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<td><strong>Author(s)</strong></td>
<td>Zhao, B.; Law, Adrian Wing-Keung; Adams, E. Eric; Er, J. W.</td>
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Formation of particle clouds

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In the literature, it has been conceptualized that a group of dense particles released instantaneously into homogeneous stagnant water would form a circulating vortex cloud and descend through the water column as a thermal. However, Wen & Nacamuli (\textit{Hydrodynamics: Theory and Applications}, 1996, pp. 1275–1280) observed the formation of particle clumps characterized by a narrow, fast-moving core shedding particles into the wake. They found clump formation to be possible even for particles in the non-cohesive range as long as the source Rayleigh number was large ($Ra > 10^3$) or, equivalently, the source cloud number was small ($Nc < 3.2 \times 10^{-2}$). This physical phenomenon has not been investigated further since the experiments of Wen and Nacamuli. In the present study, the relationship between $Nc$ and the formation process is examined more systematically. The theoretical support for cloud number dependence is explored by considering flows passing a porous sphere. Here $Nc$ values ranging from $2.9 \times 10^{-3}$ to $5.9 \times 10^{-2}$ are tested experimentally using particles with different initial masses and grain sizes, from non-cohesive to marginally cohesive. The formation processes are categorized into cloud formation, a transitional regime and clump formation, and their distinct features are presented through qualitative description of the flow patterns and quantitative assessment of the gross characteristics.

\textbf{Key words:} convection, plumes/thermals

1. Introduction

During land reclamation and dredged material disposal, large amounts of sediment are dumped into the aquatic environment. The sediment can be discharged either instantaneously (e.g. by clamshell buckets or split barges) or continuously (e.g. through pipelines). The subsequent underwater behaviour is closely related to the source conditions. In the past, these particles have been conceptualized to descend as a group and form a particle cloud (Rahimipour & Wilkinson 1992; Li 1997; Ruggaber 2000; Bush, Thurber & Blanchette 2003; Zhao \textit{et al.} 2012; Gensheimer, Adams & Law 2013; Zhao \textit{et al.} 2013a). As illustrated in figure 1, the conceptualization of the convective descent process can be categorized into three phases (Rahimipour & Wilkinson 1992): (i) a relatively short initial acceleration phase, within which the

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particles accelerate as a solid sphere; (ii) a self-preserving phase, within which the particles form a cloud growing self-similarly in shape due to turbulent entrainment; and (iii) a dispersive phase, within which the individual particles descend at their settling velocities. During the self-preserving phase, the particle cloud behaves similarly to a single-phase miscible thermal (Scorer 1957; Turner 1973), which is characterized by coherent vortex ring structures. Here, the above process is referred to as cloud formation, which is ‘thermal-like’.

As the particle cloud grows due to turbulent entrainment, the transfer of momentum to the entrained ambient stagnant water results in a continuous decay in the vortex strength and a decrease in the vertical descent velocity. During the process, the solid phase gradually separates from the entrained fluid phase as individual particles (Lai et al. 2013). The particles then settle in a dispersed manner and can easily be carried away by ambient flows. Gensheimer et al. (2013) found also that the coherent vortex structure would be completely destroyed or never be formed with the existence of strong ambient currents, while the effect of ambient waves was found to be less significant by Zhao et al. (2013a). To characterize the transition from the self-preserving phase to the dispersive phase, Rahimipour & Wilkinson (1992) defined a cloud number ($N_c$) as the ratio of the settling velocity ($w_s$) of individual particles to the characteristic circulation velocity ($w_t$) within the particle thermal,

$$N_c = \frac{w_s}{w_t} = \frac{w_s}{\sqrt{B_0/r}} \propto \frac{w_s r}{r_0 \sqrt{r_0 \Delta g}}.$$  \hspace{1cm} (1.1)

Here $w_t = \sqrt{B_0/r}$; $r$ is the bulk radius of the particle cloud underwater; $B_0 = V_0 \Delta g$ is the total kinematic buoyancy excess, where $V_0$ is the total volume of solid particles; $r_0 = [(3V_0)/(4(1-n)\pi)]^{1/3}$ is the initial equivalent radius of a sphere with the same total volume as the particle, including void space, where $n$ is the void ratio and assumed to be constant here for particles initially at the settled state; $\Delta = (\rho_s - \rho_a)/\rho_a$ is the normalized density difference; and $g$ is the gravitational acceleration. As the particle cloud descends, the bulk size grows due to the turbulent entrainment of ambient water. With a constant total buoyancy excess, $N_c$ increases as $w_s$ decreases according to (1.1). Rahimipour & Wilkinson (1992) found that the growth rate was
a function of $N_c$ when $N_c < 1$, and the rate became very small once $N_c$ reached approximately 1.5.

The formation of a particle thermal is of particular importance because of its ability to retain the dense particles inside the vortex structure, which is formed due to the internal density gradient and shear force at the density interface in a manner similar to a single-phase miscible thermal (Zhao et al. 2013b). Thus, the discrete particles are trapped inside the large eddies and descend as a group with a bulk velocity much higher than the individual particle settling velocity. The particle loss from the trailing stem has also been found to be negligible due to the high entrainment velocity at the rear of the particle thermal (Ruggaber 2000).

Owing to the transient nature of instantaneous releases, the subsequent formation process is highly dependent on the initial conditions. Wen & Nacamuli (1996) conducted an experimental study and observed particle clouds with distinct features. They formulated three non-dimensional parameters based on source conditions, and found that the formation process correlated most closely with a Rayleigh number ($Ra$) defined as follows:

\[ Ra = \frac{B_0/r_0^2}{w_s^2} = \frac{1}{N_c^2 r_{r=r_0}}, \]

(1.2)

which can be expressed in terms of the cloud number defined based on the source condition (i.e. $r = r_0$). Their experimental results showed that when $Ra < 10^3$, the particles formed a vortex ring or cloud; when $Ra$ approached $10^3$, a clump formed at the beginning but subsequently collapsed into a cloud; and when $Ra > 10^3$, the clump feature became more prominent. During the clump formation, they observed that the particles formed a narrow, fast-moving, particle-rich core that continuously shed particles into the wake, which is referred to as ‘wake-like’ here. Thus, the higher the value of $Ra$ (or the lower the value of $N_c$), the more wake-like the formation is supposed to be.

It has commonly been reported in the literature that dense particles released instantaneously would behave like a thermal, and so far the observation of clump formation and wake-like phenomena has been made only by Wen & Nacamuli (1996). It is therefore worthwhile to first review previous studies and examine why clump formation has rarely been reported. The conditions of previous experimental studies on three-dimensional particle clouds are summarized in table A.1 in the Appendix. It can be seen that most of the cases are within the cloud formation regime (i.e. $N_c > 3.2 \times 10^{-2}$ or equivalently $Ra < 10^3$ as noted by Wen & Nacamuli (1996)). However, a few cases fall within the clump formation regime according to the respective cloud numbers. Ruggaber (2000) examined conditions with cloud number down to $O(10^{-4})$ but did not observe clear clump formation. This might be attributed to the fact that in order to achieve a smooth release, he mixed the particles initially with water and then released them in suspension by constant stirring, which prevented the formation of clumps and subsequent wake-like phenomenon. Gensheimer et al. (2013) adopted the same source conditions as those of Ruggaber (2000) during their investigation of particle clouds in ambient currents. Therefore, their experimental conditions are not repeated in table A.1. Bush et al. (2003) performed experiments with $N_c$ ranging from $1.0 \times 10^{-2}$ to $2.2 \times 10^{-1}$. Their releases were from a funnel, and pulling up the plug might have affected the initial compactness and the subsequent formation process. Deguen, Olson & Cardin (2011) also reported one case with a cloud number down to $O(10^{-5})$. Their experimental images (cf. figure 5(c)) seem to suggest the existence of particle clumps, but due to the relatively small experimental
domain, vortical motion and mixing very quickly became dominant after the particles reached the bottom of a spherical flask. Some doubts also remain in regard to one of Wen & Nacamuli’s cases on cloud formation (second line under Wen & Nacamuli (1996) in table A.1, with text struck through), which had a cloud number even lower than the other clump formation cases. That particular condition is not considered here due to the limited information available. Despite its pioneering findings, the scope of the Wen & Nacamuli (1996) study was mostly qualitative and preliminary, and the selection of $Ra$ as the governing parameter was highly empirical. No further investigation was undertaken by Wen & Nacamuli or other researchers afterwards as far as we are aware.

The primary goal of the present study is to confirm the existence and raise the significance of clump formation. To further reveal the mechanism, the gross characteristics of particle clouds undergoing various formation processes are assessed through a novel integrated view approach and compared in both qualitative and quantitative ways. In the following, the cloud number dependence is first examined theoretically in § 2; the experimental methodology is introduced in § 3; experimental observations on the formation process and the behaviour of gross characteristics are described in § 4; and finally, a summary of the results is presented in § 5.

2. Theory

When dense particles are released in a settled state, they start as a clump and accelerate from rest due to gravity. The clump can continue to descend either as a particle-dense core or as a diluted and well-mixed cloud. The process essentially depends on the capability of the ambient water to flow through the solid phase. If the interstitial space between particles is small enough that it is hard for entrained water to move through, the flow tends to go around the clump and peel off some particles around the surface, which leads to wake-like clump formation. However, if the pore size is large enough, the flow goes through and destroys the clump. The details of clump breakup and dilution are too complicated to be fully resolved at the moment, so the present discussion will be limited to the onset condition of clump preservation or dilution by formalizing the problem as a flow passing a porous sphere. The level of flow resistance will be estimated by considering the hydraulic gradient across the sphere and the associated seepage velocity based on dimensional arguments. By examining two asymptotic conditions, the cloud number is shown to govern the formation of a particle cloud.

Here, the free stream velocity $U$ is assumed to be proportional to the previously defined characteristic circulation velocity ($w_t$), which is also a characteristic cloud descent velocity. With a bulk radius of $r = r_0$, it can be derived that

$$U \propto w_t \propto \frac{\sqrt{B_0}}{r_0} \propto \sqrt{r_0 \Delta g}. \quad (2.1)$$

The drag force around the porous sphere can be calculated as

$$F_D = \frac{1}{2} \rho_a C_D A U^2 \quad (2.2)$$

where $C_D$ is the drag coefficient and $A$ is the projection area. A bulk Reynolds number can be defined as

$$Re = \frac{w_t r_0}{v} = \frac{\sqrt{B_0}}{v}. \quad (2.3)$$
which is normally within the turbulent range in the context of sediment disposal in water for both laboratory experiments and field applications. Based on the experimental conditions listed in table A.1, the bulk Reynolds number can be found to vary in the range from $1.7 \times 10^3$ to $5.4 \times 10^4$, and will be much higher for field conditions with even larger buoyancy. Therefore, $C_D$ is assumed to be constant and the average head loss across the sphere can be estimated by

$$h_L = \frac{F_D}{\rho_0 g A} \propto \frac{U^2}{g}.$$  \hspace{1cm} (2.4)

It is assumed that the flow through the porous sphere is mainly driven by the hydraulic gradient, $i$, which should be expressed in the following form based on Darcy’s law:

$$i \propto \frac{h_L}{2r_0} \propto \frac{U^2}{g r_0}.$$  \hspace{1cm} (2.5)

To estimate the characteristic seepage velocity induced by the above hydraulic gradient, we will borrow notation from studies of flow through a packed sediment bed. The pressure loss during one-dimensional flow through a packed bed consists of both viscous energy loss and inertial loss, and the amount per unit length (i.e. equivalent hydraulic gradient) can be calculated by Ergun’s equation in the following format (Ergun 1952; Niven 2002; Cheng 2003):

$$i = c_1 \frac{v}{gd_p^2} u_s + c_2 \frac{1}{gd_p} u_s^2,$$  \hspace{1cm} (2.6)

where $c_1$ and $c_2$ are constants depending on the porosity and sphericity of the particles, $d_p$ is the particle diameter, and $u_s$ is the seepage velocity, which can be estimated by inversely applying the above equation. Although both terms on the right-hand side of (2.6) may be important in reality, only the asymptotic cases are discussed here. In the limit of small particle size, the nonlinear term due to inertia can be ignored. Substituting the expressions from (2.1) and (2.5) into (2.6) and rearranging yields

$$\left(\frac{u_s}{U}\right)_{Lin} \propto \left(\frac{\Delta g}{r_0} \frac{1}{\nu r_0^{1/2}}\right)^{1/2}.$$  \hspace{1cm} (2.7)

At the other extreme of large particle size, only the nonlinear inertia term is retained. Repeating the above substitution yields

$$\left(\frac{u_s}{U}\right)_{NL} \propto \left(\frac{d_p}{r_0}\right)^{1/2}.$$  \hspace{1cm} (2.8)

The seepage velocity normalized by the free stream velocity represents the relative ease of flow through the porous sphere. To the extent that this ease is inversely correlated with the tendency for clump formation, we expect wake-like behaviour to increase with decreasing $u_s/U$. It can be noted that both (2.7) and (2.8) suggest an increasing tendency for particle clumping as $r_0$ increases and $d_p$ decreases. The density difference ($\Delta$) is also important when viscous effects dominate.

The focus of the discussion will now shift back to the definition of cloud number, which, from (1.2), is given by

$$Nc = \frac{w_s}{\sqrt{B_0/r_0}} \propto \frac{w_s}{\sqrt{\Delta g r_0}}.$$  \hspace{1cm} (2.9)
For small particle sizes (i.e. Stokes flow), the settling velocity of a spherical particle can be calculated as (Lamb 1993)

\[ w_s = \frac{1}{18} \frac{\Delta g}{v} d_p^2 \propto \frac{\Delta g}{v} d_p^2. \]  

(2.10)

Substituting \( d_p \) from (2.10) into (2.7) yields

\[ \left( \frac{u_s}{U} \right)_{\text{Lin}} \propto \frac{w_s}{\sqrt{\Delta g r_0}} \propto N_c. \]  

(2.11)

For large particle sizes, the settling velocity of a spherical particle can be calculated by balancing the gravity, buoyancy force and drag force; for a constant drag coefficient, it can be expressed as

\[ w_s = \left( \frac{4 \Delta g d_p}{3 C_D} \right)^{1/2} \propto (\Delta g d_p)^{1/2}. \]  

(2.12)

Substituting \( d_p \) from (2.12) into (2.8) yields

\[ \left( \frac{u_s}{U} \right)_{\text{NL}} \propto \frac{w_s}{\sqrt{\Delta g r_0}} \propto N_c. \]  

(2.13)

Therefore, the above derivation suggests that the formation process (i.e. the preservation or dilution of a particle clump) is governed by the cloud number for the full range of particle sizes under turbulent conditions. The settling velocity \( (w_s) \) in the definition of the cloud number helps to incorporate the nonlinearity introduced by the density difference (\( \Delta \)) and to provide a smooth transition.

Wen & Nacamuli (1996) suggested that the tendency to form particle clumps increases with decreasing \( N_c \). Based on the above definition of cloud number, \( N_c \) decreases as \( w_s \) decreases (or as \( \Delta g \) or \( r_0 \) increases). Considering the limit of very small \( w_s \) (i.e. low relative velocity between the solid-particle and fluid phases), the flow in a well-mixed particle cloud will behave like the release of a single-phase buoyant fluid and form a classic buoyant vortex ring or thermal (Scorer 1957; Turner 1973) with no clumping. Therefore, we suspect that the wake-like phenomenon could be related to the tendency of particles to form clumps (even if the particle sizes are not in the cohesive range) rather than their small settling velocity. The settling velocity is simply a reflection of the combined effect of particle size and density difference. It will be shown later that non-cohesive particles indeed form clumps, as observed during the present experiments.

3. Experiments

3.1. Experimental set-up

The experiments were conducted by releasing a finite mass of heavy particles instantaneously into stagnant ambient water. As discussed earlier, the settling velocity \( (w_s) \) is a function of both particle size \( (d_p) \) and reduced gravity \( (\Delta g) \), and the total buoyancy excess \( (B_0) \) is a function of both the initial length scale \( (r_0) \) and \( \Delta g \). Thus, in laboratory experiments, the cloud number can be varied by changing any of the three parameters \( d_p, r_0 \) and \( \Delta g \). However, a change in \( \Delta g \) results in simultaneous variations of \( w_s \) and \( B_0 \) in the same direction, thus making it less flexible to vary
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\[ N_c \] by changing the densities of particles or the ambient fluid. As shown in table 1, by varying \( d_p \) and \( r_0 \) (i.e. total solid mass, \( m_0 \propto r_0^3 \)), the present study investigates cloud numbers ranging from \( 2.9 \times 10^{-3} \) to \( 5.9 \times 10^{-2} \). The experiments are divided into series-A and series-B tests based on the initial solid mass, and each series of tests covers both the cloud and clump formation regimes as proposed by Wen & Nacamuli (1996). Further investigation of \( N_c \) in a wider range is limited by the scale of the current laboratory facilities. Using the same particles and ambient medium, varying \( N_c \) by one order of magnitude requires the total buoyancy \( (B_0 \propto r_0^3) \) to vary by six orders of magnitude. We have also tested walnut shell grits with a density of 1.3–1.4 g cm\(^{-3}\). As expected, the lower density also led to a slower settling velocity for particles of the same size, which eventually did not provide significant flexibility.

Glass beads (Ballotini Impact Glass Beads manufactured by Potters Industries LLC) with four different particle sizes and a density of \( \rho_s = 2.5 \) g cm\(^{-3}\) were used. Following the manufacturer’s standards, they included particles of size B (\( d_p = 0.425–0.600 \) mm, with a median diameter of \( d_{50} = 0.513 \) mm), D (\( d_p = 0.212–0.300 \) mm, with \( d_{50} = 0.256 \) mm), AE (\( d_p = 0.090–0.150 \) mm, with \( d_{50} = 0.120 \) mm) and AH (\( d_p = 0.045–0.090 \) mm, with \( d_{50} = 0.0675 \) mm). Particles of size smaller than 2 \( \mu \)m (clay) are generally considered cohesive, while those with diameter greater than 60 \( \mu \)m are non-cohesive (Yang 2006). Therefore, the size-AH particles can be considered as marginally cohesive and the rest can be considered non-cohesive. For spherical particles with diameters ranging from 0.01 to 100 mm, the settling velocity can be calculated based on the empirical functions developed by Dietrich (1982). The corresponding median settling velocities for particles of size B, D, AE and AH were 7.13, 2.92, 0.93 and 0.35 cm s\(^{-1}\), respectively.

For series-A tests, the experiments were conducted in a glass tank of dimensions 2.85 m (length) \( \times \) 0.85 m (width) \( \times \) 0.95 m (height), with a constant water depth of 0.92 m. Dry particles were initially held inside a cylindrical tube with an inner diameter of 1.8 cm. The particles were carefully prepared so that there was no clumping before release. The bottom of the tube was sealed by a piece of latex sheet and placed 0.8 cm above the water surface. A motor-driven needle was placed along the centreline of the tube with its tip touching the membrane (following the design of Lundgren, Yao & Mansour (1992)). By activating the motor, the needle would make a 0.5 cm downward displacement and pierce the latex seal. The motion of the particles underwater was illuminated by a spotlight and recorded using a video camera (Sony HDR-XR550E) with a resolution of 1080 \( \times \) 1440 pixels at 25 frames s\(^{-1}\). The background was covered with black paper, and the glass beads appeared white in the recorded images. The data were extracted from the still water surface to a depth

<table>
<thead>
<tr>
<th>Notation</th>
<th>( m_0 ) (g)</th>
<th>( V_0 ) (cm(^3))</th>
<th>( d_{50} ) (mm)</th>
<th>( w_s ) (cm s(^{-1}))</th>
<th>( B_0 ) (g cm s(^{-2}))</th>
<th>( r_0 ) (cm)</th>
<th>( N_c )</th>
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<tr>
<td>A1</td>
<td>5.5</td>
<td>3.93</td>
<td>0.256</td>
<td>2.92</td>
<td>( 3.2 \times 10^3 )</td>
<td>0.98</td>
<td>( 5.0 \times 10^{-2} )</td>
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<tr>
<td>A2</td>
<td>5.5</td>
<td>3.93</td>
<td>0.120</td>
<td>0.93</td>
<td>( 3.2 \times 10^3 )</td>
<td>0.98</td>
<td>( 1.6 \times 10^{-2} )</td>
</tr>
<tr>
<td>A3</td>
<td>5.5</td>
<td>3.93</td>
<td>0.0675</td>
<td>0.35</td>
<td>( 3.2 \times 10^3 )</td>
<td>0.98</td>
<td>( 6.0 \times 10^{-3} )</td>
</tr>
<tr>
<td>B1</td>
<td>448</td>
<td>320</td>
<td>0.513</td>
<td>7.13</td>
<td>( 2.6 \times 10^5 )</td>
<td>4.25</td>
<td>( 5.9 \times 10^{-2} )</td>
</tr>
<tr>
<td>B2</td>
<td>448</td>
<td>320</td>
<td>0.256</td>
<td>2.92</td>
<td>( 2.6 \times 10^5 )</td>
<td>4.25</td>
<td>( 2.4 \times 10^{-2} )</td>
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<tr>
<td>B3</td>
<td>448</td>
<td>320</td>
<td>0.120</td>
<td>0.93</td>
<td>( 2.6 \times 10^5 )</td>
<td>4.25</td>
<td>( 7.7 \times 10^{-3} )</td>
</tr>
<tr>
<td>B4</td>
<td>448</td>
<td>320</td>
<td>0.0675</td>
<td>0.35</td>
<td>( 2.6 \times 10^5 )</td>
<td>4.25</td>
<td>( 2.9 \times 10^{-3} )</td>
</tr>
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</table>

\textbf{Table 1.} Experimental parameters.
of 80 cm with a spatial resolution of 0.09 cm pixel\(^{-1}\). For series-B tests, similar experimental procedures were followed except that the experiments were conducted in a different glass tank, of dimensions 2.4 m (length) \(\times\) 1.2 m (width) \(\times\) 2.2 m (height), with a greater water depth of 2.1 m; the particles were held inside a larger cylindrical tube (with inner diameter 7.8 cm) placed 3.5 cm above the water surface, and were released by pulling a board initially covering the bottom of the tube. Each test was conducted five times to ensure repeatability.

The motion of instantaneously released particles is dominated by the total buoyancy excess, \(B_0\). Characteristic scales are defined as

\[
L_n = B_0^{1/3} g^{-1/3}, \quad T_n = B_0^{1/6} g^{-2/3}, \quad U_n = B_0^{1/6} g^{1/3}
\]  

and are used to normalize the experimental results under different conditions. Substituting in the current experimental parameters, for series-A tests we have \(L_n = 1.49\) cm, \(T_n = 0.039\) s and \(U_n = 38.2\) cm s\(^{-1}\); and for series-B tests we have \(L_n = 6.46\) cm, \(T_n = 0.081\) s and \(U_n = 79.6\) cm s\(^{-1}\). Although the actual dimensions are larger, the normalized field of view for series-B tests is actually smaller than that for series-A tests. In the present study, the physical references are set at the still water surface \((z = 0)\) and the time of release \((t = 0)\), and the downward direction is chosen as positive.

3.2. Parametric analysis

As summarized in table A.1, the existing laboratory experimental studies mainly covered cases where the released particles formed a particle thermal (i.e. cloud formation). Assuming the formation of particle clumps can be triggered given a small enough cloud number, its significance in actual field conditions can then be assessed to provide some indication as to the practical importance during sediment disposal.

A parametric study was performed, with results shown in figure 2, to reveal the effects of particle size and total release volume (i.e. total mass) on \(N_c\). The density of sediment is assumed to be \(2.65\) g cm\(^{-3}\). Here, the value of \(N_c\) is primarily determined by the particle size (i.e. settling velocity). Assuming \(N_c \approx 3.2 \times 10^{-2}\) to be the threshold for clump formation, the release of fine sand (with \(d_p \approx 0.1\) mm) from a clamshell bucket (with a typical volume of \(V_0 \approx 1\) m\(^3\)) would result in wake-like behaviour. The release of larger volumes (e.g. from a split barge with a typical volume of \(V_0 \approx 1000\) m\(^3\)) would cause the wake formation to happen with even larger sediment sizes. Therefore, clump formation with small \(N_c\) is likely to be a common occurrence in field operations.

3.3. Integrated view analysis

As shown earlier in figure 1, with the formation of a particle thermal, the solid particles descend as a group with a self-preserving elliptical shape. In previous studies, the behaviour of particle clouds was usually quantified by their gross characteristics, which include the maximum radius of the elliptical particle cloud and the descent velocity of the frontal or centroid position. However, as shown in this section, the motion of particle clumps is highly irregular, and thus the identification of such characteristics can be difficult. Inspired by work of Burridge & Hunt (2012), the present study introduces an integrated approach, which provides a common framework for comparing the behaviours of particle thermals and clumps. In the post-processing, each image frame recorded by the video camera first had a constant background
image subtracted, and was then converted to grey-scale. As illustrated in figures 3 and 4, which are based on typical individual experimental runs, to assess the vertical motion, the grey-scale value on each pixel was summed horizontally to produce a column vector at each time frame; these vectors were then combined into time series and normalized to yield horizontally integrated images (i.e. the panels in the first column). Therefore, a particle thermal with minimal trailing stem appears as a narrow dark band as in figure 3(a), while the particles being shed into the wake of a particle clump stay in the water column for a much longer time and result in a wide shaded area as in figure 3(c). To trace the horizontal spreading, the same integration was performed vertically, to give the panels in the rightmost column of figure 3. The integrated images provide qualitative indications of the spatial and temporal distributions of the particles. However, a quantitative measurement of the internal particle distribution requires more advanced experimental techniques and is outside the scope of the present study.

In order to quantify the gross characteristics, the integrated images from individual runs under the same conditions are first averaged as shown in figure 5. The length and time scales are non-dimensionalized based on the characteristic scales defined in (3.1), and the solid lines in panels (b) and (d) indicate the limits of the data-sampling areas within which the results are presented and are free of boundary effects. As the formation process transits from a thermal-like regime to a wake-like regime (i.e. from high to low $Nc$), the particle-containing region becomes more elongated due to increasing particle losses into the wakes of the clumps, which is reflected in the widening trend of the dark bands in horizontally integrated images; see figure 5(a,b). It is also clear that the spread in the lateral direction weakens (with decreasing $Nc$) due to the combined effect of the higher downward velocity field induced by the fast-moving clumps and the reduced turbulent entrainment around the wake region. Another trend is observed: with a larger initial mass, there is a higher chance that the smaller particles (e.g. of size $AH$) will form multiple clumps (e.g. test B4 in figure 4(d)), which makes the disintegration process faster and the

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**Figure 2.** Parametric analysis of the effects of particle size and total release volume on the cloud number $Nc$; the contours are of constant $Nc$, and clump formation is expected below the dashed curve.
FIGURE 3. Integrated view and instantaneous images: (a) test A1, run 1, with $d_{50} = 0.256$ mm, $m_0 = 5.5$ g and $Nc = 5.0 \times 10^{-2}$; (b) test A2, run 1, with $d_{50} = 0.120$ mm, $m_0 = 5.5$ g and $Nc = 1.6 \times 10^{-2}$; (c) test A3, run 1, with $d_{50} = 0.0675$ mm, $m_0 = 5.5$ g and $Nc = 6.0 \times 10^{-3}$.

flow more regular than that of a single clump (e.g. test A3 in figure 3(c)). Together with more quantitative measurements, the various formation processes and the reason for forming multiple clumps will be discussed further in the next section.

To extract quantitative information from the averaged integrated view images, the background noise level determined based on histogram analysis was used for image segmentation and subsequent detection of the averaged front ($z_f$), tail ($z_e$) and bulk radius ($r_{\text{max}}$). A similar analysis was performed on individual runs, and the averaged standard deviations were used to represent the experimental variations, as summarized in table 2. It was found that the variation increased as the conditions transited from
Formation of particle clouds

**Figure 4.** Integrated view and instantaneous images: (a) test B1, run 1, with \( d_{50} = 0.513 \text{ mm} \), \( m_0 = 448 \text{ g} \) and \( N_c = 5.9 \times 10^{-2} \); (b) test B2, run 1, with \( d_{50} = 0.256 \text{ mm} \), \( m_0 = 448 \text{ g} \) and \( N_c = 2.4 \times 10^{-2} \); (c) test B3, run 1, with \( d_{50} = 0.120 \text{ mm} \), \( m_0 = 448 \text{ g} \) and \( N_c = 7.7 \times 10^{-3} \); (d) test B4, run 1, with \( d_{50} = 0.0675 \text{ mm} \), \( m_0 = 448 \text{ g} \) and \( N_c = 2.9 \times 10^{-3} \).
Figure 5. Averaged integrated view: (a) horizontally integrated view of series-A test; (b) horizontally integrated view of series-B test; (c) vertically integrated view of series-A test; (d) vertically integrated view of series-B test. The solid lines in (b) and (d) indicate the limits of the data-sampling area.
cloud to clump formation; this was mainly attributed to the higher irregularity and asymmetry of wake-like behaviour. The overall variation was around 10%, and the maximum variation was found in test A3, during which the particles mostly formed a single clump with a strong meandering flow pattern, as shown in figure 3(c). In the following, only averaged gross characteristics of the released particles are discussed.

4. Formation processes

The behaviour of particle clouds is commonly represented by gross characteristics, such as the trajectory of descent based on the frontal position \( z_f \) and the lateral spread based on the maximum cloud radius \( r_{\text{max}} \). During the self-preserving phase of cloud formation, these gross characteristics have been found to follow the power laws

\[
\begin{align*}
    z_f & \propto t^{0.5}, \\
    r_{\text{max}} & \propto t^{0.5},
\end{align*}
\]

(4.1)

which were derived based on the assumptions of constant total buoyancy excess and self-similarity (Scorer 1957; Turner 1973). In figures 6 and 7, \( z_f \) and \( r_{\text{max}} \) are plotted against time on a log–log scale. Data fittings are performed, and the resulting best-fit power laws (i.e. the slopes denoted by \( S \)) are compared with the above theoretical values. The solid horizontal lines indicate limits of the data-sampling areas and are labelled with their corresponding actual dimensions; the vertical dotted lines indicate the times of transition in flow regimes. In the following, the formation processes are classified into three categories – cloud formation, a transitional regime and clump formation – and are described individually in more detail.

4.1. Cloud formation

As illustrated in figures 3(a) and 4(a) (with \( N_c = 5.0 \times 10^{-2} \) and \( 5.9 \times 10^{-2} \), respectively), when the cloud number is in the higher range, the particles behave like a thermal and their motion follows the classic three-phase mode (Rahimipour & Wilkinson 1992). Immediately after water entry, the particles accelerate as a porous sphere. During this stage, due to the relative ease of flow through the pores, the solid particles become well mixed with the ambient water and the mixture starts to behave like a dense fluid. Circulation is generated by the shear force around the density interface, and the particles form an elliptical particle thermal characterized by vortex ring structures. With the entrainment at the rear of the particle thermal, there is a negligible number of particles in the trailing stem. The particle thermal descends with more or less constant total buoyancy and preserved shape. Data fittings are performed on the entire range of the data as shown in figures 6(a) and 7(a), with the slopes fixed at the theoretical value of 0.5. The coefficients of determination (denoted by \( R^2 \)) are also reported and suggest a reasonable agreement with the theoretical predictions in (4.1), considering the experimental variations. Note that the initial acceleration phase is so short that the results of data fitting are not significantly affected even if this phase is incorporated.

Table 2. Summary of experimental variations.
Figure 6. Integrated view and instantaneous images: (a) test A1, run 1, with $d_{50} = 0.256$ mm, $m_0 = 5.5$ g and $N_c = 5.0 \times 10^{-2}$; (b) test A2, run 1, with $d_{50} = 0.120$ mm, $m_0 = 5.5$ g and $N_c = 1.6 \times 10^{-2}$; (c) test A3, run 1, with $d_{50} = 0.0675$ mm, $m_0 = 5.5$ g and $N_c = 6.0 \times 10^{-3}$. The solid horizontal lines indicate limits of the data-sampling area, and the vertical dotted lines indicate the times of transition in flow regimes.

4.2. Transitional regime

As suggested by the theory introduced in § 2, the resistance of flow through the porous sphere increases as the cloud number decreases. It is observed that before entering complete clump formation, the particles first experience a transitional regime. As illustrated in figures 3(b) and 4(b) (with $N_c = 1.6 \times 10^{-2}$ and $2.4 \times 10^{-2}$, respectively), the particles initially descend and accelerate as a heavy clump due
Formation of particle clouds

Figure 7. Integrated view and instantaneous images: (a) test B1, run 1, with \( d_{50} = 0.513 \) mm, \( m_0 = 448 \) g and \( N_c = 5.9 \times 10^{-2} \); (b) test B2, run 1, with \( d_{50} = 0.256 \) mm, \( m_0 = 448 \) g and \( N_c = 2.4 \times 10^{-2} \); (c) test B3, run 1, with \( d_{50} = 0.120 \) mm, \( m_0 = 448 \) g and \( N_c = 7.7 \times 10^{-3} \); (d) test B4, run 1, with \( d_{50} = 0.0675 \) mm, \( m_0 = 448 \) g and \( N_c = 2.9 \times 10^{-3} \). The solid horizontal lines indicate limits of the data-sampling area, and the vertical dotted lines indicate the times of transition in flow regimes.
to high flow resistance through the pores; particles around the surface are peeled off, and thus the radius of the clump becomes smaller and the velocity becomes higher. Considering (2.5), the continuous reduction in size and the increment in velocity result in an increasing hydraulic gradient across the particle clump. Once beyond a certain threshold, the ambient water will flow through the pores and cause the clump to disintegrate, and the particles will behave again as a dense fluid. During the transition, the clump at the front quickly transforms into a leading thermal. The induced flow field (especially the entrainment at the rear) engulfs the particles in the wake region, and thus the particles finally descend together like a particle thermal. As shown in figures 6(b) and 7(b), transitions in flow regimes can be visually identified. Before the transition, the best-fit power laws are found to be

\[ z_f, A_2 \propto t^{0.925 \pm 0.039}, \quad r_{max, A_2} \propto t^{0.994 \pm 0.076}, \]  

(4.2a)

\[ z_f, B_2 \propto t^{1.024 \pm 0.010}, \quad r_{max, B_2} \propto t^{0.764 \pm 0.009}, \]  

(4.2b)

which indicate much faster development than expected for a particle thermal. However, after the transformation,

\[ z_f, A_2 \propto t^{0.463 \pm 0.001}, \quad r_{max, A_2} \propto t^{0.525 \pm 0.003}, \]  

(4.3a)

\[ z_f, B_2 \propto t^{0.432 \pm 0.003}, \quad r_{max, B_2} \propto t^{0.511 \pm 0.004}, \]  

(4.3b)

suggesting that the power laws fall back to the theoretical values of a thermal. In the above and subsequent fitting correlations, the power laws are reported together with their uncertainties (based on ± the respective standard error). As certain standard errors of the slopes fall even below 0.01, the values are all reported to the third decimal place for consistency. However, it should be noted that the above representations do not imply accuracy up to three decimal places.

It may be argued that the formation of particle clumps before transition is part of the initial acceleration phase of a conventional particle cloud, since eventually the particles behave like a thermal with all the particles descending together. The fundamental distinction between these two phenomena is whether the ambient water is flowing around or through the particles. During the initial acceleration phase of cloud formation, particles are well mixed with the ambient water, and they move as a ballistic volume straight downwards without the meandering and significant loss of particles into the wake that are features of a clump. It should be noted that the transition from wake-like to thermal-like behaviour is only possible if the clump disintegrates and forms a leading thermal within a short distance of the source, so that the entrainment velocity at the rear of the leading vortex is strong enough to incorporate all the particles lost in the wake. In the context of sediment disposal in ambient currents and waves, the particles initially lost in the wake may be carried laterally away from the leading clump, and thus remain as turbidity even after the formation of the sediment vortex. Therefore, the loss as turbidity can be more significant than that in stagnant ambient.

4.3. Clump formation

As the cloud number decreases further, the wake-like flow phenomenon tends to dominate the descent process. With a higher resistance of flow through the particles, clumps are more easily formed and persist for longer distances. As illustrated in figures 3(c), 4(c) and 4(d) (with \( N_c = 6.0 \times 10^{-3}, 7.7 \times 10^{-3} \) and \( 2.9 \times 10^{-3} \),
Formation of particle clouds

respectively), the leading clumps travel at faster rates compared to the tails, resulting in much more elongated shapes. When the clumps eventually disintegrate, similar transitional processes (as indicated by the changes in slopes in figures 6(c), 7(c) and 7(d)) can be observed. Before the transitions, best fits of the data yield

\[
\begin{align*}
    z_f, A_3 &\propto t^{1.121\pm0.007}, & r_{\text{max}, A_3} &\propto t^{1.119\pm0.024} \\
    z_f, B_3 &\propto t^{0.933\pm0.007}, & r_{\text{max}, B_3} &\propto t^{0.566\pm0.023} \\
    z_f, B_4 &\propto t^{0.961\pm0.007}, & r_{\text{max}, B_4} &\propto t^{0.707\pm0.015}
\end{align*}
\] (4.4a)

which are higher than 0.5 (as expected for particle thermals) and similar to those at the beginning of the transitional regime. After the transitions,

\[
\begin{align*}
    z_f, A_3 &\propto t^{0.386\pm0.031}, & r_{\text{max}, A_3} &\propto t^{0.176\pm0.001} \\
    z_f, B_3 &\propto t^{0.498\pm0.002}, & r_{\text{max}, B_3} &\propto t^{0.368\pm0.001} \\
    z_f, B_4 &\propto t^{0.515\pm0.007}, & r_{\text{max}, B_4} &\propto t^{0.336\pm0.003}
\end{align*}
\] (4.5a)

indicating that the power laws for both \(z_f\) and \(r_{\text{max}}\) drop significantly. The lateral spreading is limited by both the downward flow field induced by the dense clump and less turbulent entrainment compared to a particle thermal. It is worth mentioning the differences in the physical representations of \(z_f\) and \(r_{\text{max}}\) between the transitional regime (introduced in § 4.2) and the clump formation regime. Within the transitional regime, most of the particles initially in the wake are incorporated into the leading particle vortex during the transitional process. The development of \(z_f\) and \(r_{\text{max}}\) (i.e. the frontal position and maximum radius of the particle vortex) are thus closely correlated, which is reflected in the consistency in the times of transitions, shown by the vertical dotted lines in figures 6(b) and 7(b). On the other hand, with clump formation \(r_{\text{max}}\) is normally a measure of the width of the elongated region. Although the leading clumps eventually disintegrate, the induced entrainment velocity at the rear is too weak to engulf all the particles in the wake region. Therefore, the times of transitions of \(z_f\) and \(r_{\text{max}}\) are rather independent, as shown by the vertical dotted lines in figures 6(c), 7(c) and 7(d).

The formation of turbidity is obvious during clump formation, and thus the main environmental concern is the duration of particle suspension inside the water column. To quantify this process, the tail or end positions \(z_e\) are extracted and plotted together with \(z_f\) in figure 8. As the cloud number decreases (i.e. more wake-like conditions), the tail remains at a shallower water depth for a longer time. The variations in \(z_f\) are relatively small, except in test A3, due to the formation of a single clump. The descent velocities of \(z_e\) towards the end of the sampling domain are derived by performing linear curve fittings on the last linear portion of each individual curve (indicated by solid lines). By comparison with the settling velocities \(w_s\) listed in table 1, it can be seen that the particles in the wake are still descending at higher velocities compared to their respective \(w_s\), which can be attributed to the grouping effect of heavy particles and the downward velocity field induced by the clumps at the front.

Compared to a particle thermal, a clump generally has a fast-moving front, a slower-moving tail and a smaller rate of lateral spread. Thus, the aspect ratio (i.e. height to width ratio, defined as \((z_f - z_e)/(2r_{\text{max}}))\) for clump formation should be higher than that for cloud formation. To assess the degree of wake-like behaviour, the development of the aspect ratio is plotted in figure 9. Each curve ends at the point where the frontal position reaches the bottom of the sampling area. The typical aspect ratio of 0.72 for
a miscible thermal (Scorer 1957) is indicated by a horizontal line in each panel. The general trend is as expected, i.e. that the aspect ratio increases with decreasing cloud number as the formation process changes from cloud formation (tests A1 and B1) to transitional (tests A2 and B2) and to clump formation (tests A3, B3 and B4). It is also observed that during test A3, the particles commonly form a single clump, whereas during tests B3 and B4, multiple clumps are consistently produced in a single experiment. More importantly, the aspect ratio is found to be closely related to the number of clumps being formed. As all the particles descend as a single clump, test A3 (with $N_c = 6.0 \times 10^{-3}$) is able to achieve a maximum aspect ratio of approximately 4, which is much higher than the aspect ratios of tests B3 and B4 (with $N_c = 7.7 \times 10^{-3}$ and $2.9 \times 10^{-3}$, respectively). Test B2 is also able to achieve a high aspect ratio before transition due to the formation of a single clump. Close inspection of the water-entry process reveals that during tests B3 and B4, the particles fractured into many small clumps immediately after they entered the water. By contrast, with the same initial mass but large particle sizes, tests B1 and B2 demonstrate a very smooth solid-liquid interface after water entry. Therefore, the fracture of particles can be considered as an indication of higher resistance to the flow through the particles due to smaller pore sizes. In the present study, particles were released in dry conditions, and we
believe that the initial water content could also play an important role in the number of clumps being formed, which requires further investigation.

5. Conclusions

A systematic experimental study has been conducted to examine the process of formation of particle clouds comprising both non-cohesive and marginally cohesive particles. By varying the particle size and initial mass, the present study covers cloud numbers ranging from $2.9 \times 10^{-3}$ to $5.9 \times 10^{-2}$. A theoretical analysis of the flow passing a porous sphere is performed to support the cloud number dependence of the formation process. A detailed review of existing literature on three-dimensional particle clouds and a parametric analysis reveal the need to investigate the criterion for clump formation and its significance in terms of introducing severe particle losses as turbidity. A novel integrated view analysis approach is proposed and adopted to provide qualitative descriptions about the spatial and temporal distributions of solid particles. The formation processes are categorized into cloud formation, a transitional regime and clump formation, and distinct features are described through qualitative comparison of the flow patterns and quantitative assessment of the gross characteristics.

As the first detailed study on the formation process of particle clouds, the scope of the current work is still limited. Further research should be done to gain a more complete understanding of the parametric space associated with the cloud number (e.g. the effects of density difference and ambient viscosity). Although the present study focuses on fully turbulent conditions, as characterized by the bulk Reynolds number defined in (2.3), it should be noted that the viscous effect is also involved in the definition of cloud number through the settling velocity of individual particles. Therefore, it is worthwhile to test particles and ambient media with different properties in future studies.

The current cloud number definition is limited to particles of uniform size. Thus it is of practical interest to explore the effect of particle size distributions on the formation process of particle clouds; in fact, the authors have already initiated such studies. Looking ahead, it would be beneficial to investigate the particle dynamics under conditions of high solid concentration, especially the detailed mechanisms for clump formation and subsequent disintegration.

Acknowledgement

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Appendix

The conditions of previous experimental studies on three-dimensional particle clouds are summarized in table A.1. It should be noted that certain parameters were not reported explicitly, and thus are derived here based on other known quantities. Those derived values are marked with a shaded background in the table. The density of particles is assumed to be 2.65 g cm$^{-3}$ if not stated in the study; the air void ratio in dry particles is assumed to be 40% by total volume (Ruggaber 2000); and the particle settling velocity is estimated using the empirical functions of Dietrich (1982) for spherical particles with diameters ranging from 0.01 to 100 mm.
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**Table A.1. Summary of experimental parameters.**
REFERENCES


