b_s h_s^3 / 12 + 2Y_p [(h_p + h_s) / 2]^3 h_s^3 / 8 / 3\) for \(x_1 < x < x_2\), where \(Y_s\) and \(Y_p\) are the Young’s modulus of the substrate and piezoelectric material, respectively; \(\theta\) is the piezoelectric coupling term given by \(\theta = Y_s d_{31} b_p (h_p + h_s)\), where \(d_{31}\) is the piezoelectric constant; and \(C_p\) is the total capacitance as in the SDOF model, being given by \(C_p = 2\varepsilon_{33} b_p (x_2 - x_1) / h_p (x_2 - x_1)\) equals to the length of the piezoelectric sheets \(L_p\), where \(\varepsilon_{33}\) is the permittivity at constant strain. As in the SDOF model, \(F_i\) is the aerodynamic force due to galloping.

The transverse deflection \(w(x,t)\) can be represented as

\[
\begin{align*}
w(x,t) = \sum_{r=1}^{\infty} \phi_r (x) \eta_r (t)
\end{align*}
\]

(12)

where \(\phi_r(x)\) is the mass normalized mode shape for the \(r^{th}\) mode of the corresponding undamped free vibration problem; and \(\eta_r(t)\) is the modal coordinate. Erturk and Inman [5,46,47] presented the procedure for the exact analytical solution of \(\phi_r(x)\) for the cantilever fully and uniformly covered with piezoelectric materials. As for the cantilever beam which is partially covered by piezoelectric sheets (the case for our prototype in the experiment), Abdelkefi et al. [39] developed the Euler-Bernoulli distributed parameter model by deriving the exact analytical mode shape function to analyze the electroaeroelastic behavior of a
GPEH. Yet such analytical solution for the segmented mode shapes is quite cumbersome. Single mode (fundamental mode only) was considered in their analysis. In this paper, we obtain \( r(x) \) and \( r'(x) \) easily by finite element method in Matlab. Hermite cubic functions are employed as the interpolation functions for the beam element.

\( F_z \) is still in the form as in the SDOF model (Eq. (7)), but the attack angel is here expressed in modal coordinates:

\[
\alpha = \sum_{r=1}^{\infty} \phi_r(L) \eta_r(t) + \sum_{r=1}^{\infty} \phi_r'(L) \eta_r(t)
\]

(13)

Subsequently, introducing Eqs. (12) and (13) into Eq. (11) yields the coupled governing equations in modal coordinates:

\[
\begin{cases}
\ddot{\eta}_r(t) + 2\zeta_r \omega_r \dot{\eta}_r(t) + \omega_r^2 \eta_r(t) + \chi_r V(t) = \phi_r(L) \times \frac{1}{2} \rho \lambda h l_y U^2 \sum_{i=1}^{\infty} A_i \left[ \sum_{r=1}^{\infty} \phi_r(L) \eta_r(t) \right] + \sum_{r=1}^{\infty} \phi_r'(L) \eta_r(t) \\
\frac{V(t)}{R_t} + C_r \dot{V}(t) - \sum_{r=1}^{\infty} \chi_r \dot{\eta}_r(t) = 0
\end{cases}
\]

(14)

where \( \zeta_r \) is the modal mechanical damping; \( \omega_r \) is the undamped natural frequency of the \( r \)th vibration mode; and \( \chi_r \) is the modal electromechanical coupling term written as \( \chi_r = \theta \left[ r'(x_2) - r'(x_1) \right] \). Before proceeding to solve Eq. (14), a short explanation of how to evaluate \( \zeta_r \) is presented here. As mentioned in assumption (d), the mechanical damping which includes the internal strain rate damping and external air damping is treated with the proportional damping (Rayleigh damping) assumption. For a uniform piezoelectric beam of which the flexural rigidity \( YI \) and mass per unit length \( m \) are both uniformly distributed \([5,46,47]\), \( \zeta \) can be simply expressed as \( \zeta = c_i \omega / 2YI + c_a / 2m \). Once the damping ratios for two separate vibration modes are known, the two unknown constant damping coefficients \( c_i \) and \( c_a \) can be mathematically calculated. Yet for non-uniform piezoelectric beam which is partially covered by piezoelectric sheets as in our case, \( c_i \) and \( c_a \) are no longer constant along the beam length. Whereas, we can still assume that \( c_i l(x) \) and \( c_a(x) \) are stiffness proportional and mass proportional, respectively, and obtain two constant values of \( (c_i l/YI)_{ave} \) and \( (c_a m)_{ave} \). However, to avoid the calculation of these coefficients as well as the inaccuracy caused by the proportional damping assumption, we can obtain \( \zeta \) experimentally using the logarithmic decrement method and proceed to numerical solution of Eq. (14) for the mechanical and electric responses of the GPEH.

Assume the first \( n \)th modes are considered. Again, we introduce the state vector:
The governing equations can then be written in the state space form as

\[
X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_{2r} \\ \vdots \\ X_{2n+1} \end{bmatrix} = \begin{bmatrix} \eta_1(t) \\ \eta_2(t) \\ \vdots \\ \eta_r(t) \end{bmatrix} \quad (r = 1, 2, \ldots, n)
\]

\[
X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_{2r} \\ \vdots \\ X_{2n+1} \end{bmatrix} = \begin{bmatrix} \eta_1(t) \\ \eta_2(t) \\ \vdots \\ \eta_r(t) \end{bmatrix} \quad (r = 1, 2, \ldots, n)
\]

Like the SDOF model, Eq. (16) can be solved in MATLAB using ode45. Two cases with different numbers of modes employed will be considered in the following model comparison section. Firstly, we only consider the fundamental mode by letting \( n = 1 \) in Eq. (16), since a GPEH is oscillating near its fundamental frequency [37, 38]. Secondly, in order to get a more accurate expression for the galloping aerodynamic force, we take into account the first three modes \( (n=3) \) to investigate how the higher modes influence the overall responses of a GPEH.

4. Model comparison based on experimental validation

4.1. Experimental setup

![Figure 4](image-url)  
(a) Fabricated prototype and the installation in wind tunnel. (b) Experimental setup

| Table 1. Parameters of cantilever beam and piezoelectric sheets [36] |
### Table 2. Parameters of the bluff body for the three configurations

<table>
<thead>
<tr>
<th>Properties</th>
<th>Configuration 1</th>
<th>Configuration 2</th>
<th>Configuration 3</th>
</tr>
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<tbody>
<tr>
<td>Cross-section</td>
<td>square</td>
<td>square</td>
<td>square</td>
</tr>
<tr>
<td>Section dimensions (mm²)</td>
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<td>40×40</td>
<td>40×40</td>
</tr>
<tr>
<td>Length l_{tip} (mm)</td>
<td>150</td>
<td>150</td>
<td>100</td>
</tr>
<tr>
<td>Mass M_{tip} (kg)</td>
<td>0.0268</td>
<td>0.0228</td>
<td>0.0268</td>
</tr>
</tbody>
</table>

A prototype device is fabricated and tested in the wind tunnel following the same experimental setup procedure with the preceding study [36], and the measured results are presented to validate the aforementioned SDOF and distributed parameter models. The effectiveness of the models are compared based on the agreement between their predictions and experiments. The cantilever beam is the same with the one in [36], which is composed of a aluminium substrate with two piezoelectric sheets (DuraAct P-876.A12 from Physik Instrumente) bonded to each side of the root, being connected in parallel with the total capacitance $C_p=180\text{nF}$. A metal support is employed to fix the root of the cantilever in the wind tunnel. A bluff body is attached to the free end. According to the preceding experimental study on the cross-section geometry of the bluff body [36], a square section is employed for its best performance over other geometries in the laminar flow with the lowest cut-in wind speed and the largest output power. Three configurations are tested in the wind tunnel with different bluff body properties for adequate model comparisons. The properties of the beam and piezoelectric sheets are listed in Table 1, while those of the bluff body are shown in Table 2. The prototype and the installation in the wind tunnel are shown in Figure 4(a). Prior to the wind tunnel test, some parameters like $\zeta$, $\omega_{nc}$ and $\omega_{noc}$ need to be measured under base excitations. The damping ratio $\zeta$ for the SDOF model is the same as $\zeta_i$ in the distributed parameter model. For Configuration 1, $\zeta_1=0.005$ [36], $\zeta_2=0.015$ and $\zeta_3=0.034$; for Configuration 2, $\zeta_1=0.008$, $\zeta_2=0.019$ and $\zeta_3=0.041$; for Configuration 3, $\zeta_1=0.004$, $\zeta_2=0.012$, $\zeta_3=0.031$. During the wind tunnel test, the wind speed is measured by a pitot tube anemometer, and the voltage across the load resistance is measured by the NI 9229 DAQ module. Overall experimental setup is presented in Figure 4(b).

4.2. Model comparison
The measured and predicted values of average power $P_{\text{ave}}$ versus load resistance $R_L$ are first compared in Figure 5(a). The data presented are obtained for configuration 1 at three different wind speeds. As shown in the picture, for $U=4$, $5$ and $6$ m/s, $P_{\text{ave}}$ initially increases with $R_L$ until reaches an optimum value, then decreases when $R_L$ continues growing. The optimum $R_L$ barely changes for the three wind speeds, which are all around 105~120 kΩ. All the three models can capture this trend well, yet with some discrepancies. Also, both the single mode distributed parameter model (shorted for Distri. (mode 1) here and hereafter) and multiple mode distributed parameter model (shorted for Distri. (mode 1-3) here and hereafter) predict a shallow valley around the optimum $R_L$ for $U=4$ m/s, which is an important aspect for the influence of $R_L$ on $P_{\text{ave}}$ and will be addressed in the next section. Moreover, no obvious difference is observed between the predictions of Distri. (mode 1) and Distri. (mode 1-3).

Figure 5(b) shows the measured and predicted $P_{\text{ave}}$ versus wind speed $U$ for the three different GPEH configurations at $R_L=105$ kΩ. It is noted that $P_{\text{ave}}$ increases monotonically with $U$. Again, all the three models can predict consistent results with the experiments. As for the cut-in wind speed $U_{\text{c}}$, the SDOF model predicts better results for all the three configurations, while as for the harvested power level, no obvious superiority is observed for one model over the others since small discrepancies exist for all the three models. The main source of the error is probably the unavoidable small turbulence in the wind tunnel which disturbs the bluff body to start oscillating before the wind speed reaches the real cut-in speed. In general, the
three models can successfully predict the electroaeroelastic behavior with good agreement with the experiments. The SDOF model is advantageous for its simplicity and ease of obtaining the coupling coefficient for a fabricated prototype, while the distributed models own their merits in the better representation of the aerodynamic force and ease for parametric study. In the following study about the effects of different parameters on the output power, Distri. (mode 1) is employed to obtain the simulation results.

5. Parametric study using distributed parameter model (1st mode)

In this section, a parametric study is presented in order to better understand the electroaeroelastic behavior of the GPEH. The effects of load resistance $R_L$, wind exposure area $S_{tip}$, mass of the bluff body $M_{tip}$, and length of the piezoelectric sheets $L_p$ on the cut-in wind speed as well as the output power level are investigated.

5.1. Effects of the load resistance $R_L$

Figure 6(a) shows the variation of the average output power $P_{ave}$ with the wind speed $U$ for different load resistance $R_L$. Here, Configuration 1 is considered and its properties are shown in Table 1 and Table 2. It can be seen that the growth rate of $P_{ave}$ is greatly affected by $R_L$. The growth rate of $P_{ave}$ firstly increases with $R_L$ until 105kΩ, then decreases when $R_L$ is further increasing. An enlarged view is provided to show this trend more clearly around the threshold of galloping. It is noted that the value of $R_L$ which owns the largest growth rate of $P_{ave}$ also gives the largest cut-in wind speed $U_{cr}$. The variation of $U_{cr}$ with $R_L$ is presented in Figure 6(b). As can be seen from this curve, $U_{cr}$ increases with $R_L$ up to the maximum value then decreases with $R_L$. As for the harvested power level, the largest power is obtained with $R_L$ around 105kΩ for $U>4$m/s. Features of the optimum $R_L$ can be seen from Figure 6(c) which displays the variation of $P_{ave}$ with $R_L$ at different wind speeds. For $U=2$m/s, $P_{ave}$ is zero between 20 and 800kΩ since this speed is lower than the respective $U_{cr}$ for these $R_L$ values (Figure 6(b)). When $U$ is slightly larger than $U_{cr}$ (i.e., $U=3$ and 4m/s), a shallow valley exists around 105kΩ. When $U$ continues growing (i.e., $U=5~8$m/s), there is a peak for $P_{ave}$ between 100 and 120kΩ, where lies the optimum $R_L=105kΩ$. It is worth mentioning that $R_L$ for the valley and peak of $P_{ave}$ overlap with each other, corresponding to that with the largest $U_{cr}$. Here onwards, we regard $R_L$ as the optimum one if the largest $U_{cr}$ and power growth rate are achieved in the response of $P_{ave}$ versus $U$ (i.e., the curve for $R_L=105kΩ$ in Figure 6(a)). At the optimum $R_L$, either a valley or a peak appears in the response of $P_{ave}$ versus $R_L$ at a specific $U$. 
Figure 6. (a) Variation of the average power $P_{ave}$ with the wind speed $U$ for different load resistance. (b) Variation of the average power $P_{ave}$ with the load resistance $R_L$ for different wind speeds. (c) Variation of the cut-in wind speed $U_{cr}$ with the load resistance.

5.2. Effects of the wind exposure area of the bluff body $S_{tip}$

Figure 7 shows the effects of the wind exposure area of the bluff body $S_{tip}$ ($h \times l_{tip}$) on the electroaeroelastic behavior of the GPEH. Harvester properties are the same with those listed in Table 1 and Table 2 (Configuration 1) except the length of the bluff body $l_{tip}$. Figure 7(a) is obtained with $U=6$ m/s. It is determined that the optimum $R_L$ for the various $S_{tip}$ are equal to each other, which is reasonable since the natural frequencies are the same with equal $M_{tip}$. The variation of $P_{ave}$ with $U$ at the optimum $R_L$ for different $S_{tip}$ is shown in Figure 7(b). With increasing $S_{tip}$, the cut-in wind speed decreases while the harvested power increases, both beneficial for harvesting the wind power. This can be expected since the increasing wind exposure area results in an increase in the aerodynamic force. Moreover, the growth rate of power is not much affected by $S_{tip}$. 

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5.3. Effects of the mass of the bluff body $M_{tip}$ (or fundamental frequency of the GPEH)

Figure 8 shows the effects of the mass of the bluff body $M_{tip}$ on the electroaeroelastic behavior of the GPEH. Harvester properties are the same with those list in Table 1 and Table 2 (Configuration 1) except $M_{tip}$. The most obvious change due to varying $M_{tip}$ is in the fundamental frequency of the harvester as indicated in the legend. Figure 8(a) shows the variation of $P_{ave}$ with $R_L$ for $U=6\text{m/s}$, from which the respective optimum $R_L$ can be obtained for each $M_{tip}$. It is noted that the optimum $R_L$ varies with different $M_{tip}$ (i.e., different fundamental frequency $\omega_1$). With the respective optimum $R_L$, the variation of $P_{ave}$ with $U$ is displayed in Figure 8(b). As can be seen from this figure, the cut-in wind speed increases and the harvested power decreases with the increasing $M_{tip}$ (or the decreasing of $\omega_1$). Moreover, the growth rate of the harvested power also decreases with the increasing $M_{tip}$. It can summarized that for the GPEH with a specific piezoelectric cantilever, reducing $M_{tip}$ can achieve a better performance on extracting power from the wind.

![Figure 7](image_url)

**Figure 7.** Variation of (a) the average power $P_{ave}$ with the load resistance $R_L$ at 6m/s and (b) the average power $P_{ave}$ with the wind speed $U$ at the optimum $R_L$ for different wind exposure areas of the bluff body $S_{tip}$. 

![Figure 8](image_url)

**Figure 8.** Effects of the mass of the bluff body $M_{tip}$ on the electroaeroelastic behavior of the GPEH.
Figure 8. Variation of (a) the average power $P_{ave}$ with the load resistance $R_L$ at 6m/s and (b) the average power $P_{ave}$ with the wind speed $U$ at the respective optimum $R_L$ for different masses of the bluff body $M_{tip}$.

5.4. Effects of the length of the piezoelectric sheets $L_p$

Figure 9. Variation of (a) the average power $P_{ave}$ with the load resistance $R_L$ at 10m/s, (b) the average power $P_{ave}$ with the wind speed $U$ at the respective optimum $R_L$ and (c) the power density with the wind speed $U$ for different lengths of the piezoelectric sheets $L_p$.

Figure 9 shows the effects of the lengths of piezoelectric sheets $L_p$ on the electroaeroelastic behavior of the GPEH. Harvester properties are the same with those list in Table 1 and Table 2 (Configuration 1) except $L_p$ and $C_p$ since the latter one varies linearly with the former one when other parameters are kept unchanged. The respective optimum $R_L$ is determined from figure 9(a), which varies with $L_p$ because of the change in the total stiffness of the composite beam. Figure 9(b) shows the variation of $P_{ave}$ with $U$ at the respective optimum $R_L$. The cut-in wind speed as well as the growth rate of the harvested power increases with the increasing $L_p$. The largest $L_p$ can extract the highest power at relatively high wind speeds (>12m/s). A useful factor to evaluate the performance of the GPEH is the power density which is calculated as power per piezoelectric volume in this paper. Figure 9(c) shows the power density versus the wind speed for different $L_p$. It is noted that the power density is not monotonically increasing.
with $L_p$. At high wind speeds the highest power density is obtained by a medium value of $L_p$, which is 100mm among the considered discrete values. A maximum power density of 12.96mW/cm$^3$ is achieved at 14m/s with $L_p=100mm$. A comparison of the power density with other piezoelectric wind energy harvesters is shown in Figure 10. As can be seen from this figure, the present GPEH shows good performance in power density, especially at relatively high wind speeds. The present GPEH is less advantageous at lower wind speeds when compared to the harvesters in [15] and [37]. However, the present GPEH owns the largest overall range of wind speed for effective power generation with considerable power density. Note that the data for the present GPEH in the figure is not optimized in the aspects of $S_{tip}$, $M_{tip}$, etc.

![Figure 10](image.png)

**Figure 10.** Comparison of the power density with other piezoelectric wind energy harvesters

### 7. Conclusion

In this paper, a comparison study is presented on the performance of the modelling methods for GPEH, including the SDOF model, single mode and multi-mode Euler-Bernoulli distributed parameter models. A typical GPEH consisting of a piezoelectric cantilever attached to a square-sectioned bluff body is considered. Procedures for these modelling methods are presented, in which the aerodynamic forces are all formulated based on quasi-steady hypothesis. Subsequently, wind tunnel experiments for the fabricated prototypes are conducted to validate and evaluate these models. The results show that all these models can successfully predict the variation of the average power with the load resistance and the wind speed with. Quite small difference is observed between the single-mode and multi-mode Euler-Bernoulli distributed parameter models. The distributed parameter model owns a more rational representation of the aerodynamic force, while the SDOF model gives a better prediction on the cut-in wind speed. In general, the SDOF can predict the electroaeroelastic behavior of a GPEH device accurately enough with its advantage of simplicity and ease of obtaining the electromechanical coupling term.

The influence of the load resistance, wind exposure area and mass of the bluff body, and length of the piezoelectric sheets on the cut-in wind speed as well as the output power level of
the GPEH are investigated with a single-mode Euler-Bernoulli distributed parameter model. It is determined that $U_{cr}$ and the growth rate of $P_{ave}$ increases first with $R_L$ to a certain value, then drops with $R_L$. When $U$ is large enough the largest $P_{ave}$ is obtained with the optimum $R_L$, which owns the largest $U_{cr}$ as well. With the corresponding optimum $R_L$, increasing $S_{tip}$ and decreasing $M_{tip}$ can increase $P_{ave}$ as well as reduce $U_{cr}$. For the case of $L_p$, larger $L_p$ gives larger $U_{cr}$ and larger growth rate of $P_{ave}$ yet not guarantees larger power density (power per piezoelectric volume), which is obtained at a medium $L_p$. Compared to other piezoelectric wind energy harvesters, our present GPEH gives higher power density at high wind speeds and owns the largest range of $U$ with considerable power density. Future work involves improving the performance of the GPEH at low wind speeds.

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