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Design, implementation, and extension of thermal invisibility cloaks
Youming Zhang, Hongyi Xu, and Baile Zhang

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A thermal invisibility cloak, as inspired by optical invisibility cloaks, is a device which can steer the conductive heat flux around an isolated object without changing the ambient temperature distribution so that the object can be “invisible” to external thermal environment. While designs of thermal invisibility cloaks inherit previous theories from optical cloaks, the uniqueness of heat diffusion leads to more achievable implementations. Thermal invisibility cloaks, as well as the variations including thermal concentrator, rotator, and illusion devices, have potentials to be applied in thermal management, sensing and imaging applications. Here, we review the current knowledge of thermal invisibility cloaks in terms of their design and implementation in cloaking studies, and their extension as other functional devices. © 2015 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution 3.0 Unported License. [http://dx.doi.org/10.1063/1.4913996]

I. INTRODUCTION

Metamaterials are materials with designed artificial structures that exhibit unusual physical properties. Studies of metamaterials initiated from electromagnetic research. The ability of metamaterials to produce tailored constitutive parameters has provided effective ways to engineer electromagnetic space where the propagation of electromagnetic waves can be controlled almost at will. As one of the most influential benchmarks, invisibility cloaking has drawn a large amount of research interests in the past few years. Significant achievements of invisibility cloaking at both microwave and optical frequencies have been reached based on the theory of transformation optics and other alternative approaches. However, almost all experimental works reported so far suffered from severe limitations such as narrow bandwidth, only one polarization, impedance mismatch, and limited directions etc. which have severely hindered practical applications of optical cloaking. The fundamental reason is that bending light without disturbing the optical phase implies superluminal propagation, which is only possible in an extremely narrow band with strict constraints.

Recently the idea of cloaking (guiding field and energy flux around an object) has been extended to other physical disciplines including thermodynamics, acoustics and mechanics. Compared with electromagnetic waves which carry phase, the manipulation of heat flux is comparably easier due to its diffusion nature that involves no phase. This means that the first practical cloaking technology is likely to be achieved in heat flux manipulation which is relevant to electronic and energy industries as well as military applications.

Inspired by transformation optics, heat flux manipulation based on coordinate transformation was first studied by Fan, et al. and Chen, et al. in 2008. Later in 2010, Li, et al. adopted the coordinate transformation method for the steady heat conduction equation, based on which a bifunctional spherical cloak working for both static electric field and heat flux was numerically...
demonstrated. In 2012, Guenneau, et al. achieved a systematic theoretical extension of transformation optics to the field of thermal conduction and proposed a practical method to simplify the highly inhomogeneous anisotropic and singular parameters for a transient (before reaching thermal equilibrium) cylindrical thermal invisibility cloak which was later experimentally demonstrated by Schittny, et al. in the following year. Meanwhile, Narayana, et al. experimentally proposed to simplify the thermal invisibility cloak with homogeneous anisotropic materials working in steady state and the design was later theoretically verified in Ref. 26. Recently, as inspired by the successful demonstration of a static magnetic cloak, an alternative way to realize a thermal cloak with homogeneous isotropic structures that is similar to the previous scattering-cancellation plasmonic cloaking approach for electromagnetic waves has been experimentally demonstrated by Xu, et al. in three dimensions and Han, et al. in two dimensions, respectively. Besides the thermal invisibility cloak, other thermal devices have also been proposed with interesting functionalities such as the thermal concentrator, rotator and thermal camouflage. In this review, we will summarize general thermal cloaking approaches, different designs and experimental implementations of thermal invisibility cloaks, as well as their extension including thermal concentrator, thermal rotator and thermal camouflage.

II. APPROACHES

Let us start from the thermal continuity equation:

$$\nabla \cdot \vec{J} + \frac{\partial u}{\partial t} = 0$$

(1)

where $\vec{J}$ is the heat current density and $u$ is the local energy density. This equation describes how the heat current flow causes the local energy to change with time in a close system. Both the heat current density $\vec{J}$ and the local energy density $u$ are related to local temperature $T$ as

$$\vec{J} = -\kappa \nabla T$$

(2)

$$u = c\rho T$$

(3)

where $\kappa$ is the thermal conductivity and $c$ is the specific heat. Substitute Eq.(2) and Eq.(3) into Eq.(1), we obtain the familiar form of the heat conduction equation

$$\nabla \cdot (-\kappa \nabla T) + c\rho \frac{\partial T}{\partial t} = 0$$

(4)

which shows the possibility of controlling temperature variation by engineering the thermal conductivity.

A. Coordinate transformation approach

As the origin of coordinate transformation approach for heat manipulation, the theory of transformation-optics cloaking of light was first proposed by Pendry, et al., and Leonhardt in 2006. This theory provides a systematic mathematical tool of designing wave-controlling devices with metamaterials. In a coordinate transformation, the form of Maxwell’s equations is invariant while only the constitutive parameters and field values are changed. Therefore the full control of electromagnetic waves can be achieved with transformed constitutive parameters.

Similar to the transformation of Maxwell’s Equations, the heat conduction equation can also preserve its form before/after a coordinate transformation by varying the thermal conductivity and specific heat:

$$\nabla' \cdot (-\kappa' \nabla'T') + c'\rho' \frac{\partial T'}{\partial t} = 0$$

(5)

where

$$\kappa' = \frac{A^2 \kappa \bar{A}^T}{\det(\bar{A})}, \quad \rho' c' = \frac{\rho c}{\det(\bar{A})}$$

(6)
and $\mathbf{A}$ is the Jacobian of the coordinate transformation. Following Pendry, et al.’s work,\textsuperscript{2} as shown in Fig.1 (a), the coordinate transformations of compressing the region $r < R_2$ into a shell region $R_1 < r < R_2$ to achieve a two-dimensional cylindrical cloak and a three-dimensional spherical cloak are:

\begin{align}
\text{Cylindrical cloak: } & r' = R_1 + \frac{r (R_2 - R_1)}{R_2}, \quad \phi' = \phi, \quad z' = z \quad (7) \\
\text{Spherical cloak: } & r' = R_1 + \frac{r (R_2 - R_1)}{R_2}, \quad \theta' = \theta, \quad \phi' = \phi. \quad (8)
\end{align}

Assuming the original medium before transformation is homogeneous and isotropic with constant thermal conductivity $\kappa_0$, density $\rho_0$ and specific heat $c_0$, we can obtain the transformed conductivities and specific heat according to Eq. (6):

\begin{align}
\text{Cylindrical cloak: } & \mathbf{\kappa}' = \kappa_0 \begin{bmatrix} \kappa'_{rr} & 0 & 0 \\ 0 & \kappa'_{\phi\phi} & 0 \\ 0 & 0 & \kappa'_{zz} \end{bmatrix}, \quad \rho' c'_s = \frac{r' - R_1}{r'} \left( \frac{R_1}{R_2 - R_1} \right)^2 (\rho_0 c_0) \quad (9) \\
\text{Spherical cloak: } & \mathbf{\kappa}' = \kappa_0 \begin{bmatrix} \kappa'_{rr} & 0 & 0 \\ 0 & \kappa'_{\phi\phi} & 0 \\ 0 & 0 & \kappa'_{zz} \end{bmatrix}, \quad \rho' c'_s = \frac{R_2 - R_1}{R_2} \left( \frac{r'}{r' - R_1} \right)^2 (\rho_0 c_0) \quad (10)
\end{align}

where $\kappa'_{rr} = \frac{r' - R_1}{r'}$, $\kappa'_{\phi\phi} = \frac{r' - R_1}{r'}$, and $\kappa'_{zz} = \left( \frac{R_2 - R_1}{R_2 - R_1} \right)^2$. Similar to the constitutive parameters for optical cloaks, the transformed heat conductivities are inhomogeneous and anisotropic, and the specific heat capacities are also inhomogeneous. Both the transformed heat conductivities and specific heat capacities include singularities ($\rho' c'_s \rightarrow 0$; $\kappa'_{\phi\phi}, \rho' c'_s \rightarrow \infty$) at the inner boundary ($r' = R_1$) of the cloak. These parameters are practically difficult to achieve. Therefore, simplifications or tradeoffs must be made in concern of the practical realization of the cloak.

**B. Scattering cancellation approach**

Another approach to realize invisibility cloaking, different from transformation optics, is to use scattering cancellation to achieve a zero scattering cross section.\textsuperscript{15} The method has been further extended to static magnetic\textsuperscript{41} and electron-transport cloaks.\textsuperscript{44} In a broad sense, heat can also be
regarded as a slowly varying “wave” like a static field. Therefore, we can use a similar method to eliminate the distortion of the isotherms so that a thermal invisibility cloak in steady state can be realized.

Consider homogeneous heat flux conduction in steady state for a core-shell cloak structure in a homogeneous background medium in either cylindrical or spherical coordinates as shown in Fig.1 (b). The thermal conductivity of the background, core and shell are \( \kappa_0, \kappa_1 \) and \( \kappa_2 \), respectively. The shell has an inner radius of \( R_1 \) and an outer radius of \( R_2 \). The heat conduction equation in steady state without additional source is a Laplace equation of temperature as:

\[
\nabla \cdot (-\kappa \nabla T) = 0.
\]

Following the similar method solving the Laplace equation of static magnetic potential,\(^{41}\) Eq.(11) can also be solved for both cylindrical and spherical coordinates when setting the heat flux outside the cloak to have no distortion and the core to have zero thermal conductivity \( \kappa_1 = 0 \). The solutions are\(^{27,28}\)

\[
\begin{align*}
\text{Cylindrical cloak:} & \quad \kappa_2 = \frac{R_2^2 + R_1^2}{R_2^2 - R_1^2} \kappa_0 \\
\text{Spherical cloak:} & \quad \kappa_2 = \frac{2R_2^3 + R_1^3}{2(R_2^3 - R_1^3)} \kappa_0
\end{align*}
\]

which mean that by setting the shell conductivity \( \kappa_2 \) and the inner/outer radius ratio \( R_2/R_1 \) to satisfy Eq.(12) for cylindrical structure or Eq.(13) for spherical structure (both with an adiabatic core \( \kappa_1 = 0 \)), we can achieve thermal invisibility cloaking. It must be emphasized that the above equations are valid with the condition that the external thermal environment possesses a homogeneous temperature gradient. Therefore an inhomogeneous temperature field will deteriorate the cloak’s performance.

III. DESIGN AND IMPLEMENTATION

A. Design and implementation of thermal invisibility cloaks with coordinate transformation approach

Following the work in Ref. 22, if we consider a steady thermal conduction without additional source as in Eq.(11), the transformations of the specific heat capacities as shown in Eq.(9) and Eq.(10) are of no necessity to be considered. For a cylindrical cloak, the singularity in thermal conductivity \( \kappa'_{sb} \rightarrow \infty \) still cannot be ignored. While for a spherical cloak, there is no singularity and the components of \( \bar{\kappa}'_{sj} \) have the range of:

\[
0 \leq \kappa'_{sx} = \frac{R_2}{R_2 - R_1} \left( \frac{r' - R_1}{r'} \right)^2 \leq \kappa_0(R_2 - R_1), \quad \kappa'_{sy} = \kappa'_{sz} = \kappa_0 \left( \frac{R_2}{R_2 - R_1} \right) = \text{const}
\]

which can be realized by thermal metamaterials composed of nanoparticle-distributed medium.\(^{22}\) In Li, et al.’s design, they theoretically proposed to distribute nonspherical nanoparticles with thermal conductivity \( \kappa_m \) (after taking into account the interfacial thermal resistance of the nanoparticles) into a homogeneous medium with thermal conductivity \( \kappa_i \) in order to realize the required thermal conductivity. The effective thermal conductivity is given by the Bruggeman “shape-dependent” effective medium theory\(^{22,45}\)

\[
\frac{\kappa_m - \kappa_{eff}}{\kappa_{eff} + \Gamma(\kappa_m - \kappa_{eff})} p + \frac{\kappa_i - \kappa_{eff}}{\kappa_{eff} + \Gamma(\kappa_i - \kappa_{eff})} (1 - p) = 0
\]

where \( \kappa_{eff} \) is the effective thermal conductivity, \( \Gamma \) is the geometrical shape factors, \( p \) is the volume fraction of the nonspherical nanoparticles. Substituting the required thermal conductivity \( \kappa'_{sx} \) and \( \kappa'_{sb} \) into Eq.(15), the corresponding geometric parameters of the nanoparticles \( \Gamma \) and \( p \) can be solved. In other words, the shape parameters of nanoparticles in radial and tangential direction, as shown in Fig.2 (a), can be settled. The simulation result of this design can be seen in Fig.2 (b).
FIG. 2. Thermal invisibility cloak designs based on coordinate transformation. (a) Illustration of three-dimensional bifunctional spherical cloak design for steady state using nanoparticle doping.\textsuperscript{22} The cloak has inner radius $R_1$ and outer radius $R_2$. (b) Simulated temperature distribution of (a). The little arrows represent heat flux directions. (c)(d)(e) Simulated temperature distribution of the multilayered cylindrical thermal invisibility cloak (20 layers) at different time frame ($t_1 < t_2 < t_3$).\textsuperscript{23} Figure reprinted with permission: a, b, Ref. 22, © 2010 American Institute of Physics; c,d,e, Ref. 23, © 2012 Optical Society of America.

However the complicated shape control and three dimensional spatial distributions of the non-spherical nanoparticles are obviously challenging in experiment, which might explain its missing experimental demonstration.

Another way to simplify the transformed parameters is to reduce the dimensions from three dimensions to two dimensions. For a two-dimensional cylindrical cloak, the Jacobian of the coordinate transformation is $A = \text{diag}(\frac{R_2-R_1}{R_2}, \frac{r'-R_1}{R_2})$. The transformed conductivity and specific heat can be seen from Eq. (9). Following Ref. 23, the transformed thermal conductivity and specific heat in Eq. (9) can be simplified by dividing $\text{det}(A)$, followed by the reduced parameters as

$$\overline{\kappa}'' = \frac{\overline{\kappa}'}{\text{det}(A)} = \kappa_0 \begin{bmatrix} \kappa''_{cr} & 0 \\ 0 & \kappa''_{c\phi} \end{bmatrix}, \quad \rho''_{c} = \frac{\rho' c'}{\text{det}(A)} = \rho_0 c_0 \tag{16}$$

where $\kappa''_{cr} = \left(\frac{r'-R_1}{r}\right)^2 \left(\frac{R_1}{R_2-R_1}\right)^2$, $\kappa''_{c\phi} = \left(\frac{R_1}{R_2-R_1}\right)^2$. We notice that the reduced $\rho''_{c}$ is homogeneous and the reduced conductivity has no singularity anymore. Moreover, only one component of the reduced conductivity is inhomogeneous, and the other component is a constant. However, the reduced parameters in Eq.(16) lead to an approximation of Eq.(5) as:

$$\nabla \cdot \left( \overline{\kappa}'' \nabla T \right) = \frac{1}{\text{det}(A)} \nabla \cdot \left( \overline{\kappa}' \nabla T \right) + \kappa''_{c} \nabla \cdot \left( \frac{1}{\text{det}(A)} \nabla T \right). \tag{17}$$

This approximation can be satisfied only when:

$$\overline{\kappa}'' \nabla \cdot \left( \frac{1}{\text{det}(A)} \nabla T \right) \ll 1 \tag{18}$$
which means \( \det(\mathbf{A}) = \left( \frac{r'}{r''} \left( \frac{R_2 - R_1}{R_1} \right) \right)^2 \gg 1 \). This condition is no longer satisfied near the exterior boundary of the cloak, giving rise to the “impedance mismatch” in the diffusivity \( (D = \frac{\mathbf{A}}{\rho c}) \) between the cloak and the background medium.\(^{21}\) Nevertheless, it is still a valid approximation where the essential function of the cloak, guiding heat flow around certain cloaking area, is still maintained. Guenneau, \textit{et al.} also proposed to use discrete multilayered structure to approximate the continuous inhomogeneous parameters as shown in Fig.2 (c). The numerical simulation results of the multilayered structure at different time frames can be seen in Fig.2 (c-e).

In 2013, Schittny, \textit{et al.} experimentally demonstrated the two dimensional transient cloak based on parameters in Eq. (16).\(^{24}\) They used concentric layer-stack rings to realize the inhomogeneous anisotropic thermal conductivity. It can be shown that if a layered structure is stacked along \( y \) direction with two materials with thermal conductivity \( \kappa_A, \kappa_B \) and thickness \( d_A, d_B \) respectively, the effective heat conductivities are:\(^{24,46}\)

\[
\kappa_{xeff} = d_A \kappa_A + d_B \kappa_B, \quad \kappa_{yeff} = \frac{\kappa_A \kappa_B (d_A + d_B)}{d_A \kappa_B + d_B \kappa_A}. \tag{19}
\]

One can first divide the cloak structure into concentric rings along radial direction. Then for each ring, one can use two materials to stack along the azimuthal direction. By adjusting the sizes of each material, the corresponding parameters can be realized. In Schittny, \textit{et al.}'s work, the inhomogeneous cloak region was divided into 5 discrete rings. For each ring, Copper and PDMS were used to realize the anisotropic parameters, as shown in Fig.3 (a). As can be seen in Fig.3 (b), the copper disk at the center of the device was “cloaked” without disturbing the exterior profile of the isotherms. Slight perturbations still existed, which is because of the discrete approximation of the gradient parameters. In Ma, \textit{et al.}'s work,\(^ {34}\) the discrete approximation was also used to

FIG. 3. Experimental implementation of thermal invisibility cloaks based on coordinate transformation. (a) Schittny, \textit{et al.}'s two-dimensional transient cloak\(^ {24}\) consists of copper rings and bridges interlaid with PDMS. The copper disk in the center of the device is concealed from thermal sensing outside of the device. (b) Experimentally measured temperature distribution of (a) at \( t = 120s \). The isotherms are marked in white solid lines. (c) Ma, \textit{et al.}'s two-dimensional transient cloak\(^ {34}\) made of sub-layers with five different material ingredients. (d) Experimentally measured temperature distribution of (c) at \( t = 15\) mins. The isotherms are marked in white solid lines. Figure reprinted with permission: a, c, Ref. 24, \( \copyright \) 2013 American Physical Society; c, d, e, Ref. 34, \( \copyright \) 2013 Nature Publishing Group.
realize the inhomogeneity. The main difference, as shown in Fig. 3 (c), is that the anisotropic parameters of each ring were realized with composites composed of more materials with different volume percentages. The experimental result of the state approaching thermal equilibrium (t = 15 mins in Ref. 34) can be seen in Fig. 3 (d).

As a simplification of the designs based on coordinate transformation introduced above, Narayana, et al. first numerically and experimentally demonstrated an alternative way to realize thermal cloak (as well as designs of thermal concentrator and inverter, which will be introduced in Section IV) for steady state with homogeneous anisotropic materials.25 Examining inhomogeneous transformed parameters in Eq. (9) for a two-dimensional cylindrical cloak, one can see that $\kappa'_{cr} \kappa'_{c\phi} = \kappa_0^2$. It is shown in Ref. 26 that cloaking for steady homogeneous heat flux can be achieved by making

$$\kappa'_{cr} = \frac{1}{\kappa'_{c\phi}} = C \kappa_0$$

(20)

where $C$ is a constant which satisfies $0 < C < \frac{\log(R_1/R_2)}{\log(R_1/R_2) - 1} < 1$ and $\kappa_0$ is the thermal conductivity of the background medium. Besides, as shown in Ref. 26, the smaller $C$, the smaller the cloak can be. Compared with inhomogeneous parameters based on coordinate transformation, homogeneous anisotropic materials are easier to achieve just by stacking two homogeneous isotropic materials in the radial direction.

In Narayana, et al.’s experiments,25 40 alternating layers of natural latex rubber and baron-nitride-doped silicone elastomers were used, as shown in Fig. 4 (a), to construct the anisotropic

![Experimental implementation of thermal invisibility cloaks with homogeneous anisotropic parameters.](image-url)

**FIG. 4.** Experimental implementation of thermal invisibility cloaks with homogeneous anisotropic parameters. (a) Illustration of Narayana, et al.’s two-dimensional steady cloak,25 made of 40 alternating layers of natural latex (Material A) rubber and baron-nitride-doped silicone elastomers (Material B) in a background of agar-water. (b) Experimentally measured temperature distribution of (a) when reaching thermal equilibrium. (c) Dede, et al.’s two-dimensional steady cloak fabricated with glass-epoxy FR-4 printed circuit board.30 (d) Experimentally measured temperature distribution of (c) when reaching thermal equilibrium. Figure reprinted with permission: a, b, Ref. 25, © 2012 American Physical Society; c, d, Ref. 30, © 2013 American Institute of Physics.
material for a two dimensional cylindrical cloak working in the background of agar-water. The experimental observation is shown in Fig.4 (b). Similar design using glass-epoxy FR-4 printed circuit board\textsuperscript{30} and the experimental results are shown in Fig.4 (c) and (d), respectively. Other numerical designs using composition of natural latex rubber and thermal epoxy, and composition of wood and stainless steel can be found in Ref. \textsuperscript{26}.

Besides, in Narayana, et al.’s following work, the significance of the inhomogeneity of the parameters for the cloaking performance in transient state was recognized.\textsuperscript{35} By adopting inhomogeneous thermal conductivity and specific heat, previous cloak design for steady state\textsuperscript{25} could be potentially extended to work in transient state. The homogeneity could be realized by copper and polymide layers with spatially changing thickness ratio. Instead of simplifying parameters derived from coordinate transformation, parameter-sweeps with numerical simulations were used to fix the thickness of each polymide and copper layer.

**B. Design and implementation of thermal invisibility cloaks with scattering cancellation approach**

Recently, thermal cloak designs merely using homogeneous isotropic materials in both two-dimensional and three-dimensional cases have been demonstrated.\textsuperscript{27,28} Both works were inspired by the cloaking approach for static magnetic fields,\textsuperscript{41} the idea of which is similar to previous plasmonic cloaking using scattering cancellation,\textsuperscript{15,42}

![Experimental implementations of homogeneous isotropic thermal invisibility cloaks.](image)

**FIG. 5.** Experimental implementations of homogeneous isotropic thermal invisibility cloaks. (a) Snapshot of Xu, et al.’s ultrathin three dimensional thermal invisibility cloak\textsuperscript{27} making of copper. (b) Experimentally measured temperature distribution of (a) when reaching thermal equilibrium (t = 4.5 mins). (c) Snapshot of Han, et al.’s two dimensional bilayer thermal invisibility cloak\textsuperscript{28} consisting of alloy and polystyrene. (d) Experimentally measured temperature distribution of (c) when reaching thermal equilibrium (t = 60 mins). Figure reprinted with permission: a, b, Ref. 27, © 2014 American Physical Society; c, d, Ref. \textsuperscript{28}, © 2014 American Physical Society.
In Xu, et al.’s experiment, a single-layer three dimensional spherical thermal invisibility cloak was demonstrated, as shown in Fig. 5 (b). It could protect an air bubble from invasion of external thermal flux without disturbing the external environment. By examining the solution for spherical thermal cloak in Eq. (13), it can be found that the radius ratio $R_2/R_1$, which features the relative thickness of the shell, decreases as the thermal conductivity ratio $\kappa_2/\kappa_0$ increases. In their experiment, copper was selected as the shell material, as shown in Fig. 5 (a), due to both its high conductivity over background stainless steel and low cost. The radius ratio of the three-dimensional spherical cloak was only 1.02. Similar two-dimensional bilayer homogeneous isotropic cloak using alloy and polystyrene, as demonstrated in Han, et al.’s experiment, can be seen in Fig. 5 (c). The experimental result is shown in Fig. 5 (d). Both two experiments successfully demonstrated similar thermal cloaking performance in a bulk material environment.

Both these two experiments showed that these cloaks had fairly good cloaking performances not only for homogeneous heat flux in steady state, but also for transient state and inhomogeneous heat flux. To explain why the cloaks also worked for transient state, we shall first go back to the general form of the heat conduction equation in Eq. (4). We find that if the time varying term $c\rho \frac{\partial T}{\partial t}$ is very small, this term can be ignored. This implies that if temperature does not change very fast or the specific heat is very small, the heat conduction can be approximately regarded as steady state. In Ref. 27 and Ref. 28’s experiments, it took 4.5 minutes and 60 minutes, respectively, for the systems to reach thermal equilibrium, which implies that the temperature does not change very fast. Besides, the product of density $\rho$ and specific heat $c$, namely the specific heat capacity per volume, represents the amount of heat required to heat up an unit volume of by an unit temperature increase. Due to the different specific heat capacities per volume of cloak and the ambient medium, it is difficult to synchronize the heating up processes of the cloak and the ambient medium, if the size of the cloak is substantial. However, both those cloaks are very thin compared to the size of the hidden object. This automatically guarantees the synchronization of the heating up process of the cloak and the ambient medium. Both slowly varying temperature and the thin thicknesses of the cloaks are reasons why two cloak designs based on steady state heat conduction still had fairly good cloaking performance for transient state.

As for the inhomogeneous heat flux, it can be shown that isotropic cloaks do not actually work perfectly. If we compare the point source isotherms of the reference and the cloaks, slight differences of the isothermal lines after heat flowing through the cloak can be found. However, as Ref. 41 (calculated in its supporting online material) shows, the isotherm distortions caused by inhomogeneous heat flux can be significantly reduced when the thickness of the cloak is small. Therefore, the cloaks also worked for inhomogeneous heat flux.

Although being imperfect, the homogeneous isotropic thermal cloaks have avoided the extreme parameters of previous cloaks based on coordinate transformation. Compared with homogeneous anisotropic cloaks, a homogeneous isotropic cloak is easier to fabricate and can even be realized with ultrathin thickness which might have advantageous application potential in practice.

IV. THERMAL CONCENTRATOR, ROTATOR AND ILLUSION DEVICES

A. Thermal concentrator

A thermal cloak guides the heat flux around a certain region without changing the isothermal lines outside the cloak. A thermal concentrator, on the contrary, is meant to focus the heat flux at a certain region. The coordinate transformation method can still be used to design a thermal concentrator. However, as shown in Ref. 23, the parameters are also challenging in terms of high inhomogeneity, anisotropy and singularities. In fact, as far as thermal concentrator is concerned, the strict transient performance and the maintaining of the isothermal profiles in a certain region should be secondary. Therefore, as proposed in Ref. 25, a more practical way to realize a thermal concentrator is to use homogeneous anisotropic materials. Opposite to cloak design with anisotropic parameters $0 < \kappa'_{cr} = \frac{1}{\kappa'_{c,\phi}} = C_{k0} < \kappa_0$, thermal concentration can be realized by inverting the
thermal cloak parameters

\[ 0 < \kappa'_c \phi = \frac{1}{\kappa'_r} = C k_0 < \kappa_0 \]  

which can be realized by changing the radially stacked structure of cloak to fan-shaped stacked structure as shown in Fig.6 (a). This design has been experimentally demonstrated in Ref. 25 using composite composed of stacked natural latex rubber and silicone elastomers. The experimental result can be seen in Fig.6 (c). As further analyzed in Ref. 29, the smaller \( \kappa'_c \phi \), the more focused the heat flux in the center of the concentrator. Dele, et al. also proposed the similar structure on a printed circuit board to build a concentrator, as shown in Fig.6 (b). The experimental result is shown in Fig.6 (d). Other similar designs can be found in Refs. 29, 31, and 37.

**B. Thermal rotator**

The function of a thermal rotator is to twist the thermal flux in a certain manner. Particularly, if the heat flux is twisted by \( \pm \pi \) radians, the direction of thermal flux in the central region of the rotator could have an opposite direction with that outside the rotator, functioning as a thermal inverter. One method to design a thermal rotator is to use the coordinate transformation method. The obtained parameters after transformation are inhomogeneous and anisotropic. Another way is to sacrifice the transient state performance. Specifically, furthering the anisotropic concentrator design, a
thermal rotator for steady state can be realized simply by making a simple twisting coordinate transformation of the concentrator:\textsuperscript{25,46}

\[
\phi' = \phi + \phi_t \frac{r - R_1}{R_2 - R_1}
\] (22)

where \( \phi_t \) is the twisting angle. For a thermal inverter, \( \phi_t \) should be \( \pm \pi \), corresponding to anticlockwise twisting and clockwise twisting, respectively. Two experimental implementations are shown in Fig.6 (e)(f). As be seen in Fig.6 (g)(h), the measured temperature distribution was obviously twisted,\textsuperscript{25,30} and in the center, heat flux direction is opposite to that of outside.

C. Thermal illusion devices

Different from cloaking an object from heat flux, thermal illusion devices aim at changing the thermal signature of an object into that of another. By doing so, thermal illusions can be generated to dodge or confuse the thermal inspections. Actually, in a broad sense, a thermal cloak is also a type of thermal illusion device that changes the thermal signature of an object into that of “nothing”. In Han, et al.’s work,\textsuperscript{32} as shown in Fig.6 (i-k), a further step to the two-dimensional isotropic cloak was made by putting two insolated sectors outside the cloak. When testing the heat signature, what the inspectors “see” or “feel” is not the copper cylinder inside camouflage, but the two sectors. Another way to design a thermal illusion device, as proposed in Ref. 33, is making use of coordinate transformation twice, which could generate arbitrary-shape illusions in theory for transient state.

V. CONCLUSION AND OUTLOOK

Initial designs of thermal invisibility cloaks started from the coordinate transformation method as inspired by the idea of optical invisibility cloaking. While the bottlenecks of current optical cloaking technology mainly stem from the phase preservation in transformation optics, thermal cloaking has no such limitations as the concept of phase is inherently not involved in heat diffusion. In that sense, the most promising cloaking technology may start its application in heat flux manipulation. As we have seen in recent years, many research efforts are currently being made in this direction.

Despite the fact that heat dissipation and thermal management already have established engineering techniques and wide applications in industries, they are often the last consideration in a realistic engineering design. The advantages of thermal cloaking technology are that, first, its underlying “space” concept can be incorporated in an engineering design from the very beginning with balances from other functionalities. Moreover, traditional heat dissipation devices are bulky and difficult to use in a compact space. A thermal cloak, on the other hand, can apply in a compact space. One should note that, compared to phononic crystals that have strict requirement on periodicity and sizes of constituents (which implies a challenging fabrication at nanoscale), a thermal cloak typically uses concepts of effective medium that are only dependent on local density of constituents and insensitive to periodicity and size fluctuations.

As a new research branch in the field of metamaterials, the thermal cloaking technology still needs substantial development. Though many different designs have been proposed to achieve cloaking and other novel functionalities with metamaterials and other structured materials, they are mainly limited to “proof-of-concept” stages and lacks direct comparison with current heat dissipation and thermal management technologies. We expect that many works will be carried out to incorporate this novel thermal cloaking technology into industry-level applications.

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