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Critical route for coherent perfect absorption in a Fano resonance plasmonic system
Ming Kang, Y. D. Chong, Hui-Tian Wang, Weiren Zhu, and Malin Premaratne

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Critical route for coherent perfect absorption in a Fano resonance plasmonic system

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We develop a method for realizing coherent perfect absorption in thin metamaterial systems, based on the coupled-mode theory of Fano resonance. Coherent perfect absorption refers to the complete absorption of symmetric plane waves incident on opposite sides of the system, due to critical coupling into the dissipative degrees of freedom. Using the reflection and transmission spectra measured on a limited number of samples, our theory predicts the precise frequency and metamaterial parameter values required to achieve coherent perfect absorption. The coupled-mode theory and the design method are found to agree well with full-wave numerical simulations of a subwavelength-thickness metamaterial surface with a pair of bright and dark plasmonic modes.

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Fano resonance occurs in scattering systems possessing coupled super-radiant and sub-radiant internal states. The interference between these states results in distinctive features such as asymmetric spectroscopic line-shapes, which have been observed in a wide variety of systems.1–3 Recently, Fano resonance has been the subject of renewed interest in the field of nanophotonics, based on the idea of engineering the super-radiant and sub-radiant internal states by explicit nanophotonic structure design, instead of relying on atomic resonances in the underlying materials. Fano resonance has been realized in many nanophotonic platforms, from photonic crystals to plasmonics and metamaterials.4–8 Electromagnetically induced transparency (EIT), which is one of the most dramatic examples of Fano resonance phenomena, has been theoretically proposed and experimentally verified in microwave and optical-scale nanophotonic structures,9–17 with potential applications in slow-light waveguide, sensing, and enhancement of optical nonlinearity. In the EIT and slow-light applications, the goal is typically to create narrow resonances, which means that losses due to dissipation need to be minimized.4–11 Another possibility is to manipulate Fano resonance to control, or even maximize optical absorption.

In this letter, we derive a set of principles for designing metasurface Fano resonances for the realization of coherent perfect absorption (CPA). The CPA refers to the complete absorption of a coherent wave incident on a resonator, due to critical coupling into dissipative internal degrees of freedom.18–29 Mathematically, this corresponds to a zero eigenvalue of the scattering matrix S at a real frequency, which can be interpreted as the time-reversed counterpart of a laser at threshold.18 Recently, the CPA has been observed in a thin metamaterial system, using plane-wave illumination incident on opposite sides of the system;34 based on this demonstration, the CPA has been proposed as a mechanism for implementing coherent optical gating, and for coupling light into plasmonic modes down to the single-photon level. It has also been shown that a metamaterial film tuned to the CPA condition can be used as a platform for realizing “parity-time symmetric” physics.26 In previous works, it has been shown that the CPA occurs in a thin film when \( r = \mp 1/2 \) and \( t = 1 \pm r \) for electric (magnetic) dipole scattering (\( r \) and \( t \) are the reflection and transmission coefficients), which can be achieved by tuning the ratio of dissipation and scattering losses in the metallic elements of metasurface.26,27 However, this intuitive condition is a macroscopic prescription, involving no microscopic responses and no coupling between the metasurface modes, and cannot therefore provide a true “design route”. In other words, the previous works24,26,27 showed that it is possible to achieve a CPA-supporting metasurface, but do not provide the steps for designing such a system, apart from scanning through the space of sample parameters.

By the coupled-mode theory (CMT), we develop a design route for the CPA in a metasurface supporting a plasmonic Fano resonance consisting of interacting bright and dark modes. For a given set of CMT parameters, we obtain theoretically the frequency-dependent reflection and transmission spectra. Hence, for a given length parameter, such as the metamaterial resonator length \( a \) to be defined below, the measured spectra can be used to fit the CMT’s parameters. All the CMT’s parameters exhibit linear dependence on \( a \), which can be reliably determined by measuring the spectra for a relatively small number of samples with different \( a \). Putting this dependence into the CMT, one can then predict the precise value of \( a \) required for realizing the CPA and the
frequency at which the CPA occurs. A great deal of trial-and-error in sample fabrication can thus be bypassed. We have verified that the CMT is in good agreement with the design scheme using full-wave numerical simulations. This method can be used to realize the CPA-supporting metasurfaces in the future experiments.

The CMT model describes a pair of coupled bright and dark resonant modes. The coupling between the two modes is described by the scattering matrix relation

\[ \Omega \mathbf{q} = \mathbf{K}^\dagger \mathbf{a} \quad \text{and} \quad \mathbf{K} \mathbf{q} - \mathbf{I} \mathbf{a} = \mathbf{b}. \]  

(1)

Here, \( \mathbf{a} \) and \( \mathbf{b} \) are complex vectors giving the amplitudes for the incident and output light, respectively. The resonant modes are described by the amplitude vector \( \mathbf{q}^\dagger = (q_b, q_d)^\dagger \), where the subscripts \( b \) and \( d \) denote the bright and dark modes. The coupling between the two modes is described by

\[ \Omega = \begin{bmatrix} -if_b + if + \gamma_b + \gamma_d & -ik & -ik \\ -ik & -if_d + if + \gamma_d \\ \end{bmatrix}, \]  

(2)

where \( \gamma_b, \gamma_d, \gamma_b^d \) are the resonant frequency, scattering loss, and dissipation loss for the bright (dark) mode, respectively; and \( \kappa \) indicates the coupling between the dark and bright mode. We assume the scattering loss of the dark mode to be zero, implying that this mode cannot be directly excited by the incident light. Finally, the coupling matrix \( \mathbf{K}^\dagger \) describes how the resonant modes connect to the input and output channels as follows:

\[ \mathbf{K}^\dagger = \begin{bmatrix} \sqrt{\gamma_b^d} & \sqrt{\gamma_b} \\ 0 & 0 \end{bmatrix}. \]  

(3)

The input and output amplitudes are related by the scattering matrix relation \( \mathbf{b} = \mathbf{S} \mathbf{a} \). According to Eq. (1), we have

\[ \mathbf{S} = \begin{bmatrix} r & t \\ t & r \end{bmatrix} = -\mathbf{I} + \mathbf{K} \Omega^{-1} \mathbf{K}^\dagger. \]  

(4)

The determinant of the scattering matrix can be expressed as

\[ \det \mathbf{S} = \det \mathbf{H} / \det \Omega, \]

where

\[ \mathbf{H} = \begin{bmatrix} -if_b + if + (\gamma_b^d - \gamma_b) & -ik \\ -ik & -if_d + if + \gamma_d \end{bmatrix}. \]  

(5)

The eigenvalues of \( \mathbf{S} \) are \( \chi_\pm = r \pm t \) for eigenvectors \( \pm 1, 1 \)^\dagger. For a thin film, the condition \( t = 1 + r \) is automatically satisfied. Thus, the CPA is only induced by the symmetric scattering eigenvector \( 1, 1 \)^\dagger with the eigenvalue of \( \chi_+ = r + t = -\det \mathbf{S} \). The asymmetric eigenvector \( 1, -1 \)^\dagger has always an eigenvalue of \(-1\), implying that the existence of an intensity node on the plane of the metasurface and the zero loss. The CPA occurs when \( \chi_+ = 0 \), implying that \( \det \mathbf{H} = 0 \) as

\[ (\gamma_b^d - \gamma_b) (f_d - f) - \gamma_d^d (f_b - f) = 0, \]  

(6a)

\[ (f_d - f) (f_b - f) + \gamma_d^d (\gamma_b^d - \gamma_b) - \kappa^2 = 0. \]  

(6b)

Equation (6), as the main consequence of the CMT, indicates the critical route for realizing the CPA in a general Fano resonant system. In the special case, \( f_d = f_b \) corresponds to an EIT system, which has been used to investigate the complex eigenvalue problem of the matrix \( \mathbf{H} \) in our previous work. For a given set of the CMT parameters with \( f_d \neq f_b \), there is generally a single solution only for achieving the CPA, occurring when the frequency and the critical coupling satisfy

\[ f_{\text{CPA}} = f_d + \gamma_d^d \frac{f_b - f_d}{\gamma_b + \gamma_d - i\kappa}, \]  

(7a)

\[ \kappa_{\text{CPA}} = \left( (f_d - f_{\text{CPA}}) (f_b - f_{\text{CPA}}) + \gamma_d^d (\gamma_b^d - \gamma_b) \right)^{1/2}. \]  

(7b)

Equation (6) sets a strong limit to achieve the CPA in a planar Fano resonant plasmonic system with mirror symmetry in the propagation direction, identifying the influence of each parameter. These equations are not limited to the plasmonic systems, but are also suitable for describing other Fano systems sharing the same coupled mode, such as acoustic systems.

To verify the above CMT, we have designed and studied a realistic metasurface supporting a plasmonic Fano resonance. As shown in Fig. 1, the metasurface is a thin-film metamaterial with a thickness \( t = 30 \) nm, with an in-plane unit cell consisting of an H-shaped gold strip antenna sandwiched between two continuous gold strips. The H-shaped antenna has a length \( a \), a width \( w = 50 \) nm, an arm length \( b = 120 \) nm, and an arm width \( h = 30 \) nm. The side strips have a width \( g = 75 \) nm and a length \( d = 300 \) nm. The metasurface unit cells are arranged in a square lattice with a period \( d = 300 \) nm, and the entire metasurface is free-standing in vacuum (ensuring that no high-order diffraction modes exist below 1000 THz). The dielectric function of gold is assumed to follow the Drude model, \( \varepsilon_m = \varepsilon_\infty - f_p^2 / (\omega^2 + \gamma_p^2) \), where \( f_p = 2.18 \times 10^3 \) THz, \( \gamma_p = 1.62 \) THz, and \( \varepsilon_\infty = 9.30 \).
The reflection and transmission spectra of the metasurface are calculated using full-wave numerical simulations (finite-difference time-domain method). To achieve the critical requirements of Eq. (6), we optimize the geometric structure of the unit cell by only changing the length of the H-shaped antenna. The results are shown in Fig. 2, for the H-shaped antenna length ranging from \(a = 80\) to \(120\) nm (with all other geometry parameters fixed at the values given above), and a frequency range of 250–450 THz. The normally incident plane wave is linearly polarized, with its E field being parallel to the continuous side strips. In the reflection spectrum, for each value of \(a\), there is a clear dip at a lower frequency accompanied by a peak at a higher frequency, which is a direct signature of a Fano resonance arising from the hybridized bright and dark modes. As \(a\) increases, the Fano resonance exhibits a clear redshift.

As discussed above, the CPA can be realized when the scattering matrix eigenvalue \(r + t\) goes to zero. The values of \(\log_{10} |r + t|^2\) obtained from the numerical simulations are plotted in Fig. 3. At \(a = 86\) nm and \(f = 334.4\) THz, the logarithm intensity goes to about \(-4\), i.e., the output intensity is \(10^{-4}\) of the input when the input beams are symmetric. Away from the CPA point, \(\log_{10} |r + t|^2\) is of order unity. We emphasize that it is necessary to tune two parameters, \(a\) and \(f\), to achieve the CPA.\(^{18}\) The CPA condition cannot be satisfied by simply picking an arbitrary value of \(a\) and then tuning \(f\).

Finally, we use the numerical simulation results to verify the proposed design route for finding the CPA point. For a given set of the CMT parameters, the reflection and transmission spectra can be calculated from Eqs. (1)–(4). Because the dissipative loss in the studied metasurface is completely from the ohmic losses in gold, we simplify the model by assuming \(\gamma^d_b = \gamma^d_d\). For each value of the antenna length \(a\), we perform a nonlinear fit of \(|r|^2\) and \(|t|^2\) by the CMT, to obtain the fitted values for the dark mode frequency \(f_d\), the bright mode frequency \(f_b\), the bright mode scattering loss \(\gamma^s_b\), and the dissipative loss \(\gamma^d_{b,d}\). As shown in Fig. 4, these fitted parameters are found to be well-described by simple linear functions of \(a\). The fitted value of \(\kappa\), the coupling between the bright and dark modes, is found to be \(\kappa = 59.4\) THz, independent of \(a\).
Using the fitted parameter functions, the absorption $A(a, f) = 1 - |r|^2 - |t|^2$ and coherent absorption $A_C(a, f) = 1 - |r + t|^2$ spectra are plotted in Figs. 5(a) and 5(c). These in good agreement with the directly-obtained numerical results, plotted in Figs. 5(b) and 5(d). As the length $a$ increases, the absorption peak shows a clear redshift, and its value reaches a maximum at around $a = 86$ nm. Furthermore, using the fitted parameter functions, we can search numerically for a solution to the CPA conditions given by Eq. (6). This results in a predicted solution at $f_{CPA} = 334.4$ THz and $a = 86.0$ nm. By comparison, a numerical sweep through different finely-spaced values of $a$ gives $f_{CPA} \approx 334.4$ THz and $a \approx 86.0$ nm, which is in a good agreement. In Fig. 5(e), the absorption and coherent absorption spectra predicted by the fitted CMT (solid lines) are plotted against $f$ for the $a = 86$ nm structure, and are found to be in excellent quantitative agreement with the numerical results (dashed and dotted lines). The peak of the absorption spectrum $A_{\text{Max}} = 0.5$ coincides with the peak of the coherent absorption spectrum $A_{C\text{Max}} = 1.0$ at the same frequency $f = 334.4$ THz. This agrees with the principle, discussed in earlier works, that the maximum absorption in an infinitely thin film under single-sided illumination is 0.5.

In conclusion, we have theoretically developed a design route for realizing the CPA in Fano resonant plasmonic metasurfaces. This method is based on the CMT of a Fano resonance in a thin film, which is found to be in well agreement with the full-wave numerical simulations of a subwavelength-thickness metasurface. Due to the smooth variation of the coupled-mode parameters with geometry parameters, by taking the reflection and transmission spectra of a small number of experimental samples, one can “home in” on the point in parameter space where the system supports coherent perfect absorption. This method can be used to design absorbing metasurfaces for a variety of photonics applications.

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