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Ultra-high Q even eigenmode resonance in terahertz metamaterials
Ibraheem Al-Naib, Yuping Yang, Marc M. Dignam, Weili Zhang, and Ranjan Singh

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Dual-band terahertz metamaterials based on nested split ring resonators
Ultra-high $Q$ even eigenmode resonance in terahertz metamaterials

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We report the simultaneous excitation of the odd and the even eigenmode resonances in a periodic array of square split-ring resonators, with four resonators per unit cell. When the electric field is parallel to their gaps, only the two well-studied odd eigenmodes are excited. As the resonators are rotated relative to one another, we observe the emergence and excitation of an extremely sharp even eigenmode. In uncoupled split-ring resonators, this even eigenmode is typically radiative in nature with a broad resonance linewidth and low $Q$-factor. However, in our coupled system, for specific range of rotation angles, our simulations revealed a remarkably high quality factor ($Q \approx 100$) for this eigenmode, which has sub-radiant characteristics. This type of quad-supercell metamaterial offers the advantage of enabling access to all the three distinct resonance features of the split-ring resonator, which consists of two odd eigenmodes in addition to the high-$Q$ even eigenmode, which could be exploited for high performance multiband filters and absorbers. The high $Q$ even eigenmode could find applications in designing label free bio-sensors and for studying the enhanced light matter interaction effects. © 2015 AIP Publishing LLC [http://dx.doi.org/10.1063/1.4905478]

The terahertz (THz) gap in the electromagnetic spectrum has been an intense area of research over the last decade. Due to the shortage of efficient sources and detectors, the THz regime has been a relatively underexplored region of the electromagnetic spectrum in comparison to the microwave and optical bands. Hence, there has been a sustained effort to develop basic system components to manipulate the THz radiation such as filters, modulators, sensors, waveguides, and imaging devices.1–3 Metamaterials (MTMs) represent a revolutionary technology for the development of THz components with simple designs by employing planar structures due to their reconfigurable properties and ease of fabrication. As a result, these materials have been widely utilized at THz frequencies for the development of novel devices.4–6 They are mainly designed as a planar array of subwavelength metallic resonators, namely, the split-ring resonators (SRRs).

The concept of MTMs relies heavily on the resonance properties of the SRRs. Various MTM configurations have been proposed to manipulate the fundamental and higher order resonances in SRRs.7,8 Moreover, the influence of near and far field electromagnetic field coupling between SRRs with different orientations has been investigated by several groups.9–21 Sharp, high quality factor ($Q$-factor) resonances are of particular importance, as they can be utilized to realize narrow-band filters,22–24 slow light devices, and ultrasensitive thin-film sensors.25–29 It has been observed that conventional planar MTMs have quite low quality factors due to the high radiation and Joule losses. At THz frequencies, the resonance linewidths in MTMs are limited by radiation losses as the ohmic losses are quite low in most plasmonic metals in this frequency regime.30,31 One of the most common strategies to control radiation losses in MTMs is to break the symmetry of double split resensors; these are typically known as asymmetric split-ring resonators (ASRs).32–34 In ASRs, a Fano-like asymmetric lineshape resonance32,35 with a high $Q$ is excited. Various designs of ASRs have been proposed for filtering,24 slow-light devices,35 sensing,36,37 and lasing spasers.38 In addition to the ASRs, high $Q$ resonances can also be excited in MTMs through different coupling schemes among the meta-molecules, by forming a supercell that consists of a group of closely spaced SRRs.

Individual SRRs, such as those shown in Fig. 1, typically support odd or even eigenmodes. However, the simultaneous excitation of both even and odd eigenmodes is forbidden due to the symmetry constraints of the structure with respect to the incident electric field. At normal incidence, when the electric field is parallel to the gap of the SRRs, odd resonances are excited. The lower frequency odd...
mode is called as the fundamental inductive–capacitive (LC) resonance \((n = 1)\), and the higher order mode \((n = 3)\) is quadrupolar in nature.\(^8\) When the incident electric field is perpendicular to the gap bearing arms of the SRR, only the even eigenmode \((n = 2)\) with dipolar characteristic is excited. The dipolar \((n = 2)\) resonance mode is highly radiative in nature due to parallel oscillating currents and have low \(Q\) factors.\(^{14}\)

In this work, we report two distinct findings (a) the simultaneous excitation of the odd and the even eigenmodes and (b) ultra-high \(Q \sim 100\) subradiant dipolar resonance excitation \((n = 2)\). We achieve the simultaneous excitation of odd and even eigenmodes by choosing a quad SRR design that forms a supercell. When the four SRRs are gradually rotated around their respective centers (as indicated in Fig. 1), we observe the emergence of an ultra-sharp even eigenmode that appears between the already-excited odd eigenmodes in the transmitted spectrum. We also perform a systematic study to understand the effects of mutual interaction among the quad SRRs by sweeping the rotation angle. We find from our simulations that an ultrahigh \(Q\)-factor of 98 can be achieved for very small angle of 2° under the horizontal polarization, and vice-versa at 88° under the vertical polarization.

Three different MTM arrays with different mutual rotation angles were considered: 0°, 45°, and 75° as shown in Figs. 1(a)–1(c), respectively. Each SRR has a side length of \(l = 36 \mu m\), a width of \(w = 6 \mu m\), a gap of \(g = 3 \mu m\), and periodicity of the supercell of \(p = 120 \mu m\). They are patterned as a 200 nm gold layer on top of a 640 nm thick silicon wafer (refractive index of 3.42) using conventional photolithography. The transmission through the MTM samples was determined using a typical terahertz time-domain spectroscopy (THz-TDS) system.\(^{39}\) A reference scan was taken using a bare silicon wafer and was followed by a measurement of the fabricated MTM samples. All the measurements were performed at a normal incidence such that the electric (E) and magnetic (H) fields of the incident radiation were in the MTM plane. Two orthogonal polarizations of electric field excitation were considered, first horizontal with the electric field along the \(x\)-axis and then vertical with electric field along the \(y\)-axis.

The measured transmission spectra for horizontal and vertical polarizations for the three samples are shown in Figs. 2(a) and 2(c), respectively; while the corresponding numerical simulations are shown in Figs. 2(b) and 2(d). The simulations have been performed using the commercial software Microwave Studio CST. Its frequency domain solver, which is based on finite element method to solve the Maxwell equations, has been employed to calculate the transmission amplitude. Moreover, unit cell boundary conditions along with Floquet ports have been employed. Reasonable agreement has been obtained between the simulations and the measurements. The simulations reproduce the main features of the measurements such as the excitation of all resonances and their resonance frequencies. We attribute the discrepancy in the sharpness of the resonances between the theory and the experiments to the limited resolution of the measurements which were performed with a 17 ps time scan. We are limited to a 17 ps scan due to the Fabry-Perot reflections arising from the rear surface of the substrate.

\[Q = f_r/\Delta f,\]

FIG. 2. Measured ((a) and (c)) and simulated ((b) and (d)) amplitude transmission spectra for the three samples with different rotation angles for horizontal ((a) and (b)) and vertical ((c) and (d)) polarizations. The shaded blue, yellow, and green areas are a guide to the approximate frequency ranges of the resonances of \(n = 1, 2,\) and 3 eigenmodes, respectively.

When the exciting electric field is horizontally polarized (see Figs. 2(a) and 2(b)) and the rotation angle is 0°, the odd eigenmodes of the quad SRRs resonate, namely, the LC mode \((n = 1)\) at about 0.5 THz and the quadrupole \((n = 3)\) eigenmode at about 1.5 THz. From simulations, we find that rotating the quad SRRs by an angle as small as 1° around their centers (not shown) allow the excitation of the even eigenmode \((n = 2)\) between the aforementioned odd eigenmodes at about 1 THz. This is due to the additional asymmetry of the rotated quad SRRs with respect to the excitation field. When the rotation angle is increased to 45°, the even eigenmode is enhanced and well defined. As the rotation angle is further increased to 75°, the even eigenmode and the quadrupole mode \((n = 3)\) merge and give rise to the conventional dipolar even eigenmode, and more interestingly the LC, \(n = 1\) resonance dip becomes much narrower and shallower. The \(Q\)-factor of the LC mode, defined as \(Q = f_r/\Delta f\), where \(f_r\) is the resonance frequency and \(\Delta f\) is the full width at half maximum (FWHM), is enhanced by a factor of four at the 75° rotation angle to a final value of \(Q = 24.3\), compared to a value of \(Q = 6.2\) when the SRRs are not mutually rotated.

In the case of vertical polarization, where the exciting electric field is aligned with the \(y\)-axis (see Figs. 2(c) and 2(d)), when the rotation angle is 0°, only the even, \(n = 2\) eigenmode, which is clearly a dipole resonance, appears in the spectral response of the SRR supercell array. Rotating the quad SRRs allows the odd eigenmodes to be excited...
simultaneously, i.e., both the \( LC \) resonance at the lower frequency of 0.5 THz and the \( n = 3 \) eigenmode at the higher frequency of 1.5 THz are excited. The dipolar even eigenmode undergoes a tremendous line narrowing of its resonance, which is a transition from a highly radiating dipole mode to a subradiant dipole resonance mode. This is mainly caused by the coherent near field interaction between the quad SRRs of the supercell. The response of the 45\(^\circ\) rotated quad SRRs is identical to the other polarization which is not surprising. However, at 75\(^\circ\) rotation angle the behavior is very interesting as one can see that all three resonances are clearly excited and the even eigenmode has a very high \( Q \)-factor of 45 in the simulation. Conversely, the measured \( Q \) is only 21 due to the limited time scan. This behavior can exactly be seen at a rotation angle of 15\(^\circ\) under the horizontal polarization. The other two odd eigenmodes revealed \( Q \)-factors of 5.8 and 4.6 for the \( LC \) and quadrupole modes, respectively. The discrepancy in the experimental and simulated \( Q \)-factor for these two modes is less than 8%.

We now examine the critical angles at which the different resonances appear and disappear. To accomplish this, we simulated the quad SRRs in steps of angular rotation of 2\(^\circ\) at each polarization. The results are shown in Figs. 3(a) and 3(b), where the electric field is horizontal and vertical, respectively. In the case of a horizontal electric field, only the two odd eigenmodes are excited at an angle of 0\(^\circ\) as mentioned earlier. Interestingly, the \( LC \) eigenmode exhibits quite large narrowing until it nearly disappears at an angle of 80\(^\circ\).

The quadrupole mode red-shifts, broadens, and almost merges with the even eigenmode beyond an angle of 60\(^\circ\). The most fascinating event is the immediate appearance of the previously forbidden even eigenmode at about 1 THz with 2\(^\circ\) of angular rotation and its extremely sharp nature (\( Q = 98 \)). However, the resonance modulation depth [1 \( - \) (the transmission depth at resonance)] is quite low (2.4\%) for small rotation angles and only increases gradually. We would like to stress here that the even eigenmode is the dark eigenmode that is forbidden at a rotation angle of 0\(^\circ\) and it is only when the symmetry is broken by introducing a small angular rotation that we observe the appearance of the sharp dark eigenmode as it is very weakly coupled to the free space. As the degree of asymmetry is enhanced by increasing the angular rotation, this dark even eigenmode broadens with an enhancement in the amplitude depth. We will return to a discussion of the trade-off between the \( Q \)-factor and the amplitude modulation towards the end of the paper. The range of the rotation angles at which all the odd and even modes remain excited falls between 1\(^\circ\) and 55\(^\circ\).

For the vertical electric field case in Fig. 3(b), only the dipole mode is excited at the 0\(^\circ\) angle of rotation, since in this case, the odd modes (\( n = 1,3 \)) remain dark and unexcited. The \( LC \) eigenmode appears at the rotation angle of about 10\(^\circ\) beyond which its resonance linewidth broadens. The even dipole mode (\( n = 2 \)) transitions from a radiative low \( Q \) resonance to a subradiant high \( Q \) dipole resonance as the quad SRRs are rotated gradually. The high \( Q \) characteristic of this even eigenmode exists between angles of 56\(^\circ\) and 88\(^\circ\) (25.5 < \( Q < 98 \)). As aforementioned, the measured \( Q \)-factor at a rotation angle of 75\(^\circ\) is 21 only, while the simulated one is 45. In order to achieve a higher spectral resolution and measure such a high \( Q \)-factor resonance, a longer time scan is required and hence either free-standing sample or a thicker substrate on the order of few millimeters should be employed. Instead, continuous-wave THz spectrometers can be utilized for the measurements as they can offer a resolution of less than 1 GHz.

In order to understand the physical mechanism behind the sharpness of the even eigenmode of the quad SRRs, we investigate the simulated surface current and electric field distributions for the vertical polarization with a 75\(^\circ\) rotation angle as shown in Fig. 4 at the respective resonance frequencies, \( f_n \). The distributions clearly show the characteristic behavior of the \( n = 1,2,3 \) modes, where \( n \) denotes the eigenmode of the SRR. As can be seen in Figs. 4(a) and 4(d), respectively, the current is large at the first resonance (\( f_1 = 0.5 \) THz) and the electric field is mainly confined in the gap of the SRRs. Most notably, the current distribution in each horizontally neighboring SRRs is out-of-phase. This explains the narrow linewidths, especially at small rotation angle where there is a quite large cancellation of the dipole moment. More interestingly, at the even eigenmode resonance (\( f_2 = 0.993 \) THz), we observe parallel surface currents along the side arms of the SRR (\( n = 2 \)) as shown in Fig. 4(b) and is also evident in the field distribution in Fig. 4(e). It is important to emphasize that the surface current is out-of-phase in the neighboring SRRs in both the horizontal and vertical directions. Therefore, the cancellation of the dipole moment is maximum in this case, which explains the subradiant nature of this particular eigenmode. At the

![Figure 3](image-url)
third resonance $f_1 = 1.462$ THz, the conventional quadrupole resonance is observed. The surface current as well as the field confinement shown in Figs. 4(c) and 4(f) are quite weak in agreement with the simulated and experimental results shown in Figs. 2(c) and 2(d), respectively.

After analyzing the transmission response and the current distributions for the two polarizations, we performed a parametric study of the effect of the rotation angle ($\theta$) on the behavior of the particularly interesting even eigenmode ($n = 2$). Fig. 5 shows the modulation depth (left scale) and the $Q$ factor (right scale) versus the rotation angle when the electric field is vertically polarized. The $Q$-factor increases dramatically as the rotation angle changes from 60° to 90°, while the modulation depth decreases rapidly, going to zero for an angle of 90°. There is an obvious desire to maximize the $Q$ factor and the modulation depth of the resonances, but our results shows that there is a trade-off between these two quantities. For example, we find from our simulations that a $Q$-factor of ~100 is achievable, but only with a modulation depth of 2.4%.

To conclude, we have demonstrated a supercell for a MTM structure where even and odd eigenmodes are excited simultaneously. This design can also support extremely sharp high $Q$ even eigenmode that was previously known to be highly radiative and low $Q$ in nature. The supercell consists of four interacting SRRs that are mutually rotated. The specific even eigenmode that allows the observation of such sharp resonances has been explored in detail. We also demonstrate that we could engineer the $Q$ factor and the amplitude depth of each resonance of the SRR by controlling the mutual rotation angle of the SRRs. The MTMs that we have designed can be used for various applications, e.g., for the realization of ultrasensitive sensors and narrowband filters. The design principles we have introduced could be applied to other geometries of MTMs across large parts of the electromagnetic spectrum.

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