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A LDO Regulator with Weighted Current Feedback Technique for 0.47nF-10nF Capacitive Load

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Abstract

A Weighted Current Feedback (WCF) technique for output capacitorless low-dropout (OCL-LDO) regulator is presented in this paper. Through feedback of a weighted current, the WCF permits smart management of the output impedance as well as the gain from the inter-gain stage. Based on the Routh–Hurwitz stability criterion, the WCF can avoid the right-half plane (RHP) pole and push the left-half plane (LHP) non-dominant complex pole pair to a higher frequency. Besides, it provides good regulator loop gain and fast transient response. Validated by UMC 65-nm CMOS process, the simulation and measurement results have shown that the WCF LDO regulator can operate at a $C_L$ range from 470 pF to 10 nF with only 3.8 pF compensation capacitor. At a supply of 0.75 V and a quiescent current of 15.9 µA, the proposed circuit can support a maximum load current ($I_L$) of 50 mA. When $I_L$ switches from 0 to 50 mA in 100 ns, the output can settle within 400 ns for the whole $C_L$ range. For a case of single capacitor ($C_L = 470$ pF), the settling time is only 250 ns. The comparison results have shown that the WCF LDO regulator offers a comparable or better transient figure-of-merit (FOM) and additional merit to drive wide load capacitance range.

Keywords

Cascode Compensation, LDO regulator, Miller Compensation, Weighted Current Feedback (WCF), Wide Load Capacitance Range.
I. INTRODUCTION

Voltage regulators have been widely used to supply various function blocks in battery powered portable devices. A LDO regulator is very popular in power management IC design on the basis of its simple structure, fast response and low noise characteristic [1]. However, the LDO regulators using a μF level off-chip capacitor to achieve stable operation limit the ability for fully on-chip applications [2-6]. This turns out that the output capacitorless LDO (OCL-LDO) regulators [7-24] have received much attention recently. In System-on-Chip (SoC) environment pertaining to large scale digital circuits like DSP core(s) and memory banks, the effective supply line parasitic capacitance is large. To drive the circuits, the LDO regulator should support a wide load capacitance ($C_L$) range (few hundred pF to few nF) [25] with fast response. Furthermore, to ensure the accuracy of the LDO regulator, a multi-gain stage amplifier topology has to be adopted. Besides, under low power constraint, the output impedances of the gain stages can be high, resulting in several low frequency poles. This increases the design difficulty of the OCL-LDO regulator, especially for applications with wide range of $C_L$. In most of the reported OCL-LDO regulator designs, they usually drive a maximum $C_L$ up to tens of pF or 100 pF [11, 12, 14, 17, 18, 20]. Few designs aim at driving relatively large $C_L$ [8, 13, 16, 19, 24, 26]. Nevertheless, they have their limitations which are described in the following.

In [8], a Flipped Voltage Follower (FVF) topology is used as an output driver. It gives a very fast response with a recovering time of 0.54 ns. Since the quiescent current in this LDO regulator is 6 mA, it leads to unavoidable large quiescent power consumption. In addition, only a maximum $C_L$ of 600 pF is reported. A direct voltage spike detection technique is reported in [13] that supports a 100 pF and a 1 nF capacitive load. Since the LDO regulator utilizes a simple folded FVF topology, the loop gain is fairly limited, thus trading off some of transient performances. In [16], an active compensation scheme is realized in order to enable the LDO regulator to drive a $C_L$ up to 1 nF. However, several poles and zeros exist within the unity gain frequency ($\omega_{UGF}$), leading to complicated pole-zero tracking. Any mismatch between poles and zeros will contribute slower transient response. In [19], a current amplifier is adopted to multiply the Miller capacitor which can extend the $C_L$ driving capability to 1 nF. However, the design needs a large compensation
capacitor (tens of pF) to ensure stable operation. This may lead to relatively larger silicon area and slower transient speed. In [24, 26], a dual-summed Miller compensation is implemented. This is targeted to support a $C_L$ range up to 10 nF but at the expense of increased settling time through the in-band zero for stability.

In view of the need to support a wide $C_L$ range and good transient performance metrics under low quiescent power design objectives, a new circuit technique is demanded in the design of LDO regulators. In this paper, a weighted current feedback (WCF) technique, which aims at a wide $C_L$ range (470 pF to 10 nF) for the LDO regulator design, is proposed.

In Section II, it presents the WCF circuit technique working principle. Section III describes the inter-gain stage’s dynamic output impedance reduction as well as the small-signal model. Section IV presents the detailed stability analysis and the design strategy of WCF. This is then followed by the WCF LDO regulator circuit implementation in Section V. The experimental results, discussions and performance comparison are given in Section VI. Finally, the conclusion is drawn in Section VII.

II. PROPOSED WCF CIRCUIT TECHNIQUE

A. Conventional Multi-gain Stages in a LDO Regulator

Fig. 1 depicts the multi-gain stages in a LDO regulator. The regulator is usually a 3-stage amplifier plus a power transistor ($M_P$). It is noted that $g_{mi}$ denotes the transconductance while $R_i$ and $C_i$ are the equivalent output resistance and lumped output parasitic capacitance of the i-th gain stage, respectively. $R_O$ is the effective output resistance which includes the output resistance of power transistor as well as the loading resistance $R_L$. As can be seen from Fig. 1, the feedback system displays a four-pole ($p_{-3dB}$, $p_2$, $p_3$ and $p_O$) characteristic. In general design, $p_O$ locates at a lower frequency than that of $p_2$ and $p_3$. Hence, $p_2$ and $p_3$ must be allocated to a higher frequency to ensure the stability. The typical implementation of 2$^{nd}$ and 3$^{rd}$ gain stages is to make node $N_2$ as a low impedance node ($R_2 \approx 1/g_m$) whereas node $N_P$ as a high impedance node [10, 14, 15, 17, 20]. Since $C_P$ is large due to a large power transistor $M_P$, $p_3$ is low especially under low power constraint. As reported in [14], two parasitic poles are generated with frequency inversely proportional to
To stabilize the LDO regulator, $C_L$ must be small. Moreover, since $R_O$ is inversely proportional to $I_L$, a minimum workable $I_L$ (e.g. 3 mA) is required to ensure a small $R_O$ [14]. This restricts the LDO regulator’s applications for a wide range of $C_L$ and $I_L$. As such, a new circuit technique is needed to address this issue.

**B. Proposed Negative Current Feedback Circuit Technique**

In the proposed topology shown in Fig. 2, $R_2$ is designed to be high whereas $R_P$ is designed to be dominantly small ($\approx 1/g_m$). With this arrangement, the 3rd gain stage can be dynamically biased. This will increase the charging/discharging rate of $V_P$. As a result, the speed of the regulator is greatly improved with respect to the conventional topology (Fig. 1). It is important to note that $C_P$ and $R_2$ may be large. They lead to large $C_P R_P$ and $C_2 R_2$, respectively. Based on Routh–Hurwitz stability criterion, the large time constant $C_2 R_2$ and $C_P R_P$ may introduce a right-half-plane (RHP) pole. Therefore, a negative current feedback (NCF) technique is employed to avoid the RHP pole formation.

In the NCF block depicted in Fig. 2, the current sensor senses the voltage $V_P$ and generates a transconductance current $g_{mf} V_P$ ($g_{mf}$ is the transconductance of the current sensor). It is then fed back to the node $N_2$. Not only does the feedback current increase the bias current of 2nd stage as the first effect, it also forms a local negative current feedback loop (NCF loop in Fig. 2) as the second effect. Combining these two effects, the output impedance at node $N_2$ is reduced from $R_2$ to $R_{2f}$ ($R_{2f}$ is the negative current feedback loaded impedance at node $N_2$). As such, both $C_2 R_{2f}$ and the regulator loop gain are reduced. This permits the feedback system to fulfill the Routh–Hurwitz stability criterion. In addition, the NCF technique adds another advantage by shifting the non-dominant poles to a higher frequency. Hence the stability of the LDO regulator can be attained in the context of wide range of capacitive load and load current.

However, there are two tradeoff issues in the NCF LDO regulator. They are given as follows: (i) The gain of 2nd stage is reduced because of a smaller $R_{2f}$. This in turn reduces the total loop gain of the LDO regulator, thus sacrificing some regulation accuracy. (ii) The negative feedback current reduces the charging/discharging rate of the node $N_2$, which can reduce the transient speed. In view of the two drawbacks,
a weighted current feedback circuit technique is further proposed to tackle the limitations arising from the foundation NCF technique.

C. Proposed Weighted Current Feedback Circuit Technique

Fig. 3 shows a LDO regulator architecture using the WCF circuit technique. It comprises a fixed first gain stage, two variable gain stages (2\textsuperscript{nd} and 3\textsuperscript{rd} gain stages), a WCF block, a power transistor M\textsubscript{p}, an overshoot reduction block and a frequency compensation network.

The shadowed area embodies the 2\textsuperscript{nd}, 3\textsuperscript{rd} gain stages as well as the WCF circuit (dash enclosed box). In the WCF topology, two sense transistors (M\textsubscript{a1} and M\textsubscript{a2}, size of M\textsubscript{a2} > size of M\textsubscript{a1}) sense the same voltage V\textsubscript{p} and each generates respective feedback current (I\textsubscript{a1}, I\textsubscript{a2}) at the output of 2\textsuperscript{nd} gain stage. Moreover, the diode transistor M\textsubscript{a3} is added in series with M\textsubscript{a4} to control the operating region of M\textsubscript{a2} during the change of load current. The working principle of the WCF technique can be explained from Fig. 4. (i) At low I\textsubscript{L} (Fig. 4 (a)), both M\textsubscript{a1} and M\textsubscript{a2} are weakly biased for small negative current feedback. Each 2\textsuperscript{nd} and 3\textsuperscript{rd} gain stage works as a normal inverting amplifier. (ii) At moderated I\textsubscript{L} (Fig. 4 (b)), both M\textsubscript{a1} and M\textsubscript{a2} are designed to work in saturation region. Two feedback currents (I\textsubscript{a1}, I\textsubscript{a2} and I\textsubscript{a2} > I\textsubscript{a1}) are generated and fed back to the node N\textsubscript{2}. This results in a strong negative current feedback to the 2\textsuperscript{nd} stage. (iii) At high I\textsubscript{L} (Fig. 4 (c)), only M\textsubscript{a1} works in saturation region to give a small negative current feedback. M\textsubscript{a2} is forced to work in linear region by the two diode transistors, M\textsubscript{a3} and M\textsubscript{a4}. As such, the negative current feedback is reduced. Owing to this weighted control mechanism in the WCF technique, the impedance at the node N\textsubscript{2} as well as the gain of 2\textsuperscript{nd} stage can be dynamically managed. Therefore, with reference to the proposed NCF technique, the advantages are as follows: (i) The gain of the WCF LDO regulator can be maintained reasonably well across the whole I\textsubscript{L} range such that better regulation accuracy can be obtained. (ii) The charging/discharging rate of node N\textsubscript{2} is increased at high I\textsubscript{L}, which results in faster transient speed.

The 3\textsuperscript{rd} gain stage is loaded by a resistor R\textsubscript{X} and a diode transistor M\textsubscript{d1}. The output impedance of this gain stage (R\textsubscript{P} \approx (1/g\textsubscript{md1})//R\textsubscript{X}, g\textsubscript{md1} represents the transconductance of M\textsubscript{d1}) reduces when I\textsubscript{L} increases. In this way, the load provides an adaptive bias when I\textsubscript{L} changes. This subsequently increases the speed of 3\textsuperscript{rd} gain
Turning to the frequency compensation design, a combined frequency compensation scheme using both cascode and Miller compensation techniques is adopted. The dominant pole is mainly formed by the cascode compensation capacitor whilst a small Miller compensation capacitor is utilized to reduce the Q factor of the complex poles.

To complete the LDO architecture, an overshoot reduction block [20] is employed to reduce the overshoot magnitude and the settling time through a momentary discharging current.

III. DYNAMIC IMPEDANCE REDUCTION AND SMALL-SIGNAL MODEL

To investigate the negative current feedback loaded impedance $R_{2f}$ at the node $N_2$, the small-signal model of the WCF LDO regulator is depicted in Fig. 5. It is obtained by breaking the loop at the output node as shown in Fig. 3. In the small-signal model, $g_{mi}$, $R_i$, and $C_i$ have their usual meanings as defined in Section II-A. Particularly, $C_c$ is the cascode compensation capacitor whereas $C_m$ is the Miller compensation capacitor. $C_L$ is the load capacitance which has a value ranging from 470 pF to 10 nF. Refer to Fig. 5, the total gain of the WCF feedback loop ($A_{WCF}$) in Fig. 3 is examined. It is given as

$$A_{WCF} = g_{m3}g_{mf}R_2R_P$$

while $R_{2f}$ can be obtained as

$$R_{2f} = \frac{R_2}{A_{WCF} + 1} = \frac{R_2}{\beta} = \frac{1}{g_{m3}g_{mf}R_P + 1/R_2}$$

By comparing the output impedance $R_2$ (without loaded feedback current source) in (2), $R_{2f}$ is reduced by $\beta$ ($= A_{WCF} + 1$) times. Moreover, $R_{2f}$ can be reduced via either increasing $A_{WCF}$ or decreasing $R_2$. Using this relationship, for the WCF LDO regulator, at low $I_L$, $R_{2f}$ is large since $\beta$ is small and $R_2$ is large. At moderate $I_L$, $R_{2f}$ is significantly reduced with respect to $R_2$ due to a large $\beta$. At high $I_L$, since $R_2$ is already small due to a large current flowing in the 2nd gain stage, $R_{2f}$ is small even with a small $\beta$. 
Turning to the transfer function of the whole LDO regulator, it is derived using the following assumptions: (i) \( C_1, C_2 << C_m << C_c << C_L \); (ii) \( g_{m1}R_1, g_{m2}R_2 >> 1 \), (iii) the input resistance at the cascode compensation node, is approximately equal to \( 1/g_{mc} \) and (iv) \( R_P \) is inversely proportional to \( I_L \). Finally, the open-loop transfer function is obtained as follows:

\[
A_{op}(s) = \frac{v_{\text{fbout}}}{v_{\text{fbin}}} = -\frac{A_{DC} \left( 1 + s \frac{C_c}{g_{mc}} \right)}{1 + as + bs^2 + cs^3 + ds^4 + es^5}
\]  

(3)

where

\[
a = C_L R_O + C_c g_{m2} g_{m3} g_{mp} R_1 R_2 R_P R_O / \beta
\]

(4)

\[
b = C_m C_L R_1 R_O + C_c C_m g_{m2} g_{m3} g_{mp} R_1 R_2 R_P R_O / (\beta g_{mc})
\]

(5)

\[
c = C_m C_L R_1 (C_c / g_{mc} + C_P R_P / \beta)
\]

(6)

\[
d = C_c C_m C_P C_L R_1 R_P R_O / (\beta g_{mc})
\]

(7)

\[
e = C_c C_m C_P C_L R_1 R_2 R_P R_O / (\beta g_{mc})
\]

(8)

The DC loop gain of the LDO regulator is given by

\[
A_{DC} = g_{m1} g_{m2} g_{m3} g_{mp} R_1 R_2 R_P R_O / \beta
\]

(9)

which indicates that the loop gain is reduced by a factor of \( \beta \). This correlates well with the impedance reduction at the node \( N_2 \), namely, \( R_{2f} = R_2 / \beta \). Besides, the cascode compensation generates a zero which is expressed as

\[
z = -g_{mc} / C_c
\]

(10)

Using (3) – (10), the stability of LDO regulator can be analyzed in the following Section.
IV. STABILITY ANALYSIS AND WCF DESIGN STRATEGY

The design strategy of the WCF technique is on the basis of the stability of the LDO regulator at different $C_L$ and $I_L$ conditions. Firstly, using the Routh–Hurwitz stability criterion, the required $\beta$ (denoted as $\beta_{RH}$) can be obtained. Secondly, under a PM of $45^\circ$ constraint, the required $\beta$ (denoted as $\beta_{PM}$) is also investigated. The respective analysis is explained in the following.

As can be seen from (4) to (8), all the parameters in the transfer function are dependent on the value of $C_L R_O$. Besides, $R_O$ is inversely proportional to $I_L$. For this reason, the design strategy for $\beta$ and the stability of the regulator are analyzed in three cases: (I) $C_L R_O$ is large (low $I_L$). The first term in (4) and (5) are dominant. (II) $C_L R_O$ is moderate (low $I_L$). The first term is comparable with second term in (4) and the first term in (5) is dominant. (III) $C_L R_O$ is small (moderate $I_L$ and high $I_L$). The second term in (4) and (5) are dominant. Besides, the term $C_P R_P/\beta$ in (6) is small due to a small $R_P$. Finally, the respective simplified expression for variables from a to e is summarized in Table I.

A. Design Strategy of $\beta$ using Routh–Hurwitz Stability Criterion

Routh–Hurwitz stability criterion has been widely used in the multi-stage amplifier designs [27-29]. It is simply evaluated by constructing the Routh Table A-I in Appendix using the closed-loop transfer function. To achieve stability for the feedback system, the coefficients for $a_0-a_5$ and $b_1$, $c_1$, $d_1$ in the second column of Table A-I must be positive. Since $a_0-a_5$ is always larger than zero, the design condition for $\beta_{RH}$ can be obtained by setting $b_1$, $c_1$ and $d_1$ larger than zero respectively. Refer to the WCF LDO regulator, the closed-loop transfer function can be expressed as

$$A_{cl}(s) = \frac{A_{op}(s)}{1 - A_{op}(s)}$$

where $A_{op}(s)$ is the open-loop transfer function defined in (3). By substituting the approximated expression for variables a to e from Table I as well as (9)–(10) into (11), the Routh table parameter expansion for Large,
Moderate and Small \( C_L R_O \) cases is listed in Appendix Table A-II. Based on these parameters, \( \beta_{RH} \) for each case is analyzed as follows:

Case I): Large \( C_L R_O \) (Low \( I_L \)) condition, the Routh Table parameters are shown in Case I of Table A-II. To meet the stability criterion, the following conditions must be satisfied. They are obtained as

\[
C_m C_L R_O \left( \frac{C_c}{g_{mc}} + \frac{C_P R_P}{\beta_{RH}} \right) > 0 \quad (12)
\]

\[
C_m C_L R_O > 0 \quad (13)
\]

\[
\beta_{RH} > C_c g_{m1} g_{m2} g_{mp} R_L R_P \left( C_L g_{mc} \right) \quad (14)
\]

For (12) and (13), it is obviously valid for any \( \beta_{RH} \). As for (14), the right hand side term is inversely proportional to \( C_L \). Due to \( C_L \) ranges from 470 pF to 10 nF, \( \beta_{RH} \) is small in this case.

Case II): Moderate \( C_L R_O \) (Low \( I_L \)) condition, when \( C_L R_O \) is equal to the cascode compensation term, namely,

\[
C_c g_{m2} g_{m3} g_{mp} R_L R_P R_O / \beta = C_L R_O \quad (15)
\]

the Routh Table parameters are shown in Case II of Table A-II. As indicated, \( b_1 \) and \( c_1 \) for Case I and Case II are the same. The design conditions for \( b_1 > 0 \) and \( c_1 > 0 \) can still be expressed by (12) and (13), respectively. Since (12) and (13) are always valid for any \( \beta_{RH} \), the condition for \( \beta_{RH} \) to meet the criterion is obtained as

\[
\beta_{RH} > C_P g_{m1} R_P / (2C_c) \quad (16)
\]

From (16), the right hand side term is proportional to \( g_{m1} R_P \). Since the maximum value for \( R_P \) is \( R_X \) as explained in Section II-C, \( \beta_{RH} \) can be made small by proper sizing of \( R_P \) and \( g_{m1} \).

Case III): Small \( C_L R_O \) (Moderate and High \( I_L \)) condition, the Routh Table parameters are shown in Case III of Table A-II. To meet the criterion, the following three design requirements for \( \beta_{RH} \) must be fulfilled. They are expressed as
\[ \beta_{RH} > C_2 g_{m2} g_{m3} g_{mp} R_2^2 R_P / C_L \]  
\[ \beta_{RH} > (1 + g_{m1} / g_{mc}) C_P g_{mc} R_P / C_m \]  
\[ \beta_{RH} > \frac{C_2 g_{m2} g_{m3} g_{mp} R_2^2 R_P}{C_L} + \left(1 + \frac{g_{m1}}{g_{mc}}\right) \frac{C_P g_{mc} R_P}{C_m} \]

If the condition in (19) is met, (17) and (18) will be valid as well, but not vice versa. This suggests that (19) determines the only choice out of three \( \beta_{RH} \) inequalities. As such, at moderate \( I_L \), \( R_2 \) and \( R_P \) are fairly large. \( \beta_{RH} \) is the largest. On the other hand, at high \( I_L \), \( R_P \) is small owing to the large biasing current flowing in the diode transistor \( M_{d1} \) of 3rd gain stage. \( R_2 \) is also small due to the dynamic bias introduced by the WCF block. Thus \( \beta_{RH} \) at high \( I_L \) condition is reduced with respect to that of moderate \( I_L \) case. Based on the above analysis, Table II summarizes the \( \beta_{RH} \) at different \( I_L \) conditions. From this table, it confirms that the WCF should be made strong at moderate \( I_L \) condition whilst weak at both low and high \( I_L \) conditions.

Consider the WCF technique employed in the LDO regulator as discussed in Section II-C, the feedback factor \( \beta \) (\( \beta_{sim} \)) and \( R_{2f} \) are simulated with respect to \( I_L \). As observed in Fig. 6, the WCF technique can significantly reduce \( R_2 \), especially for moderate and high \( I_L \). Moreover, when \( I_L \) increases, \( \beta_{sim} \) increases to around 25 dB at \( I_L = 200 \mu A \) first. Then it drops below 5 dB when \( I_L \) is larger than 10 mA.

To verify that the WCF design can fulfill the stability criterion using numerical examples, the theoretical \( \beta_{RH} \) at different \( I_L \) conditions are calculated using the right hand side term of inequalities (14), (16) and (19). The design parameters and stability verification using theoretical \( \beta_{RH} \) and simulated \( \beta_{sim} \) at \( I_L = 0 \) mA, 1 mA and 50 mA are shown in Table III. It can be seen that the WCF can meet the \( \beta_{RH} \) requirement for all three \( I_L \) conditions for both \( C_L = 470 \) pF and 10 nF.

From the above analysis together with the numerical examples, the WCF technique can provide an appropriate feedback to meet the Routh–Hurwitz stability criterion for low, moderate and high \( I_L \) conditions using the design equations (14), (16) and (19). As a result, these equations provide the design guidelines for the amount of feedback in the WCF at different \( I_L \) conditions.
B. Pole and Zero Locations

The feedback factor $\beta$, $R_{2f}$, poles, zero and related parameters for Large, Moderate and Small $C_L R_O$ cases are presented in Table IV. Owing to the cascode compensation, a LHP zero, expressed in (10), is generated. It locates outside the $\omega_{UGF}$. Hence, it will not jeopardize the settling time of LDO regulator. The pole location analysis for each case is discussed as follows:

Case I − Large $C_L R_O$ (Low $I_L$): At this condition, $C_L R_O$ forms the low frequency dominant pole ($p_{-3dB} = 1/(C_L R_O)$). Since $p_{-3dB}$ locates at a very low frequency, the $\omega_{UGF}$ is small. At this juncture, all the parasitic poles locate at relative high frequencies. The LDO regulator can maintain a stable operation. This is the reason why a weak negative current feedback is designed. It also correlates well with the Routh–Hurwitz stability criterion analysis in Section IV-A that at large $C_L R_O$ condition, $\beta_{RH}$ is small using (14). In view of the weak feedback, the regulator’s gain and speed do not significantly change with respect to the regulator without WCF technique.

Case II − Moderate $C_L R_O$ (Low $I_L$): In this case, $p_{-3dB}$ is constituted by both the $C_L R_O$ and the cascode compensation. The $\omega_{UGF}$ is increased with respect to that in Case I. However, $\omega_{UGF}$ remains small and only half of its maximum value. Besides, $|p_{2,3}|_f$ is also increased slightly when compared to that in Case I. The regulator can achieve a stable operation with a small feedback as suggested in (16).

Case III − Small $C_L R_O$ (Moderate and High $I_L$): In this case, $p_{-3dB}$ is mainly constituted by the cascode compensation. Moreover, though the loop gain of regulator is reduced by a factor of $\beta$ according to (9), the dominant pole $p_{-3dB}$ frequency is increased by the same factor, which yields a constant $\omega_{UGF}$.

For $|p_{2,3}|_f$, they are reduced by a factor of $\sqrt{\beta}$. However, $|p_{2,3}|_f$ can still be designed to be higher than that of $\omega_{UGF}$ through choosing a proper value of $g_{mc}$ and $C_c$. This can be derived as

$$\frac{g_{m2} g_{m3} g_{mp} g_{mc} R_2 R_p}{\beta C_m C_L} \geq \frac{g_{m1}}{C_c}$$

which can be rewritten in a form of
$$C_c \sqrt{g_{mc}} \geq g_{m1} \sqrt{\frac{\beta C_m C_L}{g_{m2} g_{m3} g_{mp} R_s R_P}}$$

From (21), both $C_c$ and $g_{mc}$ can be increased to ensure that $|p_{2,3}|_f$ locates at a higher frequency than $\omega_{UGF}$. By substituting the design parameters from Table III with $I_L = 1$ mA to (21) as a numerical example, the calculated minimum $C_c$ is 1.5 pF. Since the designed $C_c$ is 3.5 pF in the WCF LDO regulator, it is more than the required theoretical minimum value.

Of another important design consideration, besides the WCF LDO regulator can fulfill the Routh–Hurwitz stability criterion as discussed in Case III of Section IV-A, the open-loop transfer function (3) generates a high frequency LHP complex pole pair $|p_{4,5}|_f$. Its frequency is multiplied by a factor of $\sqrt{\beta}$ with respect to that without the feedback. In this way, at moderate $I_L$, due to a large $\beta$, the complex pole pair $|p_{4,5}|_f$ is located far away from the $\omega_{UGF}$. At high $I_L$, though $\beta$ is reduced with reference to that at moderate $I_L$, due to very small $R_P$ and $R_2$, the complex pole pair $|p_{4,5}|_f$ still locates at a much higher frequency than $\omega_{UGF}$. This confirms the stability of LDO regulator at both moderate and high $I_L$ conditions.

Fig. 7 depicts the simulated open-loop gain and phase of the WCF LDO regulator for $C_L = 470$ pF and $C_L = 10$ nF under different $I_L$ conditions. It can be seen that the LDO regulator can provides a stable operation for both $C_L$ corners. Fig. 8 shows the phase margin (PM) and gain margin (GM) for $C_L = 470$ pF, 1 nF, 3.3 nF and 10 nF when sweeping $I_L$. The regulator achieves a minimum PM of 45° and a minimum GM of 11 dB. This has illustrated that the WCF LDO regulator is stable under different $C_L$ and $I_L$ combinations. The detailed explanation is discussed in next Section.

C. Phase Margin under $C_L$ and $I_L$ Variations

Refer to the PM plot in Fig. 8, for $C_L = 3.3$ nF and 10 nF, when the $I_L$ increases from 1 $\mu$A to 50 mA, the PM of LDO regulator initially decreases to around 45° and then it increases until to 100°. This observation stems from different dominant factors between $C_L R_O$ and cascode compensation term in the formation of $p_{3dB}$. The analysis can also be partitioned into three regions: (i) Under large $C_L R_O$ condition (low $I_L$), due to a small $\omega_{UGF}$, $|p_{2,3}|_f$ is located at a much higher frequency than $\omega_{UGF}$, resulting in a large PM. (ii) when $I_L$
increases, \( R_O \) reduces. The continual reduction of \( C_L R_O \) increases the \( \omega_{UGF} \). Since the load current \( I_L \) is still low, the \( |p_{2,3}|_f \) is almost constant based on Table IV. As such, the PM of regulator will keep reducing as the \( \omega_{UGF} \) becomes closer to the \( |p_{2,3}|_f \). When \( C_L R_O \) becomes moderate and comparable with cascode compensation term, the PM is approaching to the vicinity of the minimum point. (iii) When \( I_L \) further increases, \( C_L R_O \) becomes small. On the contrary, the cascode compensation becomes the dominant term, leading to the fixed \( \omega_{UGF} (=g_{m1}/C_c) \). Besides, \( |p_{2,3}|_f \) will increase since \( g_{mp} \) becomes larger. This will cause the PM to rise when \( I_L \) increases.

Based on the analysis in Section IV-A, if the feedback factor \( \beta \) can meet the design conditions stated in (14), (16) and (19) according to the Routh–Hurwitz stability criterion, \( |p_{4,5}|_f \) will be pushed to a much higher frequency than \( \omega_{UGF} \). This indicates that \( |p_{4,5}|_f \) will not influence the PM of regulator. With this assumption, the PM of feedback system can be approximated as

\[
PM \approx 90^\circ - \tan^{-1} \left( \frac{\omega_{UGF}}{Q_{p2,3}|_f \left| p_{2,3}|_f \right| 1 - \left( \frac{\omega_{UGF}}{|p_{2,3}|_f} \right)^2} \right) + \tan^{-1} \left( \frac{\omega_{UGF}}{\omega_{UGF}} \right)
\]

(22)

based on [15]. Through substituting the respective expression of \( \omega_{UGF} \), \( |p_{2,3}|_f \) and \( Q_{p_{4,5}} \) for Large, Moderate and Small \( C_L R_O \) case in Table IV into (22), the design requirement of \( \beta_{PM} \) for a minimum PM of 45\(^\circ\) is analyzed as follows:

*Case (I) – Large \( C_L R_O \) (Low \( I_L \)):* To achieve a PM \( \geq 45^\circ \), the necessary condition for \( \beta_{PM} \) to be satisfied is

\[
\beta_{PM} \geq \frac{C_m g_{m1} g_{m2} g_{mp} R_P^2 R_2 R_3}{C_L}
\]

(23)

From (23), the minimum \( \beta_{PM} (\beta_{PM_{min}}) \) is inversely proportional to \( C_L \). A large \( C_L \) gives a smaller \( \omega_{UGF} \) which subsequently reduces the feedback requirement for a PM \( \geq 45^\circ \).

*Case (II) – Moderate \( C_L R_O \) (Low \( I_L \)):* In the situation when the cascode compensation effect equals to
that of $C_L R_O$ as defined in (15), for a larger $C_L$, a larger $g_{mp}$ (implying a large $I_L$) in the left hand side is needed to balance the right hand side term. This implies that the minimum PM region will shift slightly to the right when $C_L$ increases in context of whole current range (0–50mA). This is consistent with the observation of PM plot in Fig. 8. For a $PM \geq 45^\circ$, it suggests that $\beta_{PM}$ should fulfill the condition of

$$\beta_{PM} \geq \frac{C_m C_{pm_{max}} g_{mp}^2 R_p R_P}{8C_c^2 - (2 + \frac{g_{m1}}{g_{mc}}) C_m C_c g_{m1} R_1}$$

(24)

It is noted that $I_L$ is still small even the balance point for (15) shifts to a higher $I_L$ when $C_L$ increases. Moreover, since $R_P$ is inversely proportional to $\sqrt{I_L S_{di} / S_p}$, where $S_p$ and $S_{d1}$ is the respective aspect ratio of the power transistor $M_p$ and the driving transistor $M_{d1}$ in Fig. 3, $R_P$ is approximated independent of $C_L$. As a result, $\beta_{PM_{min}}$ is almost constant for the whole $C_L$ range from (24).

Case (III) – Small $C_L R_O$ (Moderate and High $I_L$): As indicated in small $C_L R_O$ case of Table IV, $|p_{2,3}|$ is inversely proportional to $\sqrt{\beta}$. $\beta$ should not be made too large to achieve a minimum $45^\circ$ PM. The condition for the $\beta_{PM}$ becomes

$$\beta_{PM} \leq \frac{C_c^2 g_{mc} g_{m2} g_{m3} g_{mp} R_L R_P}{C_m C_L g_{m1}^2}$$

(25)

From (25), the maximum allowable $\beta_{PM}$ ($\beta_{PM_{max}}$) is proportional to $g_{mp}$. At small $C_L R_O$ case, due to large $I_L$, $g_{mp}$ becomes large, contributing a large $\beta_{PM_{max}}$. On the other hand, due to the WCF technique to reduce the feedback at large $I_L$, the design $\beta$ is not easily exceeding the $\beta_{PM_{max}}$ defined in (25).

To verify the WCF regulator can fulfill the design requirement for a minimum $45^\circ$ PM using numerical examples, $\beta_{PM_{min}}$ or $\beta_{PM_{max}}$ at different $C_L R_O$ conditions are calculated using (23)–(25). Using the design parameters in Table III, the theoretical $\beta_{PM}$ and simulated $\beta_{sim}$ for Large, Moderate and Small $C_L R_O$ are shown in Table V. It is noted that (i) for Large $C_L R_O$ case, $\beta_{PM}$ at $C_L = 3.3$ nF and $I_L = 0$ mA is chosen. This is because, for $C_L = 470$ pF and $1$ nF, the $C_L R_O$ is already comparable with the cascode compensation term at $0$ mA. For $10$ nF, $\beta_{PM_{min}}$ is smaller than that of $3.3$ nF. (ii) For Moderate $C_L R_O$ case, the worst $\beta_{PM}$ at $C_L = 470$
pF, I_L = 0 mA is chosen. (iii) For Small C_LR_O cases, the β_PM at C_L = 10 nF, I_L = 50 mA is chosen. The results in Table V indicate that the WCF can meet the β requirement for all three C_LR_O conditions. A minimum PM of 45° can be achieved as depicted in Fig. 8.

In brief, for regulator’s stability, both the Routh–Hurwitz stability criterion and 45° minimum PM should be fulfilled. By combining the design equations of β_RH in Section IV-A [(14), (16) and (19)] and β_PM in Section IV-C [(23)-(25)], the required β is summarized in Table VI. For large C_LR_O case (Case I), since C_mR_1 >> C_v/gmc, if the condition in (23) is met, (14) is valid as well. This suggests that (23) is the only choice for design inequalities. For Moderate C_LR_O case (Case II), the design equations are decided by (16) and (24) together. For small C_LR_O case (Case III), (19) and (25) gives the lower and upper bound for β_req respectively. Using this table, the design guidelines for β is investigated in details.

D. Combined Frequency Compensation and Q-Factor

The WCF LDO regulator employs a combined cascode and Miller compensation. The dominant pole is determined by the cascode compensation since it can push the non-dominant pole to a higher frequency under a large capacitive load in comparison to the Miller compensation counterpart [30]. Unfortunately, the cascode compensation easily gives gain peaking due to a large parasitic Q factor [31]. To overcome the drawback, a small Miller compensation capacitor is added in this LDO regulator so as to reduce the Q factor. This is mainly because the Q factor value is inversely proportional to the capacitance at the Miller node [31]. This can also be validated from the Q factor expressions shown in Table IV. At moderate and high I_L conditions (cascode compensation dominating), \( Q_{\mu,L} \) is inversely proportional to \( \sqrt{C_m} \). A large C_m contributes to a small Q factor which can reduce the peaking effect arising from the complex pole pair \( |p_{2,3}|_f \). However, a large C_m will reduce the frequency of \( |p_{2,3}|_f \) as well. This will jeopardize the cascode effect. For this reason, C_m is designed to be small (0.3 pF) in the WCF topology to achieve a reasonable Q factor whilst keeping the complex pole pair \( |p_{2,3}|_f \) locating outside the \( \omega_{UGF} \).
E. Minimum \( C_L \) for a Stable Operation

As discussed in Section IV−C, when \( C_L \) is 470 pF, the dominant pole is in moderate \( C_LR_O \) region at \( I_L = 0 \) mA. If the minimum \( C_L \) (470 pF) is further reduced to a smaller value, the dominant pole will directly move into small \( C_LR_O \) region. To meet the Routh−Hurwitz stability criterion, \( \beta_{RH} \) must fulfill (19) across the whole \( I_L \) range. However, at small \( I_L \), \( R_P \) becomes large and it causes a large \( \beta_{RH} \). This suggests that a very strong feedback is needed at low \( I_L \) to ensure stable operation. The gain as well as the speed of regulator will be limited. Based on the tradeoff design considerations, the output \( C_L \) of regulator is preferably to start from the mid-range capacitive load (470 pF) onwards.

V. Circuit Implementation

The complete schematic implementation of the WCF LDO regulator is shown in Fig. 9. It utilizes a folded cascode error amplifier constituted by \( (M_0 - M_8) \) as the first gain stage. The 2\(^{nd}\), 3\(^{rd}\) inverting gain stages and the WCF block employ the same structure as that in Fig. 3. The overshoot reduction block (\( C_B, R_B, M_{13} \)) senses the voltage swing at the node \( N_P \) and generates a momentary sinking current to reduce the overshoot magnitude as well as settling time of the LDO regulator.

To compare the regulation accuracy and the speed of LDO regulator with the two proposed NCF and WCF technique, a NCF LDO regulator is also built and simulated. In the NCF LDO regulator, \( M_{a3} \) in the WCF block (Fig. 9) is removed. In such an arrangement, both \( M_{a1} \) and \( M_{a2} \) are working in saturation region. This turns out that the negative current feedback is kept strong for both moderate and high \( I_L \) conditions. Fig. 10 shows the exemplary transient responses of the LDO regulator with NCF and WCF technique. Under the same load current switching condition, it can be observed that the WCF LDO regulator displays a 1.5 times smaller undershoot and overshoot, and an approximate 2 times better load regulation with respect to the NCF LDO regulator. This has demonstrated that the WCF technique addresses the limitations of the NCF and achieves a better optimization of stability, accuracy and speed.
VI. EXPERIMENTAL RESULTS AND DISCUSSIONS

The WCF LDO regulator is implemented using UMC 65-nm CMOS process. The microphotograph of the WCF regulator is shown in Fig. 11. Excluding the supply and output PADs as connectors, the active area is 0.0133 mm². At a 0.75 V supply voltage, the WCF LDO regulator can support a maximum $I_L$ of 50 mA. The dropout voltage is less than 0.2 V at full $I_L$. The measured quiescent current at zero $I_L$ is 15.9 µA. With a total 3.8 pF (3.5 pF cascode + 0.3 pF Miller) compensation capacitance, the WCF LDO regulator offers stable operation for $C_L$ ranging from 470 pF to 10 nF across the whole $I_L$ range.

It is noted that external reference voltages are used and off-chip capacitors are added to model the $C_L$ of the LDO regulator during the measurement. To test the stability and the performance of the WCF LDO regulator for the whole $C_L$ range, four standard load capacitors (470 pF, 1 nF, 3.3 nF and 10 nF) have been chosen. In addition, two $I_L$ switching cases (0 to maximum $I_L$ and 1 mA to maximum $I_L$) are used to measure the transient performance.

Fig. 12 shows the measured load transient responses of the WCF LDO regulator when $I_L$ is switching from 0 to 50 mA with an edge time of 100 ns for all four load capacitors. The supply voltage is 0.75 V and the output voltage is 0.55 V. As can be observed from the graphs, the undershoots are 113 mV, 109 mV, 98 mV, 72 mV whereas the overshoots are 29 mV, 29 mV, 27 mV, 32 mV for $C_L = 470$ pF, 1 nF, 3.3 nF and 10 nF, respectively. The undershoot becomes smaller when the load capacitor increases. This is because a larger capacitor is able to absorb a larger transient current during $I_L$ switching. In addition, due to the intelligent control of WCF and the high speed property of the 3rd gain stage, the settling time of the WCF LDO regulator is quite small. The measured settling time are 248 ns, 244 ns, 252 ns and 368 ns for $C_L = 470$ pF, 1 nF, 3.3 nF and 10 nF, respectively.

Fig. 13 shows the transient responses of the WCF LDO regulator at $V_{DD} = 0.75$ V when $I_L$ switches from 1 mA to 50 mA (vice versa) for $C_L = 470$ pF and $C_L = 10$ nF. Using an identical edge time of 100 ns, the undershoots are substantially reduced when compared with that of $I_L$ switching from 0 to 50 mA. The undershoots are 24 mV, 35 mV whereas the overshoots are 24 mV, 29 mV for $C_L = 470$ pF and 10 nF,
respectively.

Fig. 14 depicts the transient responses of the WCF LDO regulator at $V_{DD} = 1.2$ V when $I_L$ switches from 1 mA to 50 mA (vice versa) for $C_L = 470$ pF and $C_L = 10$ nF. The LDO regulator works well for a 1.2 V supply. Comparing the results with $V_{DD} = 0.75$ V in Fig. 13, the undershoot and overshoot increase by around 15 mV. This is because, at $V_{DD} = 1.2$ V, the large feedback transistor $M_{a2}$ turns off at a higher $I_L$ than that of $V_{DD} = 0.75$ V. Therefore, the regulator’s speed at high $I_L$ is slightly reduced. This leads to a small amount increment for transient undershoot and overshoot.

The line transient response at $I_L = 1$ mA, $C_L = 470$ pF is depicted in Fig. 15. When $V_{DD}$ switches from 0.75 V to 1.2 V with a 10 µs edge time, the maximum output voltage spike is 4.3 mV and the introduced increment of error voltage is only 1.8 mV. Fig. 16 shows the measured power supply rejection (PSR) of the WCF LDO regulator for $C_L = 470$ pF at different $I_L$ conditions. At 1 kHz, it can be seen that the minimum PSR is around -44 dB when $I_L = 1$ mA. This is due to some loop gain reduction from the large feedback ($\beta$) as revealed in (9). At full $I_L$, the regulator achieves a PSR of -51 dB. Fig. 17 depicts the measured output noise response of the LDO regulator at $C_L = 470$ pF and $I_L = 0$ mA. It can be seen that the noise is -98.7 dBm/Hz ($2.6 \mu V/\sqrt{Hz}$) at 100 Hz and -105 dBm/Hz ($1.25 \mu V/\sqrt{Hz}$) at 100 kHz, respectively.

Performance comparison between the WCF LDO regulator and the other reported state-of-the-art OCL-LDO regulators is presented in Table VII. With the WCF circuit technique, the LDO regulator achieves good performance metrics with an additional merit to drive a wide $C_L$ range. To compare the load transient performance, the OCL-LDO regulator figure-of-merit (FOM) [14] is adopted. This is given by

$$FOM = K \left( \frac{\Delta V_{OUT} \times I_0}{\Delta I_{OUT}} \right)$$

where $K$ is the edge time ratio and defined as

$$K = \frac{\Delta t \text{ used in the measurement}}{\text{the smallest } \Delta t \text{ among the designs for comparison}}$$
To provide a comparison, all the results in Table VII for the WCF LDO regulator is based on $C_L = 470$ pF which is regarded as the closer load capacitance value with respect to the reported works. Since some of the designs [13, 14, 16] were tested using some amount of minimum loading currents, two FOMs of the WCF LDO regulator are used for comparison. The first FOM (left column) represents the performance metric for $I_L$ switching from 0 to 50 mA while the second FOM (right column) represents the performance metric for $I_L$ switching from 1 mA to 50 mA.

As can be seen from Table VII, the WCF LDO regulator design achieves a comparable or better FOM with respect to the reported OCL-LDO regulators. In addition, it can drive a wide $C_L$ range with fast settling time and good performance metrics such as load regulation, line regulation and PSR.

**VII. CONCLUSION**

A weighted current feedback (WCF) technique is proposed in this paper. It establishes a weighted negative current feedback loop and provides an adaptive bias to the inter-gain stage. This permits smart management of the output impedance and gain of the inter-gain stage. As a result, using the WCF circuit technique and Routh–Hurwitz stability criterion to devise the design strategy to access the stability, the regulator can achieve stable operation, high accuracy and fast response simultaneously with small quiescent power consumption.

Validated by UMC 65-nm CMOS process, the simulation and measurement results have demonstrated that the WCF technique can stabilize the LDO regulator for load capacitance ranging from 470 pF to 10 nF whilst maintaining a very good transient performance metrics. The WCF regulator design reaches a comparable or better FOM with respect to the reported OCL-LDO regulators. Therefore, the WCF LDO regulator topology is useful for fully on-chip applications with wide load capacitance range.

**VIII. ACKNOWLEDGMENT**

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REFERENCES


APPENDIX

For the denominator of the closed-loop transfer function in (11), it can be represented as

\[
D(s) = A_{DC} + \left( \frac{A_{DC} C_s}{g_{mc}} + a \right) s + b s^2 + c s^3 + d s^4 + e s^5
\]  

(I-1)

\[
= a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5
\]

The Routh Table is constructed and shown Table A-I. For stable system, it should not have RHP poles. This requires the coefficients for \(a_0\)– \(a_5\), \(b_1\), \(c_1\), \(d_1\) to be positive. By substituting the approximated expression for variables a to e in Table I and (9)–(10) into (I-1), the Routh table parameters for large \(C_{LR_o}\) (Case I), Moderate \(C_{LR_o}\) (Case II) and Small \(C_{LR_o}\) (Case III) are listed in Table A-II.
Figure Captions:
Fig. 1. Conventional multi-gain stages in a LDO regulator.
Fig. 2. Proposed Negative Current Feedback (NCF) topology embedded in multi-gain stages in a LDO regulator.
Fig. 3. A LDO regulator architecture using the WCF technique.
Fig. 4. Simplified schematic structure of 2nd, 3rd gain stages and WCF for (a) low \( I_L \), (b) moderate \( I_L \), and (c) high \( I_L \).
Fig. 5. Small-signal model of the WCF LDO regulator architecture.
Fig. 6. Simulated \( \beta (\beta_{\text{sim}}) \) and \( R_{2f} \) at different \( I_L \) conditions.
Fig. 7. Simulated open-loop gain and phase at different \( I_L \) for (a) \( C_L = 470 \) pF and (b) \( C_L = 10 \) nF at \( V_{DD} = 0.75 \) V.
Fig. 8. Simulated phase margin (PM) and gain margin (GM) for \( C_L = 470 \) pF, 1 nF, 3.3 nF and 10 nF when sweeping \( I_L \) at \( V_{DD} = 0.75 \) V.
Fig. 9. Schematic of the WCF LDO regulator.
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Fig. 11. Microphotograph of the WCF LDO regulator.
Fig. 12. Measured load transient responses with \( V_{DD} = 0.75 \) V, \( V_{OUT} = 0.55 \) V for (a) \( C_L = 470 \) pF, (b) \( C_L = 1 \) nF, (c) \( C_L = 3.3 \) nF and (d) \( C_L = 10 \) nF.
Fig. 13. Measured load transient responses at \( V_{DD} = 0.75 \) V with \( I_L \) switching from 1 mA to 50 mA (vice versa) for (a) \( C_L = 470 \) pF and (b) \( C_L = 10 \) nF.
Fig. 14. Measured load transient responses at \( V_{DD} = 1.2 \) V with \( I_L \) switching from 1 mA to 50 mA (vice versa) for (a) \( C_L = 470 \) pF and (b) \( C_L = 10 \) nF.
Fig. 15. Measured line transient response at \( I_L = 1mA \) and \( C_L = 470 \) pF.
Fig. 16. Measured PSR at \( C_L = 470 \) pF for different \( I_L \).
Fig. 17. Measured output noise at \( C_L = 470 \) pF with 0 mA \( I_L \).

Table Captions:
TABLE I: APPROXIMATED VARIABLES FROM a to e FOR LARGE, MODERATE AND SMALL \( C_L R_O \) CASES
TABLE II: SUMMARY OF THE REQUIRED \( \beta_{RH} \) AT DIFFERENT \( I_L \) CONDITIONS TO MEET ROUTH–HURWITZ CRITERION
TABLE III: DESIGN PARAMETERS, STABILITY VERIFICATION USING THEORETICAL \( \beta_{RH} \) AND SIMULATED \( \beta_{\text{sim}} \)
TABLE IV: FEEDBACK FACTOR \( \beta \), POLES, ZERO, Q-FACTOR, \( \omega_{UGF} \) FOR LARGE, MODERATE AND SMALL \( C_L R_O \) CASES
TABLE V: NUMERICAL EXAMPLE FOR PM VERIFICATION USING THEORETICAL \( \beta_{PM} \) AND SIMULATED \( \beta_{\text{sim}} \)
TABLE VI: COMBINED \( \beta \) DESIGN INEQUALITIES USING \( \beta_{RH} \) AND \( \beta_{PM} \)
TABLE VII: PERFORMANCE COMPARISON WITH THE REPORTED OCL-LDO REGULATORS
TABLE A-I: ROUTH TABLE FOR A 5TH ORDER POLYNOMIAL
TABLE A-II: ROUTH TABLE PARAMETER EXPANSION FOR THE WCF LDO REGULATOR CLOSED-LOOP TRANSFER FUNCTION IN (11)
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Fig. 2. Proposed Negative Current Feedback (NCF) topology embedded in multi-gain stages in a LDO regulator.
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Fig. 4. Simplified schematic structure of 2nd, 3rd gain stages and WCF for (a) low \(I_L\), (b) moderate \(I_L\), and (c) high \(I_L\).
Fig. 5. Small-signal model of the WCF LDO regulator architecture.

Fig. 6. Simulated $\beta$ ($\beta_{\text{sim}}$) and $R_{2f}$ at different $I_L$ conditions.
Fig. 7. Simulated open-loop gain and phase at different $I_L$ for (a) $C_L = 470$ pF and (b) $C_L = 10$ nF with $V_{DD} = 0.75$ V.

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Fig. 15. Measured line transient response at $I_L = 1\ mA$ and $C_L = 470\ \text{pF}$.

Fig. 16. Measured PSR at $C_L = 470\ \text{pF}$ for different $I_L$.

Fig. 17. Measured Output Noise at $C_L = 470\ \text{pF}$ with $0\ mA\ I_L$. 
TABLE I
APPROXIMATED VARIABLES FROM a TO e FOR LARGE, MODERATE
AND SMALL C_L R_O CASES

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TABLE II
SUMMARY OF THE REQUIRED \( \beta_{RH} \) AT DIFFERENT I_L CONDITIONS
TO MEET ROUTH–HURWITZ CRITERION

<table>
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<th>Parameter</th>
<th>Low I_L</th>
<th>Moderate I_L</th>
<th>High I_L</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_s, R_2 )</td>
<td>Large</td>
<td>Moderate</td>
<td>Small</td>
</tr>
<tr>
<td>( g_{m1}, g_{m2} )</td>
<td>Small</td>
<td>Moderate</td>
<td>Large</td>
</tr>
<tr>
<td>( \beta_{RH} ) design eqn.</td>
<td>(14) and (16)</td>
<td>(19)</td>
<td>(19)</td>
</tr>
<tr>
<td>( \beta_{RH} ) value</td>
<td>Small</td>
<td>Large</td>
<td>Small</td>
</tr>
</tbody>
</table>

TABLE III
DESIGN PARAMETERS, STABILITY VERIFICATION USING
THEORETICAL \( \beta_{RH} \) AND SIMULATED \( \beta_{sim} \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>I_L = 0 mA</th>
<th>I_L = 1 mA</th>
<th>I_L = 50 mA</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g_{m1} ) (μS)</td>
<td>51</td>
<td>51</td>
<td>51</td>
</tr>
<tr>
<td>( g_{m2} ) (μS)</td>
<td>24</td>
<td>357</td>
<td>755</td>
</tr>
<tr>
<td>( g_{m3} ) (μS)</td>
<td>71</td>
<td>452</td>
<td>5290</td>
</tr>
<tr>
<td>( g_{mp} ) (μS)</td>
<td>31</td>
<td>2.03e4</td>
<td>3.1e5</td>
</tr>
<tr>
<td>( R_1 ) (kΩ)</td>
<td>131</td>
<td>131</td>
<td>131</td>
</tr>
<tr>
<td>( R_2 ) (kΩ)</td>
<td>617</td>
<td>617</td>
<td>617</td>
</tr>
<tr>
<td>( R_v ) (kΩ)</td>
<td>342</td>
<td>22.2</td>
<td>6.46</td>
</tr>
<tr>
<td>( C_c ) (pF)</td>
<td>3.5</td>
<td>3.5</td>
<td>3.5</td>
</tr>
<tr>
<td>( C_m ) (pF)</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>( C_p ) (μF)</td>
<td>26</td>
<td>31</td>
<td>30</td>
</tr>
<tr>
<td>( C_r ) (pF)</td>
<td>2.58</td>
<td>3.22</td>
<td>3.09</td>
</tr>
</tbody>
</table>

Theoretical \( \beta_{RH} \) and Simulated \( \beta_{sim} \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \beta_{RH} ) design eqn.</th>
<th>( \beta_{RH} ) (dB)</th>
<th>( \beta_{sim} ) (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{bc} )</td>
<td>(16)</td>
<td>470 pF</td>
<td>10 nF</td>
</tr>
<tr>
<td>( \beta_{RH} ) value</td>
<td>10.6</td>
<td>-29.4</td>
<td>19.5</td>
</tr>
<tr>
<td>( \beta_{sim} ) (dB)</td>
<td>6</td>
<td>22</td>
<td>3.3</td>
</tr>
</tbody>
</table>

\( \beta_{sim} > \beta_{RH} \) for stability | Yes | Yes | Yes |
**TABLE IV**
FEEDBACK FACTOR $\beta$, POLES, ZERO, Q-FACTOR, $\omega_{UGF}$ FOR LARGE, MODERATE AND SMALL $C_L R_O$ CASES

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case I: Large $C_L R_O$</th>
<th>Case II: Moderate $C_L R_O$</th>
<th>Case III: Small $C_L R_O$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feedback</td>
<td>Low $I_L$</td>
<td>Moderate $I_L$</td>
<td>High $I_L$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$g_{m}S_{m}R_{r}R_{p}+1$</td>
<td>$S_{m}S_{m2}S_{m2}R_{r}R_{r}R_{r}/\beta$</td>
<td>$-g_{m}/C_{r}$</td>
</tr>
<tr>
<td>$A_{DC}$</td>
<td>$-1/(C_{r}R_{p})$</td>
<td>$-1/(2C_{r}R_{p})$</td>
<td>$-\beta/(C_{r}S_{m}S_{m}R_{r}R_{r}R_{r})$</td>
</tr>
<tr>
<td>$p_{z1}$</td>
<td>$\sqrt{\beta g_{m}[(\beta C_{r}+C_{r}S_{m}R_{r})C_{r}R_{r}]}$</td>
<td>$\sqrt{2\beta g_{m}[(\beta C_{r}+C_{r}S_{m}R_{r})C_{r}]}$</td>
<td>$\beta(C_{r}C_{r}R_{r}R_{r})/(\beta C_{r}C_{r})$</td>
</tr>
<tr>
<td>$p_{z1}$</td>
<td>$\sqrt{\beta C_{r}+C_{r}S_{m}R_{r})}/(C_{r}C_{r}R_{r}R_{r})$</td>
<td>$\sqrt{\beta C_{r}+C_{r}S_{m}R_{r})}/(C_{r}R_{r}R_{r})$</td>
<td>$\sqrt{\beta C_{r}R_{r})/(C_{r}R_{r})}$</td>
</tr>
<tr>
<td>$Q_{Pz1}$</td>
<td>$\sqrt{(\beta C_{r}+C_{r}S_{m}R_{r})C_{r}R_{r}}/\beta (C_{r}R_{r})$</td>
<td>$\sqrt{(\beta C_{r}+C_{r}S_{m}R_{r})C_{r}R_{r}}/\beta (C_{r}R_{r})$</td>
<td>$\sqrt{(\beta C_{r}R_{r})/(C_{r}R_{r})}$</td>
</tr>
<tr>
<td>$\omega_{UGF}$</td>
<td>$g_{m}S_{m}S_{m2}S_{m2}R_{r}R_{r}/(2C_{r})$</td>
<td>$g_{m}/(2C_{r})$</td>
<td>$g_{m}/C_{r}$</td>
</tr>
</tbody>
</table>

**TABLE V**
NUMERICAL EXAMPLE FOR PM VERIFICATION USING THEORETICAL $\beta_{PM}$ AND SIMULATED $\beta_{sim}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Large $C_L R_O$</th>
<th>Moderate $C_L R_O$</th>
<th>Small $C_L R_O$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_L$</td>
<td>3.3 nF</td>
<td>470 pF</td>
<td>10 nF</td>
</tr>
<tr>
<td>$I_L$</td>
<td>0 mA</td>
<td>0 mA</td>
<td>50 mA</td>
</tr>
<tr>
<td>$\beta_{PM}$ Design Eqn.</td>
<td>(23)</td>
<td>(24)</td>
<td>(25)</td>
</tr>
<tr>
<td>$\beta_{PM}$ (dB)</td>
<td>-2.9</td>
<td>3.3</td>
<td>50.6</td>
</tr>
<tr>
<td>$\beta_{sim}$ (dB)</td>
<td>6</td>
<td>6</td>
<td>3.3</td>
</tr>
<tr>
<td>Criterion For 45° PM</td>
<td>$\beta_{sim} \geq \beta_{PM}$</td>
<td>$\beta_{sim} \geq \beta_{PM}$</td>
<td>$\beta_{sim} \leq \beta_{PM}$</td>
</tr>
<tr>
<td>Meet Criterion</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Simulated PM</td>
<td>63°</td>
<td>52°</td>
<td>97°</td>
</tr>
</tbody>
</table>

**TABLE VI**
COMBINED $\beta$ DESIGN INEQUALITIES USING $\beta_{RH}$ AND $\beta_{PM}$

<table>
<thead>
<tr>
<th>Case</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$\beta \geq C_{r}S_{m}S_{m}S_{m2}S_{m2}R_{r}R_{r}/C_{r}$</td>
</tr>
<tr>
<td>II</td>
<td>$\left[\beta &gt; \frac{C_{r}S_{m}S_{m2}R_{r}}{2C_{r}}\right]$ &amp; $\left[\beta \geq \frac{C_{r}C_{r}S_{m}S_{m2}R_{r}}{8C_{r}^{2}-(2+g_{m}/g_{m})C_{r}C_{r}S_{m}S_{m2}}\right]$</td>
</tr>
<tr>
<td>III</td>
<td>$C_{r}S_{m}S_{m2}S_{m2}R_{r}R_{r}^{2}$ &amp; $\left(1+\frac{g_{m}}{g_{m}}\right)\frac{C_{r}S_{m}S_{m2}R_{r}}{C_{r}} &lt; \beta \leq \frac{C_{r}C_{r}S_{m}S_{m2}S_{m2}R_{r}}{C_{r}C_{r}S_{m}S_{m2}}$</td>
</tr>
</tbody>
</table>

### TABLE VII

**Performance Comparison with the Reported OCL-LDO Regulators**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>[8]</th>
<th>[11]</th>
<th>[13]</th>
<th>[14]</th>
<th>[16]</th>
<th>[17]</th>
<th>[18]</th>
<th>[19]</th>
<th>[20]</th>
<th>[21]</th>
<th>This Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Technology (μm)</td>
<td>0.09</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
<td>0.09</td>
<td>0.09</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
<td>0.065</td>
<td>0.11</td>
</tr>
<tr>
<td>Chip Area (mm²)</td>
<td>0.098</td>
<td>0.12</td>
<td>0.155</td>
<td>0.019</td>
<td>0.005</td>
<td>0.098</td>
<td>0.064</td>
<td>0.4</td>
<td>0.017</td>
<td>0.21</td>
<td>0.0133</td>
</tr>
<tr>
<td>$V_{IN}$ (V)</td>
<td>1.2</td>
<td>3</td>
<td>0.95-1.4</td>
<td>0.75-1.2</td>
<td>1.2</td>
<td>1.2</td>
<td>2.5-4</td>
<td>1.2-1.5</td>
<td>1.2</td>
<td>1.8-3.8</td>
<td>0.75-1.2</td>
</tr>
<tr>
<td>$V_{OUT}$ (V)</td>
<td>0.9</td>
<td>2.8</td>
<td>0.7-1.2</td>
<td>0.5-1</td>
<td>1</td>
<td>1</td>
<td>2.35</td>
<td>1</td>
<td>1</td>
<td>1.6-3.6</td>
<td>0.55</td>
</tr>
<tr>
<td>Dropout Voltage (mV)</td>
<td>300</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>$I_0$ (μA)</td>
<td>6000</td>
<td>65</td>
<td>43</td>
<td>8</td>
<td>408</td>
<td>28-380.1</td>
<td>7</td>
<td>45</td>
<td>0.9-82.4</td>
<td>41.5</td>
<td>15.9* - 487</td>
</tr>
<tr>
<td>$I_{OUT}$ (max) (mA)</td>
<td>100</td>
<td>50</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>50</td>
<td>100</td>
<td>200</td>
<td>50</td>
</tr>
<tr>
<td>Total On-Chip Cap. (pF)</td>
<td>600</td>
<td>21</td>
<td>6</td>
<td>7</td>
<td>1.8</td>
<td>10</td>
<td>7.5</td>
<td>41</td>
<td>4.5</td>
<td>43.2</td>
<td>4.1</td>
</tr>
<tr>
<td>Load Cap. Range (F)</td>
<td>600p</td>
<td>0-100p</td>
<td>0-100p, 1n</td>
<td>0-50p</td>
<td>0-1n</td>
<td>0-100p</td>
<td>0-100p</td>
<td>0-100p</td>
<td>0-100p</td>
<td>0-100p</td>
<td>40p</td>
</tr>
<tr>
<td>Line Reg. (mV/V)</td>
<td>N/A</td>
<td>23</td>
<td>N/A</td>
<td>3.78</td>
<td>4.3</td>
<td>0.39</td>
<td>1</td>
<td>N/A</td>
<td>4.7</td>
<td>8.9</td>
<td>4</td>
</tr>
<tr>
<td>Load Reg. (mV/mA)</td>
<td>1.8</td>
<td>0.56</td>
<td>0.4</td>
<td>0.1</td>
<td>0.003</td>
<td>0.0782</td>
<td>0.08</td>
<td>N/A</td>
<td>0.3</td>
<td>0.108</td>
<td>0.18</td>
</tr>
<tr>
<td>PSR @1kHz (dB)</td>
<td>N/A</td>
<td>-57</td>
<td>N/A</td>
<td>-44</td>
<td>-56</td>
<td>-49.8</td>
<td>N/A</td>
<td>N/A</td>
<td>-58@10kHz</td>
<td>N/A</td>
<td>-51</td>
</tr>
<tr>
<td>Settling Time (μs)</td>
<td>N/A</td>
<td>15</td>
<td>3</td>
<td>5</td>
<td>N/A</td>
<td>N/A</td>
<td>-0.15</td>
<td>-4</td>
<td>6</td>
<td>0.65</td>
<td>0.25</td>
</tr>
<tr>
<td>$I_{L,min}$ (mA) †</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>0.05</td>
<td>1</td>
<td>0</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>$A_{I_{OUT}}$ (mA)</td>
<td>100</td>
<td>50</td>
<td>99</td>
<td>97</td>
<td>100</td>
<td>100</td>
<td>99.95</td>
<td>49</td>
<td>100</td>
<td>199.5</td>
<td>50</td>
</tr>
<tr>
<td>$V_{OUT}$ (mV)</td>
<td>90</td>
<td>90</td>
<td>70</td>
<td>114</td>
<td>35</td>
<td>105</td>
<td>243</td>
<td>70</td>
<td>68.8</td>
<td>385</td>
<td>113</td>
</tr>
<tr>
<td>Edge Time (μs)</td>
<td>0.0001</td>
<td>1</td>
<td>1</td>
<td>0.1</td>
<td>0.01</td>
<td>1</td>
<td>0.5</td>
<td>1</td>
<td>300</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>Edge Time Ratio K</td>
<td>1</td>
<td>10000</td>
<td>10000</td>
<td>1000</td>
<td>100</td>
<td>10000</td>
<td>5000</td>
<td>10000</td>
<td>3000</td>
<td>5000</td>
<td>1000</td>
</tr>
<tr>
<td>FOM</td>
<td>0.0054</td>
<td>1.17</td>
<td>0.304</td>
<td>0.0094</td>
<td>0.014</td>
<td>0.294</td>
<td>0.085</td>
<td>0.643</td>
<td>0.0019</td>
<td>0.4</td>
<td>0.036 0.0079</td>
</tr>
</tbody>
</table>

* Quiescent current includes the current consumption of bias circuit. † The minimum $I_L$ used to test the transient performance. # Estimated area.

### TABLE A-I

**Root Table for a 5th-Order Polynomial**

<table>
<thead>
<tr>
<th>$s^5$</th>
<th>$a_5$</th>
<th>$a_3$</th>
<th>$a_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^4$</td>
<td>$a_4$</td>
<td>$a_2$</td>
<td>$a_0$</td>
</tr>
<tr>
<td>$s^3$</td>
<td>$b_1 = (a_3a_4 - a_2a_3)/a_4$</td>
<td>$b_2 = (a_3a_4 - a_2a_1)/a_4$</td>
<td>0</td>
</tr>
<tr>
<td>$s^2$</td>
<td>$c_1 = (a_3b_1 - a_2b_2)/b_1$</td>
<td>$c_2 = a_3$</td>
<td>0</td>
</tr>
<tr>
<td>$s$</td>
<td>$d_1 = (b_2c_1 - a_2c_2)/c_1$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>$e_1 = a_0$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
### Table A-II

**Routh Table Parameter Expansion For The WCF LDO Regulator Closed-Loop Transfer Function In (11)**

<table>
<thead>
<tr>
<th>Par.</th>
<th>Case I (Large $C_lR_o$)</th>
<th>Case II (Moderate $C_lR_o$)</th>
<th>Case III (Small $C_lR_o$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>$g_{s\alpha}g_{s\alpha}g_{s\alpha}g_{s\alpha}R_RR_R_R_O/\beta$</td>
<td>$g_{s\alpha}g_{s\alpha}g_{s\alpha}g_{s\alpha}R_RR_R_R_O/\beta$</td>
<td>$g_{s\alpha}g_{s\alpha}g_{s\alpha}g_{s\alpha}R_RR_R_R_O/\beta$</td>
</tr>
<tr>
<td>$a_1$</td>
<td>$C_lR_O$</td>
<td>$(2+g_{s\alpha}/g_{s\alpha})C_lR_O$</td>
<td>$(1+g_{s\alpha}/g_{s\alpha})C_lR_O$</td>
</tr>
<tr>
<td>$a_2$</td>
<td>$C_\alpha C_lR_R_O$</td>
<td>$C_\alpha C_lR_R_O$</td>
<td>$C_\alpha C_lR_R_O$</td>
</tr>
<tr>
<td>$a_3$</td>
<td>$C_\alpha C_lR_R_O(C_\alpha/g_{s\alpha} + C_R_r/\beta)$</td>
<td>$C_\alpha C_lR_R_O(g_{s\alpha})$</td>
<td>$C_\alpha C_lR_R_O(g_{s\alpha})$</td>
</tr>
<tr>
<td>$b_1$</td>
<td>$C_\alpha C_lR_R_O(C_\alpha/g_{s\alpha} + C_R_r/\beta)$</td>
<td>$C_\alpha C_lR_R_R_R_O(g_{s\alpha})$</td>
<td>$(C_\alpha - C_\alpha g_{s\alpha}g_{s\alpha}g_{s\alpha}g_{s\alpha}^2 R_R_r/\beta)C_\alpha C_lR_R_O(g_{s\alpha})$</td>
</tr>
<tr>
<td>$b_2$</td>
<td>$C_\alpha C_lR_R_O$</td>
<td>$(2+g_{s\alpha}/g_{s\alpha})C_lR_O$</td>
<td>$(1+g_{s\alpha}/g_{s\alpha})C_lR_O$</td>
</tr>
<tr>
<td>$c_1$</td>
<td>$C_\alpha C_lR_R_O$</td>
<td>$[z C_\alpha/g_{s\alpha} - (1+g_{s\alpha}/g_{s\alpha})C_R_R_r/\beta]C_\alpha C_lR_R_O(g_{s\alpha})/\beta(z)$.</td>
<td>$C_\alpha C_lR_R_O(g_{s\alpha})$</td>
</tr>
<tr>
<td>$c_2$</td>
<td>$a_4$</td>
<td>$a_3$</td>
<td>$a_3$</td>
</tr>
<tr>
<td>$d_1$</td>
<td>$C_lR_O - C_\alpha g_{s\alpha}g_{s\alpha}g_{s\alpha}g_{s\alpha}R_RR_R_R_O/\beta g_{s\alpha}$</td>
<td>$C_lR_O\left[2 - \frac{C_lR_O g_{s\alpha}}{\beta C_\alpha}\right]$</td>
<td>$\left[1 + \frac{g_{s\alpha}}{g_{s\alpha}}\right]z C_\alpha/g_{s\alpha} - \left[1 + \frac{g_{s\alpha}}{g_{s\alpha}}\right]C_R_R_r/\beta C_\alpha C_lR_R_O(g_{s\alpha})/\beta - \frac{2 C_\alpha g_{s\alpha} R_R_R_O}{g_{s\alpha}}z C_\alpha/g_{s\alpha} - (1+g_{s\alpha}/g_{s\alpha})C_R_R_r/\beta$</td>
</tr>
</tbody>
</table>

* $z = C_\alpha - C_\alpha g_{s\alpha}g_{s\alpha}R_R_R_O R_R_r/\beta$