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# Material strength consideration in the design optimization of nonlinear energy harvester

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## Abstract

Cantilever based piezoelectric energy harvesting from ambient vibrations is a viable solution for powering wireless sensors and low power electronic devices. For realization of such technology, it is imperative to design the energy harvester with higher power output and wider operating bandwidth. The main practical constraints on the design of harvester are system mass, volume and strength of the material. In pursuit of better performance, material strength has yet been considered in designing nonlinear energy harvesters in the literature. This paper focuses on the design optimization of nonlinear energy harvester with magnetic oscillator within the limits of allowable strain on piezoelectric material. Parametric study is carried out to find the optimal configuration of nonlinear energy harvester. Experiments show that, compared to the linear configuration, the optimized nonlinear energy harvester achieves higher power output and wider bandwidth with maximum strain on piezoelectric material below the allowable limit.

## Keywords

Material strength, piezoelectric energy harvesting, optimization, nonlinear harvester, magnetic interaction

## Introduction

Over the last decade, vibration energy harvesting has emerged as a potential technology for running low-powered electronic devices and wireless sensor nodes. With the increased emphasis on wireless sensor enabled monitoring applications and advancement in low-power-consuming devices, vibration energy harvesting garnered a lot of research interest. So often, energy harvesters are designed as cantilever beams with one or two piezoceramic layers covering the beam entirely or partially. Usually, a tip mass is attached to adjust the natural frequency of the harvester. Such configuration gives the maximum power only when excited at its resonant frequency. Many attempts have been made to broaden the frequency bandwidth via different approaches (Tang et al., 2010; Twiefel and Westermann, 2013). Magnetic interaction has been extensively used for broadband energy harvesting. Originally, Challa et al. (2008) used the magnetic force to alter the stiffness of the beam for tuning the natural frequency of the device. Mechanical strain induced by magnetic force was used to adjust the natural frequency of the harvester to adaptively match the excitation frequency in the literature. However, a large portion of the harvested power was utilized for the detection of frequency change and for automatic tuning (Challa et al., 2008; Zhu et al., 2008). Recently, Aboufotouh et al. (2013) designed a self tuning resonator which autonomously adjusted the gap between the two magnets so as to match the harvester frequency to the excitation frequency. In this study, tuning of frequency from 4.7Hz to 9Hz was achieved by varying the gap between magnets from 66 to 16.7mm. However, beat phenomenon and reduction in the output were observed with slight positioning error of magnets.

1 Magnetic interaction is also exploited to induce nonlinear oscillations for widening the operating  
2 bandwidth. Mann and Sims (2009) designed and analyzed an electromagnetic energy harvester  
3 which uses nonlinear oscillations of magnetic levitation to obtain useful power output in the  
4 frequency range of 8-13Hz when excited at  $4\text{m/s}^2$ . Stanton et al. (2009) modeled and  
5 experimentally validated a nonlinear energy harvester with bi-directional tunability. Magnetic  
6 interaction around the tip mass has been tuned to obtain both hardening and softening responses  
7 and thus achieving wider bandwidth in both directions. Erturk et al. (2009) reported 200%  
8 increase in open-circuit voltage amplitude using a bistable piezomagnetoelastic structure at a  
9 rather high base acceleration of  $0.5g$ . Very recently, Zhou et al. (2013) changed the orientation  
10 of magnets to improve the bandwidth of a nonlinear energy harvester. A frequency range of 4-  
11 22Hz with large voltage amplitude was obtained by changing the orientation of magnets from  $0$  to  
12  $90^\circ$ . Tang and Yang (2012) proposed a nonlinear energy harvester with magnetic oscillator to  
13 improve both power output and operational bandwidth. Under a harmonic excitation of magnitude  
14  $2\text{m/s}^2$ , 100% increase in the operational bandwidth and 41% increase in the power output have  
15 been reported. Other efforts related to broadband energy harvesting can be found in the  
16 comprehensive review articles by Pellegrini et al. (2012) and Harne and Wang (2013).

17  
18 Structural integrity and durability over long time are important considerations in designing energy  
19 harvester for real life applications. Inherent brittleness and fatigue due to the cyclic  
20 electromechanical loading on PZT material might cause abrupt failure of the device if material  
21 strength is not considered in the design process. In the aforementioned efforts on the design and  
22 development of nonlinear energy harvesters, material strength has not been considered in the  
23 process of achieving higher power output and wider bandwidth. Acceleration magnitudes  
24 considered in some of the works are really high which can cause premature failure of the  
25 harvester if strength constraint is not imposed in the design process (Erturk et al., 2009; Zhou et  
26 al., 2013). On the other hand, an unchecked design may result in a situation where maximum  
27 stress or strain is well below the allowable limit leading to poor utilization of the material.  
28 Wickenheiser (2011) attempted to design an energy harvester with fundamental frequency of  
29 1.95Hz using vibrations from human walking. He arrived at a beam length of 570mm to meet the  
30 maximum strain requirements of  $< 0.1\%$  and concluded that designing a compact resonant  
31 harvester from the vibrations of human walking is not possible. Shafer and Garcia (2013) derived  
32 an expression for the maximum tolerable input acceleration that a linear energy harvester can  
33 sustain based on the ultimate strength of the material and used this expression to obtain the  
34 maximum harvested power corresponding to that acceleration. Though this study provided the  
35 margin of safety of energy harvester for a given acceleration input, a more pragmatic approach  
36 would be optimizing the power output of an energy harvester for a given vibration input until the  
37 stress or strain in the material reaches its allowable limit.

38  
39 Strength of some commonly used piezoelectric materials is given in Table 1. Fatigue strength  
40 values given in Table 1 are the peak values of stress allowed on the material when subjected to  
41 alternate fields or stresses. It is clearly evident from the table that the strength of piezoelectric  
42 materials rapidly decreases in case of cyclic loading and the values are much lower compared to  
43 the strength of substrate materials such as steel, aluminum, brass etc. This fact accentuates the  
44 importance of material strength of piezoelectric material in designing the energy harvester. Many  
45 researchers investigated the fatigue behavior of piezoelectric materials. Failure of piezoelectric  
46 material depends on its chemical composition, degree of brittleness (hard or soft), poling direction  
47 and also on the material used for electrodes (Schäufele and Heinz, 1996). Fett et al. (1999)  
48 investigated the behavior of soft PZT material under tensile, compression and 4-point bending  
49 tests. Narita et al. (2007) studied experimentally the influence of electric fields, poling direction  
50 and electrode material on the cyclic fatigue behavior of PZT material. Anton et al. (2010) carried  
51 out 3-point bending tests on monolithic piezoceramics PZT-5A and PZT-5H, single crystal

piezoelectric PMN-PZT and commercially packaged QuickPack devices and reported the estimates of their strengths. The results of this study can be used as the basis for the design of piezoelectric energy harvesters.

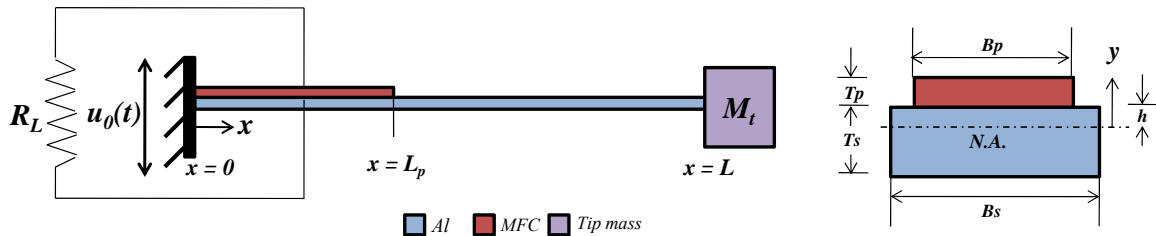
**Table 1. Material strength of commonly used piezoelectric materials.**

Material	Manufacturer	Product specification	Material strength	
PZT Bimorph	Morgan Advanced Materials	PZT 5A series	75.8MPa (static)	27.6MPa <sup>(1)</sup> (fatigue)
PZT patch	Channel Industries, Inc.	PZT 5804 Navy II	82.7MPa (static)	48.3MPa <sup>(2)</sup> (fatigue)
PZT wafer	MIDE Vulture	V22BL	800ppm <sup>(3)</sup>	
MFC patch	Smart materials	M2807-P2	1000ppm <sup>(4)</sup>	
PMN-PT patch	TRS electrostrictors	PMN-15	85MPa <sup>(5)</sup>	

In this work, an optimal configuration of nonlinear piezoelectric energy harvester with magnetic oscillator is designed considering material strength limit. Firstly, an electromechanical model of the energy harvester is introduced with better formulation for estimating magnetic interaction in place of the commonly used dipole-dipole formulation in the literature. The maximum strain on the cantilever beam is evaluated assuming that material strains are within their linear elastic limits. Parametric study is conducted by varying the beam length and the distance between magnets to achieve higher power and bandwidth and an optimal configuration is chosen. Experiments are then carried out to validate the results from theoretical modeling.

### Linear Energy Harvester

A typical piezoelectric energy harvester (PEH) is designed as a cantilever beam with a patch of piezoelectric material bonded at the fixed end and a proof mass at the free end (Figure 1). In this study, aluminum (T6) is used as substrate material and a piece of piezoelectric macro fiber composite (MFC) (Smart Materials Corp., model: M2807-P2) is bonded to the beam at its root. The portion of the aluminum beam with the patch of MFC will be referred to as composite section throughout this paper. The energy harvesting circuit with a simple resistive load  $R_L$  is considered as the main focus is on the structural aspects of the energy harvester. The governing electromechanical equations for the PEH subjected to base motion  $u_0(t)$  are derived considering single-mode approximation and using the Hamiltonian principle following similar procedure described by Hagwood et al. (1990), Sodano et al. (2004) and Elvin and Elvin (2009).



**Figure 1.** Schematic representation of a cantilever linear PEH.

$$1 \quad M \ddot{u}(t) + C \dot{u}(t) + K u(t) + \Theta V(t) = F(t) \quad (1)$$

$$2 \quad \frac{V(t)}{R_L} + C^s \dot{V}(t) - \Theta \dot{u}(t) = 0 \quad (2)$$

3 In equations (1) and (2),  $V(t)$  is the voltage across the resistor  $R_L$ ;  $u(t)$  is the modal displacement  
 4 in single mode approximation, where displacement  $w(x,t)=u(t)\phi(x)$ ;  $M$ ,  $K$ ,  $C$  and  $\Theta$  are the modal  
 5 mass, stiffness, damping and electromechanical coupling terms, respectively;  $C^s$  is the clamped  
 6 capacitance of the piezoelectric patch; and  $F(t)$  is the mechanical forcing due to the base  
 7 acceleration. The damping coefficient  $C$  is usually measured experimentally. The definitions of  
 8 other terms are given below. Throughout this paper subscripts  $s$ ,  $p$  and  $c$  corresponds to substrate,  
 9 piezoelectric material and composite sections, respectively.

$$10 \quad M = \int_0^L m_*(x) \phi(x)^2 dx + M_t \phi(L)^2 \quad (3)$$

$$11 \quad K = \int_0^L EI_*(x) \phi''(x)^2 dx \quad (4)$$

$$12 \quad \Theta = - \int_{v_p} y \phi''(x) e_{31} \psi(y) dv_p \quad (5)$$

$$13 \quad F(t) = - \left( M_t \phi(L) + \int_0^L m_*(x) \phi(x) dx \right) \ddot{u}_0(t) \quad (6)$$

14 Here,  $m$  and  $EI$  are mass per unit length and flexural rigidity of the beam. The '\*' corresponds to  
 15 the specific portion of the beam, either composite section or substrate.  $M_t$  is the tip mass.  $e_{31}$  is  
 16 piezoelectric stress constant. The spatial variable  $x$  is measured from the root and  $y$  is measured  
 17 from the neutral axis of the beam, as shown in Figure 1.  $L$  is the length of the beam. The function  
 18  $\psi(y)$  defines the electric field over the piezoelectric element which is assumed to be constant  
 19 (Sodano et al., 2004). The integral in the equation (5) is evaluated over the volume of the  
 20 piezoelectric material ( $v_p$ ) in the composite section.  $\phi(x)$  is the mode shape normalized to unity  
 21 corresponding to the first natural frequency of the beam and expressed in two parts as the  
 22 cantilever beam is covered partially by the piezoelectric material.

$$23 \quad \phi(x) = \begin{cases} \phi_c(x) & \text{for } 0 \leq x \leq L_p \\ \phi_s(x) & \text{for } L_p \leq x \leq L \end{cases} \quad (7)$$

24 Differential eigenvalue problem for this configuration is

$$25 \quad \begin{cases} (EI)_c \phi_c^{iv} - m_c \omega^2 \phi_c = 0 \\ (EI)_s \phi_s^{iv} - m_s \omega^2 \phi_s = 0 \end{cases} \quad (8)$$

26 The solution form for the mode shape is

$$27 \quad \begin{cases} \phi_c(x) = A_1 \cos(\beta_1 x) + B_1 \sin(\beta_1 x) + C_1 \cosh(\beta_1 x) + D_1 \sinh(\beta_1 x) \\ \phi_s(x) = A_2 \cos(\beta_2 x) + B_2 \sin(\beta_2 x) + C_2 \cosh(\beta_2 x) + D_2 \sinh(\beta_2 x) \end{cases} \quad (9)$$

28 with the following geometric and natural boundary conditions to determine the values of  $\beta_1$  and  
 29  $\beta_2$  which give non- trivial  $\phi(x)$ :

$$30 \quad \begin{cases} \phi_c(0) = 0, \phi_c'(0) = 0, \phi_c(L_p) = \phi_s(L_p), \phi_c'(L_p) = \phi_s'(L_p) \\ (EI)_c \phi_c''(L_p) = (EI)_s \phi_s''(L_p), (EI)_c \phi_c'''(L_p) = (EI)_s \phi_s'''(L_p) \\ (EI)_s \phi_s''(L) = I_t \omega^2 \phi_s'(L), (EI)_s \phi_s'''(L) + \omega^2 M_t \phi_s(L) = 0 \end{cases} \quad (10)$$

1 where  $I_t$  is the rotational inertia of tip mass  $M_t$ .

## 2 Nonlinear Energy Harvester

3 Tang and Yang (2012) proposed a nonlinear energy harvester in which the conventional PEH was  
 4 aided by magnetic oscillator to achieve both wider bandwidth and higher power output. As shown  
 5 in Figure 2, a subsystem of magnetic mass-spring-damper was introduced in the enclosure  
 6 carrying a PEH with a magnet at its tip. The dynamics of PEH is altered by the presence of the  
 7 magnetic oscillator, resulting in improvement of bandwidth and power. For theoretical study, the  
 8 magnetic oscillator is assumed as an additional cantilever with a tip mass carrying a magnet. The  
 9 distance  $d$  between the two embedded magnets strongly influences the nonlinear characteristics of  
 10 PEH. The electromechanical equations of this nonlinear harvester are given below:

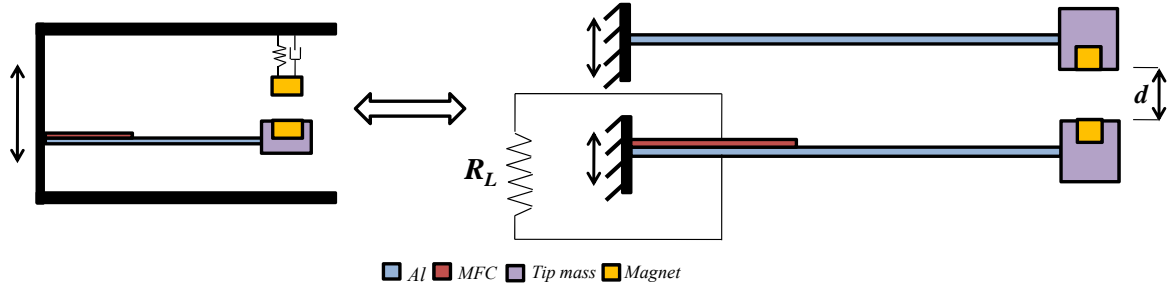
$$11 \quad M_1 \ddot{u}_1(t) + C_1 \dot{u}_1(t) + K_1 u_1(t) + \theta V(t) - F_{mag} = F_1(t) \quad (11)$$

$$12 \quad M_2 \ddot{u}_2(t) + C_2 \dot{u}_2(t) + K_2 u_2(t) + F_{mag} = F_2(t) \quad (12)$$

$$13 \quad \frac{V(t)}{R_L} + C^s \dot{V}(t) - \theta \dot{u}_1(t) = 0 \quad (13)$$

14 where subscripts 1 and 2 refer to the parameters of PEH and magnetic oscillator, respectively.  
 15 The effective modal terms for each beam are determined as explained in the previous section.  
 16  $F_{mag}$  is the magnetic force which varies with the relative displacement between the PEH and the  
 17 oscillator. In their study, Tang and Yang (2012) approximated the magnetic force as interaction  
 18 between dipoles. In this article, a better formulation is used to account for the magnetic force  
 19 which predicts the response of PEH more accurately.

20



21

22 **Figure 2.** Schematic representation of nonlinear piezoelectric energy harvester with magnetic  
 23 oscillator.

## 24 Magnetic Force

25 For analyzing nonlinear energy harvesters with magnetic interaction, the dipole-dipole  
 26 approximation has been used extensively in the literature (Challa et al., 2008; Stanton et al., 2009;  
 27 Tang et al., 2012; Stanton et al., 2010). The force between the magnetic dipoles is given by:

$$28 \quad F_{dipole} = \frac{3 \mu_0 m_1 m_2}{2\pi d^4} \quad (14)$$

29 where  $m_1$  and  $m_2$  are the moments of magnetic dipoles;  $\mu_0$  is the vacuum permeability. The dipole  
 30 model is accurate only when the distance between magnets is sufficiently large. As shown in the  
 31 later part of this section, the dipole model gives quite erroneous results within the distances  
 32 normally used in energy harvester designs. Experiments have been carried out and results have

1 been curve fitted to establish accurate relation between the magnetic force and the separation  
2 distance (Mann and Sims, 2009; Alphenaar et al., 2010; Aboulfotoh et al., 2013; Zhou et al.,  
3 2013). The curve fitted expressions were used in governing equations to study the behavior of the  
4 harvester. Though this method evaluates the magnetic interaction accurately, conducting  
5 experiments is time consuming. Experiments have to be repeated if magnets of different  
6 dimensions and strength are used. Thus, it is highly desired to have a good formulation to  
7 evaluate the magnetic force. In this section, a closed-form expression for computing the force  
8 between cylindrical magnets available in the literature is presented and compared with the  
9 extensively used dipole model. Additionally, other available solutions for evaluating the  
10 interaction between permanent magnets are discussed in the view of growing interest in the usage  
11 of magnetic interaction for broadband energy harvesting.

12 The closed-form expression of the interaction force between two identical and axially aligned  
13 magnetic disks was developed to calculate levitation force for radial and axial bearing  
14 configurations (Furlani, 2001). There are various techniques to analyze permanent magnets such  
15 as representing magnets by equivalent magnetic charges or currents, application of the Maxwell's  
16 stress tensors, the principle of virtual work and the use of Kelvin's formula (Agashe and Arnold,  
17 2008). In his study, Furlani (2001) modeled the magnets as volumes of distributed magnetic  
18 charges and the force between the two charge distributions was computed. In this model, the  
19 magnets are assumed to be of fixed and uniform polarization. Integral expressions in the model  
20 to solve for the magnetic field and force are evaluated by discretizing the magnets into mesh of  
21 elements and contribution from each element is summed up. Due to the geometric symmetry of  
22 the magnets, equations are derived in cylindrical coordinates. The force between two identical  
23 and axially aligned magnetic disks is given as:

$$24 \quad F_{mag} = \frac{\mu_0}{4\pi} J^2 \left\{ \frac{\pi(R_2^2 - R_1^2)}{N_r N_\theta} \right\}^2 N_\theta \sum_{q=0}^2 \sum_{i=1}^{N_r} \sum_{i'=1}^{N_r} \sum_{j'=1}^{N_\theta} \frac{(-1)^q \alpha_q d_q}{[r_i^2 + r_{i'}^2 - 2r_i r_{i'} \cos(\theta_{j'}) + h_q^2]^{3/2}} \quad (15)$$

25 where  $J$  is the saturation magnetization;  $R_1$  and  $R_2$  are the inner and outer radius of magnet,  
26 respectively;  $N_r$  and  $N_\theta$  are the number of elements in the radial and circumferential directions,  
27 respectively. Other parameters are defined as follows:

$$28 \quad \theta_{j'} = (j' - 1)\Delta\theta + \frac{\Delta\theta}{2} \quad (j' = 1, 2, \dots, N_\theta),$$

$$\Delta\theta = \frac{2\pi}{N_\theta},$$

$$r_i = \frac{R(i+1) + R(i)}{2} \quad (i = 1, 2, \dots, N_r),$$

$$r_{i'} = \frac{R(i'+1) + R(i')}{2} \quad (i' = 1, 2, \dots, N_r),$$

$$R(i+1) = \sqrt{R(i)^2 + \frac{R_2^2 - R_1^2}{N_r}} \quad (i = 1, 2, \dots, N_r),$$

$$R(i'+1) = \sqrt{R(i')^2 + \frac{R_2^2 - R_1^2}{N_r}} \quad (i' = 1, 2, \dots, N_r),$$

1

$$d_q = d + q t_q \quad (q=0,1,2),$$

$$\alpha_q = \begin{cases} 1 & q = 0, \\ 2 & q = 1, \\ 1 & q = 2. \end{cases}$$

2

where,  $t_q$  is the thickness of magnet. Since the above solution is obtained through discretization, its accuracy depends on the number of elements used. It is found out that with mesh resolution of  $N_r=25$  and  $N_\theta=25$ , the discretization error in the evaluation of magnetic force is of the order 0.001N. Higher mesh resolution unnecessarily increases the computational time as the magnetic force is repeatedly calculated in solving the governing equations numerically.

7

To evaluate the accuracy of different models, the magnetic forces calculated using the above closed-form solution and the dipole model are compared with the results from finite element analysis (FEA), which is carried out with magnetostatic module in Comsol Multiphysics using 3D models of magnets. Cylindrical Neodymium (NdFeB) magnets of radius 2mm, thickness 3mm and magnetization 828KA/m are used for simulation. The effective magnetic moment computed assuming magnets as dipoles is  $0.0192\text{Am}^2$ . Force values computed using equations (14), (15) and FEA for a series of separation distances are compared in Figure 3.

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From Figure 3, it is very clear that the closed-form solution given by the charge model matches well with the FEA results whereas the dipole model predicts much higher values when the separation distance becomes smaller. Force predicted by dipole model starts deviating from results of FEA and closed-form solution considerably when the spacing between magnets is 7mm. At 5mm of separation distance between magnets, error from the dipole model is almost 200%. Note that, the separation distance  $d$  shown in Figure 2 is the distance between magnets at rest. During the motion of beams under base acceleration, tip masses along with embedded magnets oscillate and interact at even closer distances. Higher base acceleration results in closer spacing of magnets around 5mm or less and the dipole model in such case would result in erroneous results. Hence it is recommended that the dipole model should not be used when the distance between magnets is close during the motion (less than 7mm) so as to avoid inaccurate prediction of response of the energy harvester. Moreover, the force expression (15) derived in terms of magnet material properties, dimensions and spacing of magnets is convenient to use, which facilitates rapid parametric studies.

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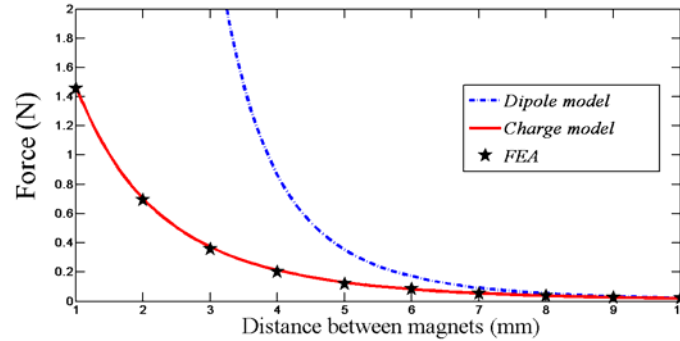
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Cylindrical and cuboidal magnets in various configurations have been used for broadband energy harvesting. There are other analytical solutions in the literature for computing the force between permanent magnets. Agashe and Arnold (2008) derived analytical expressions for axial and lateral forces between cylindrical and cuboidal magnets as functions of separation distance and dimensions of magnets. The closed form expressions were derived using the Kelvin's formula and the equivalent currents approach. The analytical expressions involving hypergeometric expansion used for evaluating the elliptic integrals are complex and computationally more expensive than the closed form expression given by Furlani (2001). Ravaud et al. (2010) has given analytical expressions for magnetic parameters such as field, force, torque and stiffness in cylindrical magnets and coil. The expressions are based on the elliptic integrals without any simplification and the integrals need to be solved in Matlab or Mathematica. These expressions are more accurate than the other closed form expressions discussed above as there is no simplifications used for evaluating the elliptical integrals. However, the repeated evaluation of elliptical integrals while solving the dynamic equations (11) to (13) is highly computationally expensive. Furthermore, the closed form expression given by Furlani (2001) gives very good accuracy for the separation distances considered in this work. Robertson et al. (2011) presented the original equation proposed by Ravaud et al. (2010) in a more compact form using fewer



1 parameters but it still requires the evaluation of elliptical integrals. Therefore, the closed form  
 2 expression given by Furlani (2001) is used in this work as it is computationally less expensive  
 3 and easy to implement.



4  
 5 **Figure 3.** Force between cylindrical magnets versus separation distance.

## 6 **Power Output of Linear and Nonlinear PEH**

7 Using the closed-form expression for magnetic force presented in the last section and equations  
 8 (11) to (13), dynamics of nonlinear energy harvester is studied in this section. The governing  
 9 differential equations are written in the state space form and solved by numerical integration in  
 10 MATLAB. The state space form of the governing equations is given in Appendix.. Attractive  
 11 configuration of magnets is considered for studying the behavior of the harvester. Natural  
 12 frequencies of 27.8 Hz and 24.4 Hz are considered for PEH and magnetic oscillator, respectively,  
 13 similar to the original design proposed by Tang and Yang (2012). Harmonic base acceleration  
 14  $a_{rms}$  of magnitude  $2m/s^2$  is used in this work. In fact, other frequency range and different base  
 15 acceleration can be considered depending upon the input vibration characteristics for which the  
 16 harvester has to be designed. The geometric parameters of the beams considered for simulation  
 17 are also of the original design and are given in Table 2. The performance of harvester is evaluated  
 18 for two different separation distances between magnets i.e.  $d=10$  mm and 8 mm. Additionally, the  
 19 maximum strain on the piezoelectric material will be evaluated for the considered configurations  
 20 in the subsequent section. The results will prompt for a parametric study to find the nonlinear  
 21 harvester configuration that meets the material strength criteria and also has better performance.

22 The tip masses weighing 4.6g and 6g are used to achieve natural frequencies of 27.8Hz and  
 23 24.4Hz for the PEH and magnetic oscillator, respectively. While deriving the modal terms, the tip  
 24 mass is assumed as point mass with  $I_t=0$ . It is assumed that rotations of tip masses are small  
 25 enough such that the magnets are axially aligned. A load resistance of 400k $\Omega$  is considered in the  
 26 simulation. Nonlinear behavior of the harvester is studied when magnets are separated by 8mm  
 27 and 10mm. The separation distance (8mm or 10mm) refers to the distance between magnets when  
 28 the beams are stationary. Smaller distances are not considered because of snapping action of  
 29 magnets. Usually, magnets snap each other (sudden movement) at a certain critical distance. In  
 30 this design, the attraction force between magnets is resisted by the beams ( $K*\delta$ ). Here,  $K$  is the  
 31 stiffness of the beam and  $\delta$  is the tip deflection. When the two tips are close enough (during  
 32 motion) so that magnetic force is more than the resistance from beams, the two magnets snap  
 33 each other. This makes the energy harvester useless and large force is required to separate  
 34 magnets. Therefore, it is very crucial to design harvesters such that the relative spacing during  
 35 motion is sufficiently large enough to avoid snapping action of magnets.

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**Table 2. Parameters of nonlinear energy harvester with magnetic oscillator.**

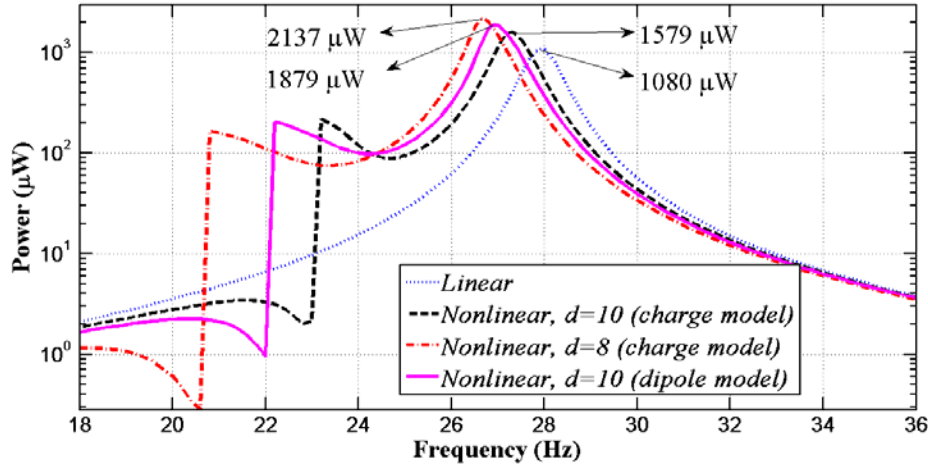
Parameter	Piezoceramic	Substrate
Material	MFC 2807-P2	Aluminum
Length ( $L_p, L$ ) (mm)	28	70
Width ( $B_p, B_s$ ) (mm)	7	10
Thickness ( $T_p, T_s$ ) (mm)	0.2	0.6
Elastic modulus ( $E_p, E_s$ ) (GPa)	30.336	68.9
Mass density ( $\rho_p, \rho_s$ ) ( $\text{kg/m}^3$ )	5440	2700
Piezoelectric constant ( $e_{31}$ ) ( $\text{C/m}^2$ )	-5.16	---
Capacitance ( $C^S$ ) (nF)	12.4	---

Figure 4 shows the power output from the linear and the nonlinear PEH. Power delivered to the electrical load is given by  $V_{rms}^2/R_{L_s}$  where  $V_{rms}$  is the RMS voltage output obtained from solving the governing equations (11) to (13). It is conspicuous that PEH with magnetic oscillator provides both wider bandwidth and higher output power when compared to linear PEH. The main features of nonlinear harvester are reflected in the increased power output at the resonant frequency and the wider bandwidth because of the presence of another peak near the resonance frequency of magnetic oscillator. The maximum power output from the nonlinear PEH is  $1579\mu\text{W}$ , showing 46% increase when compared to the maximum power output from the linear PEH ( $1080\mu\text{W}$ ). The bandwidth corresponding to  $100\mu\text{W}$  power output has also shown 30% increase from the linear PEH (3Hz) to the nonlinear PEH (3.9Hz). The reason for enhancement in performance is due to the dynamics of added magnetic oscillator to the system. When the frequency is swept near resonant frequencies of beams (27.8Hz and 24.4Hz), energy from one beam (oscillator) is transferred to the other (harvester) through magnetic interaction, resulting in better performance. Power and bandwidth are further increased through intensification of magnetic nonlinearity by reducing the spacing between magnets from 10 to 8mm. For  $d=8\text{mm}$  nonlinear configuration, the maximum power output of  $2137\mu\text{W}$  and the bandwidth of 4.3Hz have been achieved, showing 98% and 43% increase, respectively, as compared to the linear configuration.

The presence of nonlinear magnetic force makes harvester a duffing model. But, the nonlinear behavior of a duffing model becomes apparent only when the harvester experiences significant amplitude. The base acceleration considered in this work is  $2\text{ m/s}^2$  (RMS) which is not sufficient to induce higher amplitudes. From Figure 4, it is evident that the bending of response curves of the harvester which is hall mark feature of the duffing oscillator is not observed. However, the magnetic force influences linear stiffness and results in shift of the resonant frequency. The attractive magnetic force has a softening effect on the resonant frequency of the harvester. The peak of power output slightly moves towards the left of the resonant frequency of linear PEH as the attraction force between magnets reduces the stiffness of the beam. The shift in the resonant frequency increases with closer spacing of the magnets as expected. The resonant frequencies of nonlinear harvester shift by 0.6 Hz and 1.2 Hz to the left of linear harvester for  $d=10$  and  $d=8$  mm, respectively. The second peak of power output corresponding to the resonant frequency of the oscillator also shifts to the left. It should be mentioned that the higher power output and wider bandwidth are achieved not because of the nonlinear stiffness induced by the magnetic force, but the transfer of energy from the oscillator to the harvester near resonant frequencies.

The power output using the dipole formulation is also plotted for the purpose of comparison. For the case of  $d=10$  mm, power output value computed using dipole model ( $1879\mu\text{W}$ ) is much higher compared to the value computed using charge model ( $1579\mu\text{W}$ ). Power output of the case

1  $d=8\text{mm}$  using dipole model is not plotted as the model predicts higher magnetic force, resulting in  
 2 higher tip deflections. Near the resonant frequency of PEH or oscillator, the initial spacing of  
 3 magnets (8mm) is closed by relative displacement of magnets ( $u_1-u_2$ ) and resulted in sticking of  
 4 the magnets. For the same configuration, charge model predicts maximum relative displacement  
 5 as 3.36mm and maximum power output of  $2137\mu\text{W}$ . The large error in prediction of the power  
 6 and displacement concludes that the dipole model cannot be used when magnets are interacting at  
 7 closer distances.



8

9 **Figure 4.** Power output from the linear and the nonlinear energy harvesters at  $a_{rms}=2\text{m/s}^2$  from  
 10 simulation.

## 11 Maximum strain in PEH

12 In this section, we derive the maximum strain on the cantilevered PEH for material strength  
 13 consideration in design. The prediction of power output would be meaningless if the maximum  
 14 strain in the PEH exceeds the allowable limit. Generally piezoceramics have lower strength than  
 15 the substrate materials and are more susceptible to failure due to their brittleness. Hence we  
 16 analyze the maximum strain in the piezoelectric material. Assuming that the mechanical  
 17 perturbations are small and the material is under small electric field conditions, the constitutive  
 18 relations for the piezoelectric material are (IEEE Standard, 1987):

$$19 \quad \begin{bmatrix} \epsilon \\ D \end{bmatrix} = \begin{bmatrix} s^E & d^c \\ d^d & \epsilon^\sigma \end{bmatrix} \begin{bmatrix} \sigma \\ E^f \end{bmatrix} \quad (16)$$

20 In the above equation,  $\epsilon$  is the strain which depends on both stress ( $\sigma$ ) and electric field ( $E^f$ ).  
 21 Strain and electric displacement ( $D$ ) are related to stress and electric field through elastic  
 22 compliance ( $s^E$ ), dielectric permittivity ( $\epsilon^\sigma$ ) and piezoelectric constants ( $d^c$ ,  $d^d$ ). The superscripts  
 23 on elastic compliance and dielectric permittivity denote that these constant are evaluated at  
 24 constant electric field and constant stress respectively. The superscripts on piezoelectric constants  
 25 refer to direct and converse effect of piezoelectric materials but the constants are numerically the  
 26 same.

27 In deriving the maximum strain on the cantilever beam, we assume that the aluminum substrate  
 28 and MFC patch are perfectly bonded together and the effect of epoxy layer between them is  
 29 neglected. The axial strain in the cantilever beam can be written from the constitutive equations  
 30 (16) as

1 
$$\epsilon_x = s^E \sigma_x + d_{31} E^f \quad (17)$$

2 The first term on the right-hand side in the above equation refers to the strain contribution from  
 3 the deflection of the beam. The compliance is equal to the inverse of elastic modulus ( $s^E=1/E_x$ )  
 4 and the bending stress is given by the flexure formula ( $\sigma_x=-M_z y/I$ ). Sign convention of bending  
 5 moment ( $M_z$ ) is that the positive moment produces positive curvature. The bending moment can  
 6 be approximated as  $K\delta(L-x)$ . The tip deflection of the beam can be obtained as  $u(t)\phi(L)$ . With  
 7 the above relations, the first term in equation (17) can be written as

8 
$$s^E \sigma_x = \frac{-Ku(t)\phi(L)y(L-x)}{(EI)_x} \quad (18)$$

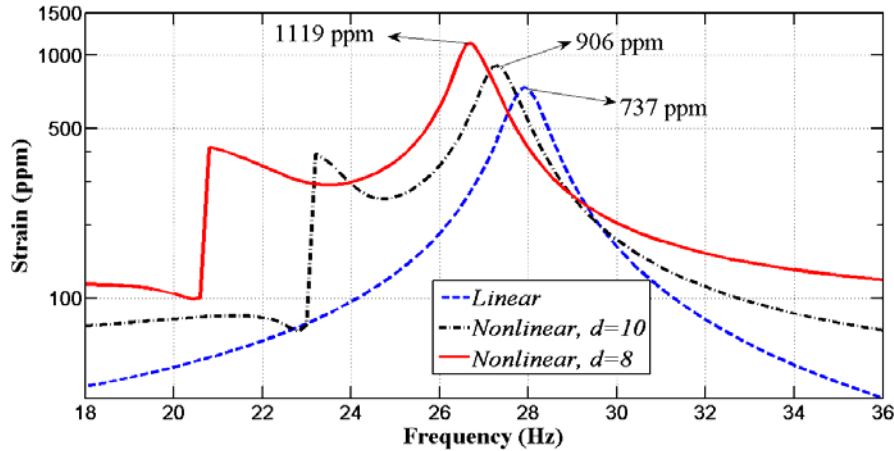
9 The second term in equation (17) is the strain caused by the electric field across the piezoelectric  
 10 element.  $d_{31}$  is the piezoelectric constant for 31 mode, given as  $d_{31}=e_{31}/E_p$ , where  $E_p$  is the elastic  
 11 modulus of piezoelectric material in the axial direction. The electric field across the piezoelectric  
 12 element is assumed to be uniform, given as  $E^f=-V(t)/T_p$  (Sodano et al., 2004). The maximum  
 13 strain occurs on the farthest layer from neutral axis at the root of beam, given by:

14 
$$\epsilon_{max} = \frac{-Ku_{max}\phi(L)(h+T_p)L}{(EI)_c} - \frac{e_{31}V}{E_p T_p} \quad (19)$$

15 where

16 
$$h = \frac{T_s}{2} \left[ \frac{1 - \frac{E_p B_p}{E_s B_s} \left(\frac{T_p}{T_s}\right)^2}{1 + \frac{E_p B_p T_p}{E_s B_s T_s}} \right]$$

$$(EI)_c = \frac{1}{3} E_s B_s (T_s - h)^3 + \frac{1}{3} E_s B_s h^3 + \frac{1}{3} E_p B_p (T_p + h)^3 - \frac{1}{3} E_p B_p h^3$$



17  
 18 **Figure 5.** Maximum strain in piezoelectric material of linear and nonlinear harvesters excited at  
 19  $a_{rms}=2m/s^2$  from simulation.

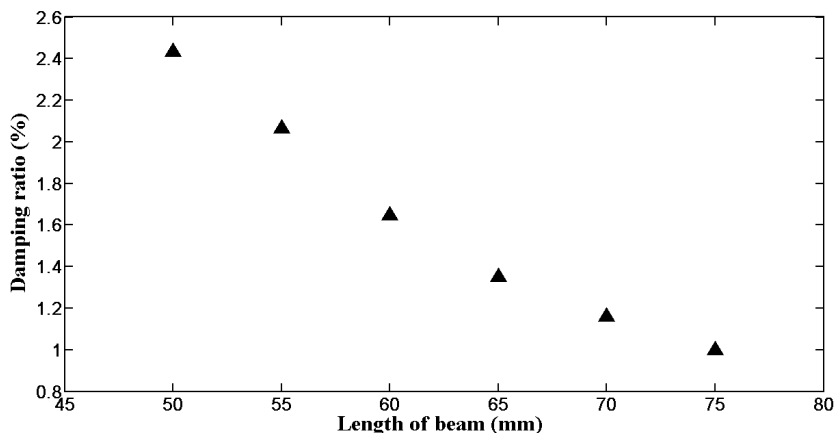
20 The experiments performed at NASA Langley Research Center has shown no reduction in the  
 21 performance of MFC actuator up to 100 million cycles under in-plane strains of -500 to 1500ppm  
 22 (Wilkie et al., 2002). Though MFC can sustain up to tensile strain of 4500ppm<sup>(4)</sup>, the durability of  
 23 the material under cycling loading decreases rapidly with higher amplitudes of strain. The

1 operating life time of MFC actuator reduces from  $10^9$  cycles to  $10^7$  cycles when applied voltage  
 2 increases from 1kV to 2kV (peak-to-peak)<sup>(4)</sup>. Furthermore, the assumptions in using the  
 3 piezoelectric constitutive equations and in deriving the maximum strain on the beam require the  
 4 material to be straining under the elastic limit. Taking into account above all the considerations,  
 5 1000ppm<sup>(4)</sup> (both in tension and compression) is considered as the upper limit of the allowable  
 6 strain in this work. The fatigue strength of the aluminum (T6) substrate used in this study is  
 7 1400ppm<sup>(6)</sup> which is higher than the strength of MFC. Moreover, the distance of the extreme  
 8 aluminum layer from the neutral axis (0.337mm) is less than the distance of the extreme  
 9 piezoceramic layer (0.463mm) from the neutral axis in the composite section of the beam. Hence  
 10 aluminum undergoes less strain than the piezoceramic material and operates within the allowable  
 11 limit. It is advisable to use specially treated metal sheets for substrates which have higher fatigue  
 12 strength.

13 The maximum strain in piezoelectric material on cantilever beams of the linear and nonlinear  
 14 PEH is shown in Figure 5. It is clear from figure that the linear PEH and the nonlinear PEH with  
 15 10mm magnetic spacing are operating within the allowable limit of strain. But, the maximum  
 16 strain of the nonlinear PEH with 8mm magnet spacing exceeds the upper limit. Closer spacing of  
 17 magnets (8mm) provides better performance in terms of power and bandwidth (Figure 4), but it  
 18 increases the strain in the piezoelectric material to an unacceptable value. On the other hand, the  
 19 nonlinear PEH with 10mm magnet spacing is safe but inferior in performance. Therefore,  
 20 parametric study is desired to seek an optimal configuration of nonlinear PEH that gives better  
 21 performance and at the same time operates within the allowable strain limit.

## 22 Parametric study

23 In this parametric study, performance characteristics of nonlinear PEH for various beam lengths  
 24 are evaluated. For each length of the beam, the tip masses are adjusted such that the natural  
 25 frequencies of PEH (27.8Hz) and magnetic oscillator (24.4Hz) are kept unchanged. Again, two  
 26 cases of separation distance between magnets i.e. 8mm and 10mm are considered and the base  
 27 acceleration is  $2\text{m/s}^2$ , same as the previous sections. The values of damping ratio, required to  
 28 investigate the behavior of PEH, are experimentally measured for different beam lengths using  
 29 the logarithmic decrement method (Leonard, 2001). The experimentally measured damping  
 30 values are shown in Figure 6.



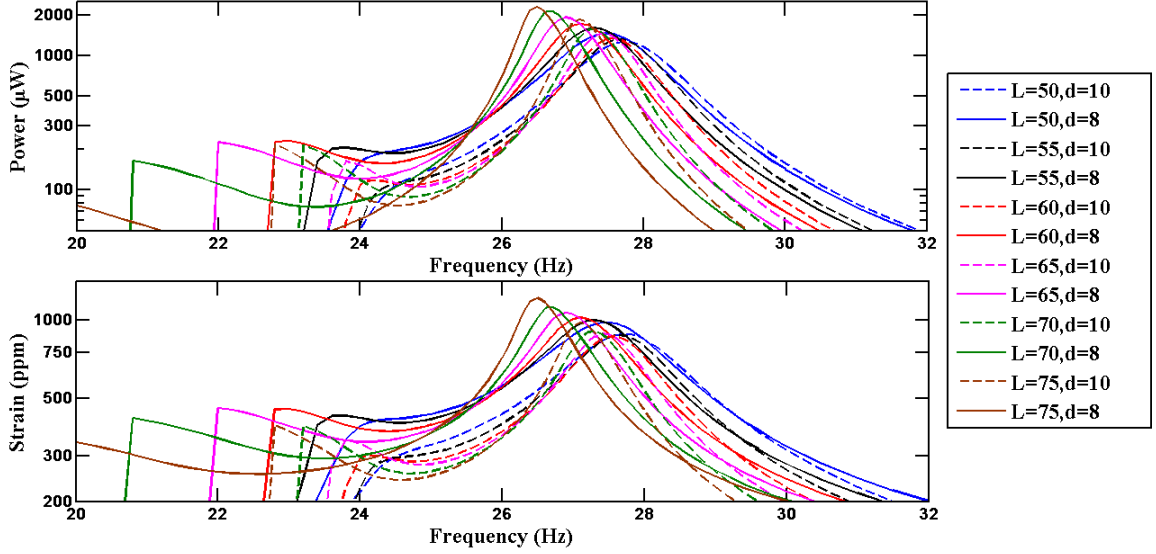
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**Figure 6.** Damping ratio for different beam lengths.

1 Figure 7 illustrates the power output and the maximum strain for various beam lengths. For the  
2 sake of legibility, the results for nonlinear PEH with  $d=8\text{mm}$  and  $d=10\text{mm}$  are only plotted. For  
3 better comparison, the results of linear PEH and two cases of nonlinear PEH with different beam  
4 lengths are also summarized in Table 3. In parametric study, the initial beam length of 75 mm is  
5 chosen and it is reduced in steps of 5 mm until the criterion on the maximum strain is met. The  
6 beam length of 75 mm is chosen to show that the further increase in beam length beyond the  
7 original length of 70 mm will result in higher strain on the material. As the length of the beam  
8 increases, its stiffness decreases, resulting in higher deflection (strain) for the given vibration  
9 input. Hence, the parametric study is carried out by shortening the beam length to reduce the  
10 strain on material. An interesting finding from Figure 7(a) is that the peak power reduces with the  
11 shortening of beam, so is the strain on the beam. This behavior can be attributed to the  
12 diminishing influence of magnetic force on the increased stiffness of shorter beams. Another  
13 noticeable feature in Figure 7(a) is the increase in the average power near the resonant frequency  
14 of magnetic oscillator with the shortening of beam. In addition, longer beams have more  
15 significant shift in the natural frequencies of PEH and magnetic oscillator because of magnetic  
16 interaction, resulting in a dip of power in between the peaks. As the beam length reduces, the shift  
17 in resonant frequencies is not so significant. Moreover, the slightly higher damping of shorter  
18 beams helps in broadening of bandwidth and thus results in achieving better power output without  
19 having a dip in between the two peaks. The bandwidth given in Table 3 corresponds to the  
20 frequency range with more than  $100\mu\text{W}$  power output (required by a wireless sensor) (Tang et al.,  
21 2010). With shortening of beams, the trade-off for reduction in the maximum power output is  
22 increase in the bandwidth. The improvement in bandwidth is minor for beam lengths less than  
23 60mm. As shown in Figure 7(b), the maximum strain in the piezoelectric material increases with  
24 the beam length. For all the beams in the parametric with  $d=10\text{ mm}$ , the maximum strain is below  
25 the acceptable limit but they have lower bandwidth when compared to their counterparts with  $d=8$   
26 mm. For the beam lengths of 50mm and 55mm with  $d=8\text{ mm}$ , the maximum strain is within the  
27 acceptable limit. Out of these two alternatives, the beam length of 55mm with 8mm of spacing  
28 between magnets is taken as the optimal configuration as it has better combination of peak power  
29 and bandwidth. It is worth mentioning that the configuration  $L=75\text{ mm}$ ,  $d=10\text{ mm}$  has higher  
30 power output than the optimal configuration ( $L=55\text{ mm}$ ,  $d=8\text{ mm}$ ) but has much lower bandwidth.

31 A constraint on the system mass affects the performance of the harvester and imposes a upper  
32 limit on the energy harvested (Kim et al., 2010; Shafer et al., 2012; Langley, 2014). The tip  
33 masses required for various lengths of the beams to keep natural frequencies of main beam and  
34 oscillator beam at 27.8 and 24.4 Hz, respectively, are also given in Table 3. It is quite evident  
35 from the table that the tip mass value increases with shortening of beam to keep same natural  
36 frequencies for the PEH and the oscillator. In this study, while choosing optimal configuration  
37 ( $L=55\text{ mm}$ ,  $d=8\text{ mm}$ ), only constraint on the material strength is considered. In case, the  
38 constraint on the system mass is also imposed and if the considered geometric parameters do not  
39 meet criteria of both system mass and material strength, a more comprehensive study is required.  
40 In such scenario, various thicknesses of beams along with various lengths should be considered in  
41 the parametric study to choose the optimal configuration. However, in the present work, only the  
42 constraint on the material strength of piezoelectric material is taken into consideration.



1

2 **Figure 7.** (a) Power and (b) strain curves for various configurations of nonlinear energy  
 3 harvesters excited at  $a_{rms}=2\text{m/s}^2$  and with  $d=8$  and 10mm.

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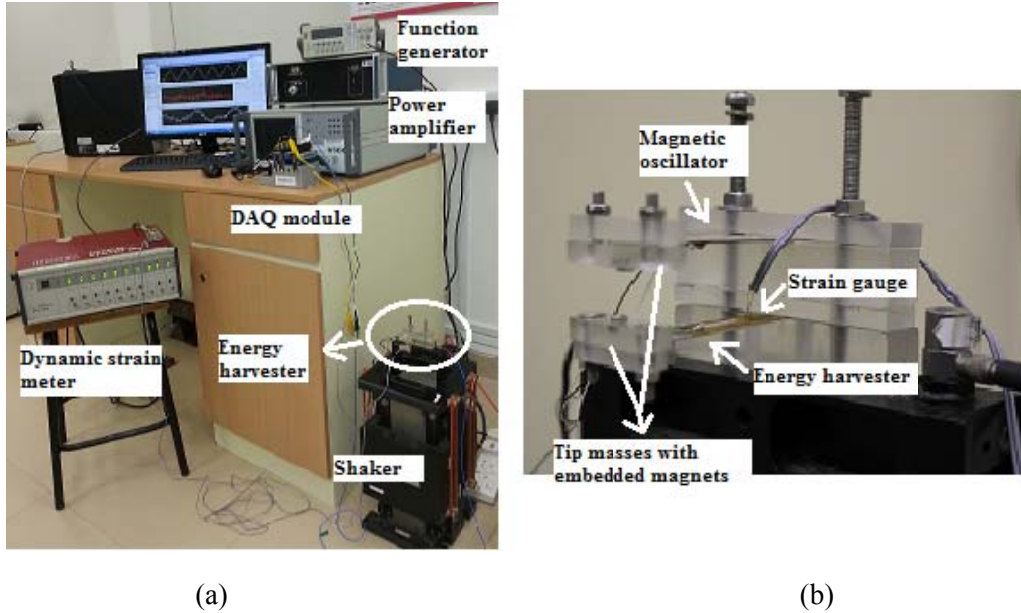
**Table 3. Summary of results of parametric study.**

$L$ (mm)	Linear harvester			Nonlinear harvester, $d=10$ mm			Nonlinear harvester, $d=8$ mm			Tip mass (g)	
	Max power (µW)	Band width (Hz)	Strain (ppm)	Max power (µW)	Band width (Hz)	Strain (ppm)	Max power (µW)	Band width (Hz)	Strain (ppm)	Main beam	Oscillator beam
75	1153.6	2.7	748.3	1868.6	3.5	975.9	2315.7	3.6	1214.8	3.63	4.79
70	1079.9	3	737.0	1579.2	3.9	906.3	2137.5	4.3	1119.0	4.61	6.06
65	1085.3	3.5	753.8	1465.2	5.8	884.8	1926.0	7.1	1065.6	5.93	7.70
60	1076.7	3.9	768.4	1349.2	5.8	862.8	1725.9	6.7	1019.5	7.74	10.1
55	1108.4	4.7	801.5	1313.4	5.9	872.4	1600.0	6.7	997.4	10.3	13.4
50	1119.7	5.5	832.5	1261.8	6.3	880.6	1462.1	6.8	977.1	14.1	18.2

5

## 6 **Experimental results**

7 A nonlinear PEH is fabricated with the optimal configuration determined from the parametric  
 8 study, i.e.,  $L=55\text{mm}$  and  $d=8\text{mm}$ . For comparison purpose, experiment is also carried out on  
 9  $d=10\text{mm}$  and linear PEH. The overview of the experimental setup is shown in Figure 8(a). The  
 10 base excitation to the nonlinear harvester is provided by the combination of a vibration exciter  
 11 (APS 113), a function generator (RIGOL DG1011) and a power amplifier (APS model 116). The  
 12 nonlinear PEH with magnetic oscillator is shown in Figure 8(b). The magnets are embedded in  
 13 the tip masses attached to the end of the beams. Figure 8(b) shows the strain gauge (FLA 5-23-  
 14 5L) used for measuring the strain on MFC patch. The strain data is acquired by dynamic strain  
 15 meter (DRA 101C) using a resolution of 0.2ms. Experiments are conducted at a harmonic base  
 16 acceleration of  $2 \text{ m/s}^2$  (RMS). The RMS voltage is measured across the resistor while sweeping  
 17 the frequency and power delivered to the resistor is calculated using the formula  $V_{rms}^2/R_L$ .



2  
3  
5  
6

(a) Entire experimental setup (b) Prototype of nonlinear energy harvester with magnetic oscillator

In Figure 9, power output of linear and nonlinear harvesters is plotted. Results of optimal configuration ( $L=55\text{mm}$ ) are compared with the results of the previous study where beam length of  $70\text{mm}$  was used (Tang and Yang, 2012). The power output from the linear configurations is shown in Figure 9(a). The optimal configuration has 21% higher power output in linear case. The power output from the linear harvester at resonance is proportional to mass, coupling coefficient and inversely proportional to damping (Erturk and Inman, 2008). It is obvious that the beam with  $L=55\text{mm}$  has higher tip mass than the beam with  $L=70\text{mm}$  to match natural frequency ( $27.8\text{Hz}$ ). Additionally, coupling coefficient increases as the length of the beam decreases because of the increased slope of the mode shape in composite section. Though the optimal beam has higher damping (Figure 6), the combination of higher tip mass, coupling coefficient and damping results in the increased power output. It is obvious from Figure 9(b) that the optimal configuration has higher peak power and wider bandwidth as compared to the beam length of  $70\text{mm}$ . The optimal configuration with  $L=55\text{mm}$  and  $d=8\text{mm}$  produces a peak power of  $1658\mu\text{W}$  with a useful bandwidth of  $6\text{Hz}$ , and has higher power output than the configuration of  $L=70\text{mm}$  and  $d=10\text{mm}$  for the most of the bandwidth. For  $L=55\text{mm}$ , the natural frequency of the linear harvester is achieved at  $27.7\text{Hz}$  compared to the simulated value of  $27.8\text{Hz}$ . The presence of magnetic nonlinearity shifted the natural frequency from  $27.7\text{Hz}$  to  $27.3\text{Hz}$  and  $26.3\text{Hz}$  for  $d=10\text{mm}$  and  $8\text{mm}$ , respectively. The experimentally measured strain in the piezoelectric material for linear and nonlinear harvesters is shown in Figure 10. The maximum strain ( $1023\text{ppm}$ ) for the optimal configuration is just above the allowable limit ( $1000\text{ppm}$ ) which is acceptable.

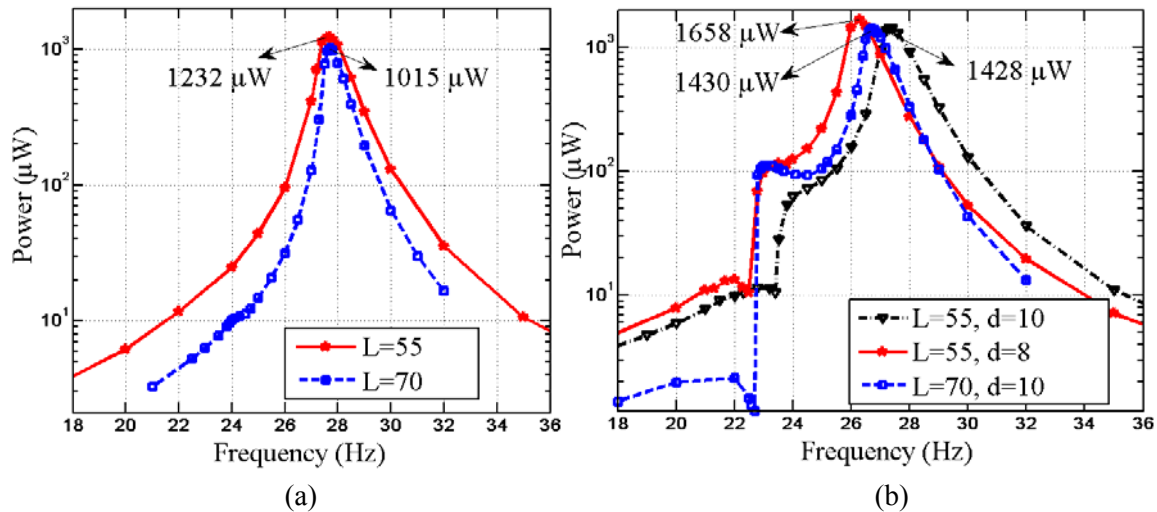
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Finally in Figure 11, the comparison between theoretical and experimental power output from the optimal configuration for both linear and nonlinear PEH is shown. The power outputs from theoretical modeling and experiment are in good agreement for the linear PEH with 10% error in prediction of maximum power output. For the nonlinear PEH, the overall trend and power output is captured quite well by theoretical modeling, but there is an error of  $0.9\text{Hz}$  in predicting resonant frequency. The peak power ( $1658\mu\text{W}$ ) and maximum strain ( $1023\text{ppm}$ ) of optimal configuration from the experiment are slightly higher than the theoretical predictions which are  $1600\mu\text{W}$  and  $997\text{ppm}$ , respectively. This is because of the slightly higher tip mass used in the



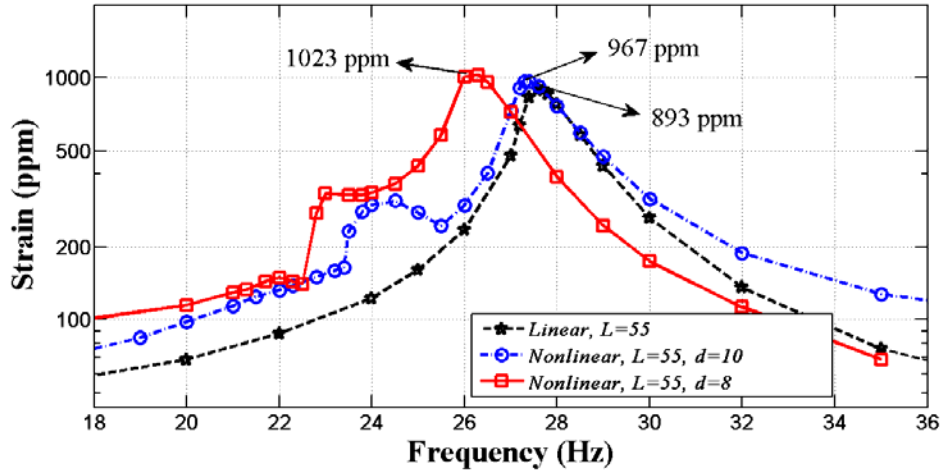
1 experiment than in the theoretical modeling to compensate for the additional stiffness due to the  
 2 epoxy layer in between the substrate and piezoelectric material. The thickness of epoxy layer  
 3 increases the distance of extreme piezoelectric layer from neutral axis, which results in excess of  
 4 strain. In theoretical modeling, the damping ratio measured for linear case has also been used for  
 5 nonlinear case. That is not exactly true as the damping ratio value changes for nonlinear case  
 6 because of higher amplitudes and changes in the stiffness of beam because of magnetic  
 7 interaction. Though extreme caution has been taken in conducting the experiments, a slight error  
 8 in the alignment of magnets or the gap between magnets could affect the results. Moreover, the  
 9 assumption of axial alignment (magnetization axis) of magnets during vibration is not so accurate  
 10 because of considerable rotations of tip masses at the free end of cantilever beams. Additionally,  
 11 the tip mass is assumed to be point mass without rotational inertia which is not the case in  
 12 experiment. Considering these sources of error, we can say experimental results are in very good  
 13 agreement with theoretical prediction.

14 Su et al. (2014) designed and experimentally tested a bistable dual-cantilever piezoelectric energy  
 15 harvester which consists of an inner and an outer beam with magnets attached to their tips. Under  
 16 a base acceleration of  $3 \text{ m/s}^2$ , the inner beam has produced a useful power output of  $100 \mu\text{W}$   
 17 between 14 to 16 Hz, with a peak power output of approximately 0.8 mW. The outer beam has  
 18 useful power output between 9 to 11 Hz and a peak power output of 1 mW. Another design by  
 19 Zhu et al. (2013), a buckled-beam piezoelectric energy harvester with a midpoint magnetic force  
 20 produced an open circuit voltage of 5V (0-peak) between 20 to 23 Hz at a base acceleration of  
 21 0.2g. When compared with the above designs, the proposed optimal nonlinear harvester with  
 22 magnetic oscillator generates useful power output between 23 to 29Hz with a much higher peak  
 23 power of 1.66 mW under a base excitation of  $0.2g_{\text{rms}}$  ( $2.83 \text{ m/s}^2$ ).



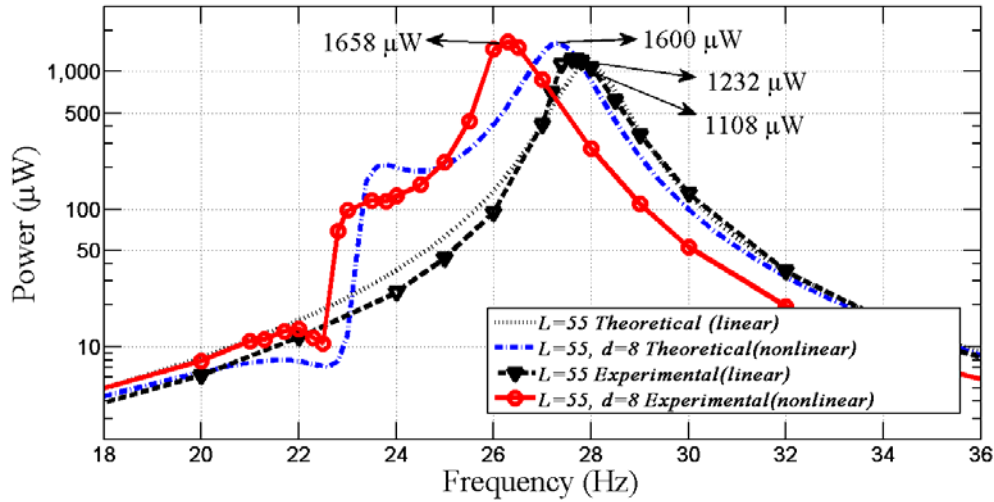
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**Figure 9.** Measured power output from (a) Linear and (b) Nonlinear energy harvesters excited at  $a_{\text{rms}}=2\text{m/s}^2$ .



1

2 **Figure 10.** Measured strain on the piezoelectric material of linear and nonlinear energy harvesters  
 3 excited at  $a_{rms}=2m/s^2$ .



4

5 **Figure 11.** Experimental and theoretical power outputs of linear and nonlinear energy PEHs  
 6 excited at  $a_{rms}=2m/s^2$ .

7 **Conclusions**

8 The authors believe that quest for enhancing the performance of nonlinear energy harvesters is  
 9 not meaningful without taking material strength into consideration in the design process. In this  
 10 paper, the performance of a nonlinear energy harvester with magnetic oscillator is optimized  
 11 while imposing constraint on the material strength. The charge model is employed to calculate the  
 12 force between permanent magnets and it is shown that the dipole model would give highly  
 13 erroneous results when the distance between magnets is 7mm or less. The maximum strain on the  
 14 cantilever beam is evaluated assuming small mechanical and electrical perturbations. Parametric  
 15 study is carried out by varying the beam length and spacing between magnets of PEH and  
 16 magnetic oscillator to seek the optimal configuration of nonlinear energy harvester. Experiments  
 17 are carried out to validate the results of theoretical modeling. At a base acceleration of  $2m/s^2$   
 18 (RMS), the optimal energy harvester generates a peak power of  $1658\mu W$  with a useful bandwidth  
 19 of 6Hz and the maximum strain in piezoelectric material is 1023ppm.

1

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4 not-for-profit sectors.

## 5 **Notes**

- 6 1. Morgan Advanced Materials. Properties of PZT bimorph available at : [http://www.morganelectroceramics.com/products/piezoelectric/piezo-bimorphs/r6001\\_piezo\\_brochureweb.pdf](http://www.morganelectroceramics.com/products/piezoelectric/piezo-bimorphs/r6001_piezo_brochureweb.pdf) (accessed on 22 October 2013)
- 7  
8  
9 2. Channel Industries, Inc. Properties of PZT patch available at : [http://www.channelindustries.com/\\_includes/chan\\_cat.pdf](http://www.channelindustries.com/_includes/chan_cat.pdf) (accessed on 22 October 2013)
- 10  
11 3. MIDE Vulture. Properties of PZT wafer available at : [http://www.mide.com/pdfs/Vulture\\_Datasheet\\_001.pdf](http://www.mide.com/pdfs/Vulture_Datasheet_001.pdf) (accessed on 22 October 2013)
- 12  
13 4. Smart materials. Properties of MFC patch available at : <http://www.smart-material.com/MFC-product-main.html> (accessed on 22 October 2013)
- 14  
15 5. TRS electrostrictors. Properties of PMN-PT patch available at : <http://www.trstechnologies.com/pdf/Electrostrictors.pdf> (accessed on 22 October 2013)
- 16  
17 6. <http://asm.matweb.com/search/SpecificMaterial.asp?bassnum=MA6061t6>
- 18

## 19 **References**

- 20 Aboulfotouh NA, Arafa MH and Megahed SM (2013) A self-tuning resonator for vibration energy  
21 harvesting. *Sensors and Actuators A: Physical*  
22
- 23 Agashe JS and Arnold DP (2008) A study of scaling and geometry effects on the forces between  
24 cuboidal and cylindrical magnets using analytical force solutions. *Journal of Physics D: Applied*  
25 *Physics*, 41(10), 105001.  
26
- 27 Akoun G and Yonnet JP (1984) 3D analytical calculation of the forces exerted between two  
28 cuboidal magnets. *Magnetics, IEEE Transactions on*, 20(5), 1962-1964.  
29
- 30 Al-Ashtari W, Hunstig M, Hemsell T and Sextro W (2012) Frequency tuning of piezoelectric  
31 energy harvesters by magnetic force. *Smart Materials and Structures*, 21(3), 035019.  
32
- 33 Anton SR, Erturk A and Inman DJ (2010) Strength analysis of piezoceramic materials for  
34 structural considerations in energy harvesting for UAVs. *In Proc. of SPIE Vol* (Vol. 7643, p.  
35 76430E).  
36
- 37 Challa VR, Prasad MG, Shi Y and Fisher FT (2008) A vibration energy harvesting device with  
38 bidirectional resonance frequency tunability. *Smart Materials and Structures*, 17(1), 015035.  
39
- 40 Elvin NG and Elvin AA (2009) A general equivalent circuit model for piezoelectric generators.  
41 *Journal of Intelligent Material Systems and Structures*, 20(1), 3-9.  
42
- 43 Erturk A, Hoffmann J and Inman DJ (2009) A piezomagnetoelastic structure for broadband  
44 vibration energy harvesting. *Applied Physics Letters*, 94(25), 254102-254102.  
45

1 Erturk A and Inman DJ (2008) On mechanical modeling of cantilevered piezoelectric vibration  
2 energy harvesters. *Journal of Intelligent Material Systems and Structures*, 19(11), 1311-1325.  
3  
4 Erturk A and Inman DJ (2011) *Piezoelectric energy harvesting*. Wiley.  
5  
6 Fett T, Munz D and Thun G (1999) Tensile and bending strength of piezoelectric ceramics.  
7 *Journal of Materials Science Letters*, 18(23), 1899-1902.  
8  
9 Furlani EP (2001) *Permanent magnet and electromechanical devices: materials, analysis, and*  
10 *applications*. Elsevier.  
11  
12 Hagood NW, Chung WH and Von Flotow A (1990) Modelling of piezoelectric actuator dynamics  
13 for active structural control. *Journal of Intelligent Material Systems and Structures*, 1(3), 327-  
14 354.  
15  
16 Harne RL and Wang KW (2013) A review of the recent research on vibration energy harvesting  
17 via bistable systems. *Smart Materials and Structures*, 22(2), 023001.  
18  
19 Kim M, Hoegen M, Dugundji J and Wardle BL (2010) Modeling and experimental verification of  
20 proof mass effects on vibration energy harvester performance, *Smart Materials and Structures*,  
21 19(4), 045023.  
22  
23 Langley RS (2014) A general mass law for broadband energy harvesting, *Journal of Sound and*  
24 *Vibration*, 333(3), 927-936.  
25  
26 Leonard M (2001), *Fundamentals of vibrations*. Prentice Hall.  
27  
28 Lin J, Lee B and Alphenaar B (2010) The Magnetic Coupling of a Piezoelectric Cantilever for  
29 Enhanced Energy Harvesting Efficiency. *Smart Materials and Structure*, 19:045012.  
30  
31 Mann BP and Sims ND (2009) Energy harvesting from the nonlinear oscillations of magnetic  
32 levitation. *Journal of Sound and Vibration*, 319(1), 515-530.  
33  
34 Narita F, Shindo Y and Saito F (2007) Cyclic Fatigue Crack Growth in Three-Point Bending PZT  
35 Ceramics under Electromechanical Loading. *Journal of the American Ceramic Society*, 90(8),  
36 2517-2524.  
37  
38 Pellegrini SP, Tolou N, Schenk M and Herder JL (2012) Bistable vibration energy harvesters: A  
39 review. *Journal of Intelligent Material Systems and Structures*.  
40  
41 Ravaut R, Lemarquand G, Babic S, Lemarquand V and Akyel C (2010) Cylindrical magnets and  
42 coils: Fields, forces and inductances. *Magnetics, IEEE Transactions on*, 46(9), 3585-3590.  
43  
44 Robertson W, Cazzolato B and Zander A (2011) A simplified force equation for coaxial  
45 cylindrical magnets and thin coils. *Magnetics, IEEE Transactions on*, 47(8), 2045-2049.  
46  
47 Schäufele AB and Heinz Härdtl K (1996) Ferroelastic properties of lead zirconate titanate  
48 ceramics. *Journal of the American Ceramic Society*, 79(10), 2637-2640.  
49

1 Shafer MW and Garcia E (2013) Fundamental power limits of piezoelectric energy harvesters  
2 based on material strength. In *SPIE Smart Structures and Materials+ Nondestructive Evaluation*  
3 *and Health Monitoring* (pp. 86880D-86880D). *International Society for Optics and Photonics*.  
4

5 Shafer MW, Bryant M and Garcia E (2012) Designing maximum power output into piezoelectric  
6 energy harvesters, *Smart materials and structures*, 21(8), 085008.  
7

8 Sodano HA, Park G and Inman DJ (2004) Estimation of electric charge output for piezoelectric  
9 energy harvesting. *Strain*, 40(2), 49-58.  
10

11 Standards Committee of the IEEE Ultrasonics, Ferroelectrics and Frequency Control Society  
12 (1987) IEEE standard on piezoelectricity, IEEE, New York.  
13

14 Stanton SC, McGehee CC and Mann BP (2010) Nonlinear Dynamics for Broadband Energy  
15 Harvesting: Investigation of a Bistable Piezoelectric Inertial Generator. *Physica D*, 239:640653.  
16

17 Stanton SC, McGehee CC and Mann BP (2009) Reversible hysteresis for broadband  
18 magnetopiezoelectric energy harvesting. *Applied Physics Letters*, 95:174103.  
19

20 Su WJ, Zu J and Zhu Y (2014) Design and development of a broadband magnet-induced dual-  
21 cantilever piezoelectric energy harvester. *Journal of Intelligent Material Systems and Structures*,  
22 25(4), 430-442.  
23

24 Tang L, Yang Y and Soh CK (2010) Toward broadband vibration-based energy harvesting.  
25 *Journal of Intelligent Material Systems and Structures*, 21:1867.  
26

27 Tang L, Yang Y and Soh CK (2012) Improving functionality of vibration energy harvesters using  
28 magnets. *Journal of Intelligent Material Systems and Structures*, 23(13), 1433-1449.  
29

30 Tang L and Yang Y (2012) A nonlinear piezoelectric energy harvester with magnetic oscillator.  
31 *Applied Physics Letters*, 101(9), 094102-094102.  
32

33 Twiefel J and Westermann H (2013) Survey on broadband techniques for vibration energy  
34 harvesting. *Journal of Intelligent Material Systems and Structures*.  
35

36 Wickenheiser AM (2011) Design optimization of linear and non-linear cantilevered energy  
37 harvesters for broadband vibrations. *Journal of Intelligent Material Systems and Structures*,  
38 22(11), 1213-1225.  
39

40 Wilkie WK, High J and Bockman J (2002) Reliability testing of NASA piezocomposite actuators.  
41 Proceedings of the 8th International Conference on New Actuators, Bremen, Germany.  
42

43 Zhu D, Roberts S, Tudor J and Beeby S. (2008) Closed Loop Frequency Tuning of a Vibration-  
44 based Microgenerator, *Proceedings of Power MEMS, Sendai, Japan*, pp. 229232.  
45

46 Zhu Y and Zu JW (2013) Enhanced buckled-beam piezoelectric energy harvesting using midpoint  
47 magnetic force, *Applied Physics Letters*, 103(4), 041905.  
48

49 Zhou S, Cao J, Erturk A and Lin J (2013) Enhanced broadband piezoelectric energy harvesting  
50 using rotatable magnets. *Applied Physics Letters*, 102(17), 173901-173901.  
51

1

## 2 **Appendix**

3 Assuming variable substitution as below:

4  $q_1(t) = u_1(t), q_2(t) = \dot{u}_1(t),$

5  $q_3(t) = u_2(t), q_4(t) = \dot{u}_2(t),$  (20)

$$q_5(t) = V(t)$$

6 Using above variable substitution, the governing equations (11) to (13) can be written in state  
7 space form as

$$\dot{q}_1(t) = q_2(t),$$

$$\dot{q}_2(t) = -\frac{C_1}{M_1}q_2(t) - \frac{K_1}{M_1}q_1(t) - \frac{\theta}{M_1}q_5(t) + \frac{F_{mag}}{M_1} + \frac{F_1(t)}{M_1},$$

8  $\dot{q}_3(t) = q_4$  (21)

$$\dot{q}_4(t) = -\frac{C_2}{M_2}q_4(t) - \frac{K_2}{M_2}q_3(t) - \frac{F_{mag}}{M_2} + \frac{F_2(t)}{M_2},$$

$$\dot{q}_5(t) = \frac{\theta}{C^s}q_2(t) - \frac{1}{RC^s}q_5(t)$$

9