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Author(s)	Qian, Kemao
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Links between fringe pattern analysis and digital image correlation: windowed, optimal and tracking (WOT)

Qian Kemao

School of Computer Engineering, Nanyang Technological University, Singapore 639798,

mkmqian@ntu.edu.sg

ABSTRACT

A windowed, optimal and tracking (WOT) strategy for measurement data analysis is described, explained and discussed. Many fringe pattern analysis and digital image correlation algorithms adopted this strategy, and thus their similarities and links are revealed. Although this strategy is not new, highlighting it is believed to be helpful for future algorithm development.

Keywords: Fringe pattern analysis, digital image correlation, local analysis, optimal estimation, tracking

1. INTRODUCTION

Optical measurement probes a specimen by certain means and acquires signals. The desired information is often encoded in the signals and thus should be decoded subsequently. Fringe patterns and speckle patterns are two fundamental and extensively used types of signals that carry the information in full-field measurement. Fringe patterns have a well-defined mathematical form from which the inherent phase distribution can be traced. Speckle patterns, on the contrary, are characterized by their randomness, which can be used as a local identity to trace the motion. At first glance, these two types of patterns are very different. They are on the two extremes of the signals: one is deterministic and the other is random.

Naturally, their data analysis techniques are also very different. For fringe pattern analysis (FPA), the most impressive, effective and widely used techniques are the phase-shifting technique [1] and the Fourier transform technique [2]. The phase-shifting technique extracts phase pixel by pixel, while the Fourier transform technique transforms the whole image into the frequency domain. The technique associated with speckle patterns is the digital image correlation (DIC), which matches a subset of data in the first image to a deformed subset in the second image [3, 4]. We call that the phase-shifting is pixel-wise, the Fourier transform technique is global, and DIC is local or block-wise.

It seems that FPA and DIC have little in common. In this paper, however, we focus on their similarities. These similarities are shown to be natural. This paper is an extension of [5].

2. GENERAL SIMILARITIES IN MEASUREMENT DATA ANALYSIS

As has been mentioned, the phase-shifting technique is pixel-wise; the Fourier transform is global; while the DIC is block-wise. In fact, the pixel-wise, block-wise and global processing approaches are applicable to both FPA and DIC.

- **Pixel-wise DIC** An image has many pixels. A megapixel image becomes common. However, these pixels usually only have 256 gray levels to represent their intensity, making each pixel less unique and difficult to be traced. If the number of pixels is much smaller and they can differentiate themselves, pixel-wise matching is possible. Optical motion capture may be seen as such an example, where a few markers are traced [6].
- **Global DIC** Imagine that we have two speckle patterns with a uniform translation between them. Such a translation will be reflected in the Fourier spectrum [7]. In other words, DIC could be global if the translation is simple enough. In fact, the global Fourier transform technique for FPA is only valid under a condition: the fringe components should be band-limited [2].
- **Block-wise FPA** Phase-shifting technique is pixel-wise. When the fringe patterns are noisy, the phase-shifting result is also noisy. Filtering is then introduced, which is often block-wise [8]. The Fourier transform technique is

global, but a block-wise extension, windowed Fourier transform, can improve the result [9-11]. Block-wise processing is widely used in FPA, as shown by the references in [11].

As both FPA and DIC serve the information decoding for optical measurement, they should satisfy the general requirements of being accurate, automatic and accelerated (A3) [11], as discussed below.

- High accuracy and precision is a natural demand in optical measurement. It is so important that it will be further discussed in Sec. 3.
- Automaticity means that a data analysis technique achieves high success rate with little or no human interaction. An ideal data analysis technique is a black box to users. This is much like the mobile devices today: we use it without asking how it works. The discussions on window size setting in both FPA [12] and DIC [13] provide such an example.
- Acceleration asks for higher speed and ideally real time. The attempts for higher speed in FPA through intelligent soft implementations [14] and hardware realization [15] and in DIC through intelligent soft implementation [16] and hardware realization [17] are to satisfy this common acceleration requirement.

3. A WOT STRATEGY FOR MEASUREMENT DATA ANALYSIS

Since the high accuracy is the most important requirement among A3, we describe a windowed, optimal and tracking (WOT) strategy to achieve it.

3.1 Windowed processing

Although FPA and DIC are able to perform full-field measurement, the underlying probing technique is point-wise. The probed and captured signal at one point is independent of other points. It is ideal if the subsequent data analysis is pixel-wise, so that the final result at one pixel is not influenced by other pixels and thus has the highest spatial resolution. White light scanning interferometry works this way and is able to get the spatially discontinuous profile [18].

In many cases, however, pixel-wise data analysis does not work satisfactorily. In FPA, if the phase-shifting technique cannot be applied conveniently, the so-called ill-posed problem may arise, which makes the phase extraction difficult or impossible. Even when the phase-shifting is introduced, the signal is often noisy and needs further improvement. In DIC, we have already mentioned that pixel-wise processing is not feasible for too many pixels in one image. All these problems stem from insufficient information at a single pixel.

It is natural to seek other pixels' help to get more information for the current pixel being processed. Two types of nearby pixels are helpful. The first type includes neighboring pixels that are geometrically nearby to the processed pixel. These pixels indeed have similar shapes as the current pixel if the shape is smooth, or have similar deformation in a continuous medium. The second type consists of pixels with similar intensity regardless of their locations, thus nonlocal [19-21]. Such pixels have been shown to be very useful in image processing. We do not consider them in the current strategy with the following reasons: first, their rationale and effectiveness for measurement accuracy and precision rather than visual appeal need further understanding and validation; second, they help on denoising but not directly on demodulation; and third, good processors are available without nonlocal pixels [11].

To summarize, in order to process a pixel at (x_0, y_0) , a block of its neighboring pixels will be involved. An easy way to take out the block of data is by windowing the image. Given an image $f(x, y)$ and a window function $g(x, y)$, the block of data centering at (x_0, y_0) is simply

$$f(x, y; x_0, y_0) = f(x, y)g(x - x_0, y - y_0). \quad (1)$$

We thus process this block and get the result for the pixel (x_0, y_0) . The same process is repeated on other pixels by sliding the window until all the pixels in the image have been processed. We call it windowed processing.

3.2 Local models and model parameter optimization

Given a block of data, we want to find the most prominent feature and use it to model the data. In measurement, we usually do not have much idea of the data – that is why we measure. A manufactured specimen is often smooth, but may have discontinuities if it consists of a few pieces. Consequently, the probed signal is often smooth, but may have discontinuities. We temporally ignore the discontinuities and stick to the smoothness. By doing so, we still keep a big portion of the specimens and their signals.

We use $d(x,y)$ as a general smooth function to represent the fields involved in FPA and DIC, including fringe intensity, background intensity, fringe amplitude, phase, u -displacement, v -displacement, and so on. This smooth function can be approximated as a polynomial by Taylor expansion around (x_0, y_0) ,

$$d(x, y; \mathbf{p}) \approx \sum_{n=0}^N \left\{ \frac{1}{n!} \left[(x-x_0) \frac{\partial}{\partial x} + (y-y_0) \frac{\partial}{\partial y} \right]^n d(x_0, y_0) \right\}, \quad (2)$$

where N is an integer and \mathbf{p} is a model parameter vector consisting of all partial derivatives. Such a polynomial can be used as the signal model, abbreviated as a dN model for simplicity. For example, the $d2$ model can be written as

$$d(x, y; \mathbf{p}) \approx d(x_0, y_0) + \frac{\partial d(x_0, y_0)}{\partial x} (x-x_0) + \frac{\partial d(x_0, y_0)}{\partial y} (y-y_0) + \frac{1}{2} \frac{\partial^2 d(x_0, y_0)}{\partial x^2} (x-x_0)^2 + \frac{\partial^2 d(x_0, y_0)}{\partial x \partial y} (x-x_0)(y-y_0) + \frac{1}{2} \frac{\partial^2 d(x_0, y_0)}{\partial y^2} (y-y_0)^2, \quad (3)$$

where

$$\mathbf{p} = \left[d(x_0, y_0), \frac{\partial d(x_0, y_0)}{\partial x}, \frac{\partial d(x_0, y_0)}{\partial y}, \frac{\partial^2 d(x_0, y_0)}{\partial x^2}, \frac{\partial^2 d(x_0, y_0)}{\partial x \partial y}, \frac{\partial^2 d(x_0, y_0)}{\partial y^2} \right]^T, \quad (4)$$

Getting the $d0$ and $d1$ models is straightforward.

The advantage of having models is now obvious. The numbers of parameters in $d0$, $d1$ and $d2$ models are 1, 3 and 6, respectively. Given a block of data with, say, $41 \times 41 = 1681$ pixels, we only need no more than 6 parameters to fully describe them, and the description is usually very close to the real data.

Once we have the model, we estimate the model parameters. The optimal estimation can be obtained by minimizing the difference between the model and the observed data in the least squares sense,

$$\hat{\mathbf{p}} = \arg \max_{\mathbf{p}} \left| f_{\text{model}}(x, y; \mathbf{p}) - f(x, y; x_0, y_0) \right|^2 \quad (5)$$

where $\hat{\mathbf{p}}$ indicates the estimated parameter vector. This approach can be used for both denoising and demodulation.

3.3 Tracking

The word “tracking” is taken from “regularized phase-tracking” [22], but the tracking function has actually been used in path-dependent phase unwrapping techniques much earlier. Tracking means that after one pixel has been treated, its neighboring pixels are processed. In the above windowing and optimization steps, all the windows are independent. The tracking step merges these independent local results into a global field. The globalization step plays two important roles:

- **Ambiguity removal** The previous result may have some ambiguities which should be removed. The purpose of phase unwrapping is to remove the order ambiguity [23, 24]. The fundamental assumption in phase unwrapping is that the true phase is continuous. Since for each point, its continuity is defined on a small area containing this point, moving from this point to its neighboring points is a natural choice. For our discrete images, we move from a pixel to its neighboring pixels. Subsequently, a path is formed. Another ambiguity is the sign ambiguity appearing in demodulation of a single fringe pattern [25].

- **Initialization** Although each window can be optimized independently, if there are a number of model parameters to be optimized, a good initial value is necessary. The most effective solution is to borrow the result from a neighbor that has already been processed [26, 27].

In both roles, the result of an already treated pixel is used as the reference value for the current pixel. We would rather trust a neighbor who can provide a reliable reference. Such an idea leads to the quality-guided tracking, namely, the processing flows from a high quality pixel to a low quality pixel.

4. THE WOT STRATEGY IN FRINGE PATTERN ANALYSIS AND DIC

In FPA, the windowed Fourier ridges (WFR) algorithm is an obvious product of the WOT strategy. Firstly, the word “windowed” is explicitly given in the name of the algorithm. Secondly, the word “ridges” highlights that the result is “optimized”. Thirdly, it is followed by the quality-guided phase unwrapping technique for “tracking” [24]. The regularized phase tracking (RPT) algorithm also processes a fringe pattern window by window, optimizes a local energy function, and uses a fringe-follower for tracking [28]. There are more algorithms described in detail in [11] and thus omitted here. It will be interesting to see whether and how the WOT strategy is used in these algorithms.

In DIC, the u - and v - displacements are estimated in subsets and thus it is “windowed”. In each window, these two displacements have been modeled as zeroth, first and second order polynomials, which are denoted as u_0v_0 , u_1v_1 , u_2v_2 models, respectively. A higher order model fits the real displacement data better, at the cost of increased implementation complexity and longer computing time. The u_1v_1 model was dominantly used. The development of computing capability enables reasonably fast optimization convergence, which makes the u_2v_2 model more popular than before [29]. Taking the u_2v_2 model as an example, we simply replace d in Eq. (3) by u to get the u -displacement, and then by v to get the v -displacement. The parameter estimation can be written as

$$\hat{\mathbf{p}} = \arg \max_{\mathbf{p}} \left| F(x, y; x_0, y_0) - G[x + u(x, y; \mathbf{p}), y + v(x, y; \mathbf{p})] \right|^2, \quad (6)$$

where F and G are two speckle patterns before and after deformation, respectively. The model parameter vector \mathbf{p} includes 12 parameters, 6 for u and 6 for v . This vector is directly optimized from Eq. (6) so the estimated vector is considered optimal. Tracking is used for initialization purpose, as there is not ambiguity problem in DIC. The line scanning was quite often used, but a more reasonable and effective quality-guided tracking path has been proposed [27]. We thus see clearly that DIC also adopts the WOT strategy.

5. DISCUSSIONS

In Sec. 4, the similarities between FPA and DIC are elucidated by showing that the WOT strategy is used in both of them. In this section, a few interesting points are discussed.

- **Window selection** Selecting a suitable window shape is a common problem in both FPA [11] and DIC [30], so is selecting a suitable window size [12, 13].
- **Optimization** The optimization step is most challenging among the three steps of the WOT. This step can be further divided into two sub-steps: selecting a proper model (what to optimize) and selecting an optimization technique (how to optimize). The importance of a good model is highlighted in [31]. Optimization techniques have been well developed [32]. Generally speaking, what to optimize is more challenging than how to optimize.
- **Optimal results** Given a window of pixels, the optimization step attempts to find an optimal estimation. Whether the result is optimal depends on a few factors: (i) the data model should be able to reflect the measured data. If the data is quadratic, but the data model is linear, the optimization result may be optimal with respect to the energy function, but it is not optimal to the problem itself. As an example, if the real phase is linear, the WFR result is optimal, but if the real phase is quadratic, the WFR result is sub-optimal; (ii) a smaller window size is preferred if such a window is able to fulfill the task. In the dN model, a smaller N requires a smaller window size; (iii) some algorithms try to simplify the optimization process, resulting in sub-optimal results [33, 34].
- **Sub-optimal results** Sub-optimal results are not necessarily bad. If they satisfy the measurement requirement on accuracy and precision, they should be considered as good results. In DIC, optimal results have always been pursued.

- **Tracking in FPA** Tracking can be performed in two ways: blended tracking and post-tracking. In the blended tracking, the windows are processed sequentially. When a window is processed, it is tracked to a new window. Usually the initial value is transferred from the previous pixel to the current pixel. If there is an ambiguity problem, it is solved simultaneously. The RPT uses blended tracking. The whole process is sequential. In the post-tracking, all windows are processed independently, followed by post-tracking to remove ambiguities. In this case, borrowing an initial value from a processed neighbor for optimization is not required. This is possible when, for example, exhaustive search is used [10]. In this process, the windowed processing is parallelizable, but the tracking is sequential.
- **Tracking in DIC** In DIC, since there is no ambiguity problem, the tracking is only for good initial value setting and thus it is blended [27]. If the initial value can be set without transferring from another pixel, the tracking is then not necessary. In a recent work, the initial values for all windows are independently obtained by a local Fourier transform [7], which enables the DIC without tracking, or called path-independent DIC [35].
- **Discontinuities** The functions we concern are treated as smooth but sometimes they are not. The nice thing is that, when discontinuities exist, the optimization process converges differently. More iterations are needed for converges, or no convergence can be achieved. Thus the number of iterations (NI) [26] can be used to identify the discontinuities. The discontinuity problem in both FPA and DIC is still being investigated.
- **Differences** Although very similar, FPA and DIC are two different techniques. For example, in FPA, we look at the same pixel location with the changing intensity, while in DIC, we look at the same intensity with the changing pixel location; the ambiguity removal is needed in FPA, but not in DIC; the intensity interpolation is needed in DIC, but not in FPA.

6. CONCLUSIONS

We analyze that the adoption of windowing, optimizing and tracking (WOT) strategy in measurement data analysis. This strategy has indeed been used in many algorithms in fringe pattern analysis and digital image correlation. The similarities between fringe pattern analysis and digital image correlation are naturally shown. Some other general similarities are also discussed. With these similarities, links between FPA and DIC are built.

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REFERENCES

- [1] Creath, K., "Phase-measurement interferometry techniques," [Progress in Optics], Vol. XXVI, 349-393, Elsevier, (1988).
- [2] Takeda, M. Ina, H. and Kobayashi, S., "Fourier-transform method of fringe-pattern analysis for computer-based topography and interferometry," J. Opt. Soc. Am. 72, 156-160 (1982).
- [3] Sutton, M. A., Orteu, J. and Schreier, H. W., [Image Correlation for Shape, Motion and deformation measurement], Springer, (2009).
- [4] Pan, B., Qian, K., Xie, H. and Asundi, A., "Two-dimensional digital image correlation for in-plane displacement and strain measurement: a review," Meas. Sci. Technol. 20, 062001 (2009).
- [5] Kemao, Q., Bing, P., Feng, L., Soon, S. H. and Asundi, A., "Links between regularized phase tracking/windowed Fourier ridges for fringe pattern analysis and digital image correlation," Proc. SPIE 7375, 737527 (2009).
- [6] Motion capture, http://en.wikipedia.org/wiki/Motion_capture, (accessed May 2014).
- [7] Chen, D. J., Chiang, F. P., Tan, Y. S. and Don, H. S., "Digital speckle-displacement measurement using a complex spectrum method," Appl. Opt. 32, 1839-1849 (1993).

- [8] Aebischer, H. A. and Waldner, S., "A simple and effective method for filtering speckle-interferometric phase fringe patterns," *Opt. Comm.* 162, 205-210 (1999).
- [9] Kemao, Q., "Windowed Fourier transform for fringe pattern analysis," *Appl. Opt.* 43, 2695-2702 (2004).
- [10] Kemao, Q., "Two-dimensional windowed Fourier transform for fringe pattern analysis: principles, applications and implementations," *Opt. Lasers Eng.* 45, 304-317 (2007)
- [11] Kemao, Q., [Windowed Fringe Pattern Analysis], SPIE Press (2013).
- [12] Kemao, Q., "On window size selection in the windowed Fourier ridges algorithm," *Opt. Lasers Eng.* 45, 1186-1192 (2007).
- [13] Pan, B., Xie, H., Wang, Z., Qian, K. and Wang, Z., "Study on subset size selection in digital image correlation for speckle patterns," *Opt. Express* 16, 7037-7048 (2008).
- [14] Kai, L. and Kemao, Q., "Fast frequency-guided sequential demodulation of a single fringe pattern," *Opt. Lett.* 35, 3718-3720 (2010).
- [15] Gao, W., Huyen, N. T. H., Loi, H. S. and Kemao, Q., "Real-time 2D parallel windowed Fourier transform for fringe pattern analysis using Graphics Processing Unit," *Opt. Express* 17, 23147-23152 (2009).
- [16] Pan, B. and Li, K., "A fast digital image correlation method for deformation measurement," *Opt. Lasers Eng.* 49, 841-847 (2011).
- [17] Singh, A. and Omkar, S. N., "Digital Image Correlation using GPU Computing Applied to Biomechanics," *Biomedical Science and Engineering* 1, 1-10 (2013).
- [18] Ma, S., Quan, C., Zhu, R., Tay, C. J., Chen, L. and Gao, Z., "Micro-profile measurement based on windowed Fourier transform in white-light scanning interferometry," *Opt. Comm.* 284, 2488-2493 (2011).
- [19] Buades, A., Coll, B. and Morel, J.-M., "A non-local algorithm for image denoising," *IEEE conf. Computer Vision and Pattern Recognition 2005*, Vol. 2, 60-65 (2005).
- [20] Fu, S. and Zhang, C., "Fringe pattern denoising using averaging based on nonlocal self-similarity," *Opt. Comm.* 285, 2541-2544 (2012).
- [21] Wielgus, M. and Patorski, K., "Non-local fringe image filtration: a new interferometric data filtration paradigm?" *Photonics Letters of Poland* 4, 66-68 (2012).
- [22] Servin, M., Marroquin, J. L. and Cuevas, F. J., "Demodulation of a single interferogram by use of a two-dimensional regularized phase-tracking technique," *Appl. Opt.* 36, 4540-4548 (1997).
- [23] Ghiglia, D. C. and Pritt, M. D., [Two-Dimensional Phase Unwrapping: Theory, Algorithm and Software], John Wiley & Sons, Inc., (1998).
- [24] Kemao, Q., Gao, W. and Wang, H., "Windowed Fourier filtered and quality guided phase unwrapping algorithm: on locally high-order polynomial phase," *Appl. Opt.* 49, 1075-1079 (2010).
- [25] Wang, H. and Kemao Q., "Frequency guided methods for demodulation of a single fringe pattern," *Opt. Express* 17, 15118-15127 (2009).
- [26] Kai, L. and Kemao Q., "A generalized regularized phase tracker for demodulation of a single fringe pattern," *Opt. Express* 20, 12579-12592 (2012).
- [27] Pan, B., "Reliability-guided digital image correlation for image deformation measurement," *Appl. Opt.* 48, 1535-1542 (2009).
- [28] Servin, M., Marroquin J. L. and Cuevas, F. J., "Fringe-follower regularized phase tracker for demodulation of a closed-fringe interferograms," *J. Opt. Soc. Am. A* 18, 689-695 (2001).
- [29] Gao, Y., Cheng, T., Su, Y., Xu, X., Zhang, Y. and Zhang, Q., "High-efficiency and high accuracy digital image correlation for three-dimensional measurement," *Opt. Lasers Eng.*, submitted (2014).
- [30] Wang, M., Cen, Y., Hu, X., Yu, X., Xie, N., Wu, Y., Xu, P. and Xu, D., "A weighting window applied to the digital image correlation method," *Opt. & Laser Technol.* 41, 154-158 (2009).
- [31] Elad, M., "Sparse and redundant representation modeling – what next?" *IEEE Signal Processing Letters* 19, 922-928 (2012).
- [32] Madsen, K. and Nielsen, H. B., [Introduction to Optimization and Data Fitting], DTU, www.imm.dtu.dk (2010).
- [33] Gorthi, S. S. and Rastogi, P., "Analysis of reconstructed interference fields in digital holographic interferometry using the polynomial phase transform," *Meas. Sci. Technol.* 20, 375307 (2009).
- [34] Gorthi, S. S., Rajshkhar, G. and Rastogi, P., "Strain estimation in digital holographic interferometry using piecewise polynomial phase approximation based method," *Optics Express* 18, 560-565 (2009).
- [35] Jiang, Z., Kemao, Q., Miao, H., Yang, J. and Tang, L., "Path-independent digital image correlation with high accuracy, speed and robustness," *Opt. Lasers Eng.*, submitted (2014).