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Survey of Recent Progress in Networked Control Systems

Ke-You YOU¹ Li-Hua XIE²

Abstract: With the rapid development of sensing, information processing, and communication technologies, the research in the emerging area of networked control systems (NCSs) has attracted considerable attention in the research community. The purpose of this survey is to provide a review of the state-of-the-art of such research. We particularly discuss various network conditions required for different control purposes, such as the minimum rate coding for stabilizability of linear systems in the presence of time-varying channel capacity, the critical packet loss condition for stability of the Kalman filter with intermittent observations, network topology for coordination of networked multi-agent systems, as well as event-based sampling for energy and communication efficiency. The common goal of discussion on these topics is to reveal the effect of the communication network on the operation of the networked systems.

Keywords: Networked control systems (NCSs), linear systems, data rate, packet loss, event-based control, consensus

Networked control systems (NCSs) are spatially distributed systems wherein the control loops are closed through some form of communication network as shown in Fig. 1. The communication network connects the spatially distributed components, such as actuators, sensors and controllers. This universal feature makes it possible to design large-scale systems in an asynchronous manner which results in advantages of NCSs over conventional control systems including low cost of installation, flexibility in system implementation, and ease of maintenance. Examples of practical significance of NCSs include sensor networks, industrial control networks, multiple vehicle coordination, and micro-electro-mechanical systems (MEMS)^[1], where the aim is to control one or more dynamical systems by deploying a shared network for data exchange. From the biological point of view, a gene regulatory network can also be identified as an NCS, consisting of a network of interacting genes and associated proteins.

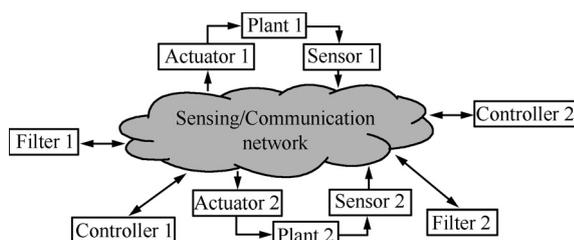


Fig. 1 General NCS architecture

The incorporation of a network in the feedback loop makes the analysis and design of an NCS complex since in most problems, estimation/control interacts with communication in various ways. This has raised fundamentally new challenges in communications, signal processing, and control/estimation dealing with the network operation and the overall system's quality. Recently, a wide range of research has been reported in the literature, focusing on problems

related to the minimum data rate requirements in digital feedback channels, packet loss conditions in lossy channels, network topology for coordination of multi-agent systems, event-based sampling for networked systems, and etc. They are collectively called control over networks, which has been identified as one of the key research directions in control^[2]. The objective of this survey is to provide a snapshot assessment of the state-of-the-art of such research respectively from the information, network, graph and sampling theoretic approaches.

1 Overview of survey

The present survey attempts to cover multiple aspects of research problems in NCSs.

In Section 2, we investigate the properties of the existing networks adopted in NCSs since the development of the control technology in NCSs is mainly motivated by the type of networks used.

In Section 3, a brief history of the evolution of control system technologies is reviewed. It gives us a clear picture on how the control technologies are affected by instrumentation for implementing control systems. The use of digital computers drives researchers to study the digital control technology, while the use of networks in the feedback loop forms the basis of research on NCSs, which also makes it possible to develop large-scale systems by using distributed control.

In Section 4, we discuss the minimum data rate problem for stabilization of linear systems over noiseless and noisy feedback channels, respectively. For the noisy case, we focus on a digital channel subject to random packet losses. We establish the data rate theorem from the information-theoretic approach.

In Section 5, a network-theoretic approach is adopted to derive the stability condition for Kalman filtering with random packet losses. We discuss a random down-sampling method to deal with the temporal correlations of the packet loss process.

In Section 6, consensus control of multi-agent systems which consist of multiple interacting linear systems is discussed. The goal is to reveal the joint effect of agent dynamics and network topology on consensus under a common control protocol.

In Section 7, event-based control for cooperation of multi-agent systems is revisited in the context of NCSs. The benefit of using the event-based control is to reduce the number of transmissions to satisfy the communication resource con-

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straints.

In Section 8, we draw some concluding remarks and suggest several potential research topics for the reader.

2 Limited capacity network

The advantages of NCSs result from the insertion of some form of network in the feedback loop to remotely control systems, which may also pose limitations on the quality of system operation. Inspired by this, it is helpful to provide a brief description of the current network architecture or protocols used in NCSs.

Current candidate networks for NCS implementations are DeviceNet^[3], Ethernet^[4], and FireWire^[5], to name a few. Each network has its own protocols that are designed for a specific range of applications since the behavior of an NCS largely depends on the quality of the underlying network, including finite transmission rate, delays, packet losses, and so on.

Any communication network channel is only able to carry a finite number of bits information per unit time. This introduces significant constraints on the operation of NCSs due to the possible low resolution of the transmitted information. For instance, in current and future generations of MEMS arrays in which there can be as many as 10^4 to 10^6 actuators or sensors on a single chip, and on a larger physical scale it is of central interest in closing feedback loops through wireless link as BluetoothTM or IEEE 802.11 (b) and using feedback network protocols such as controller area network (CAN)^[1]. Although the total amount of information in bits per unit time may be large, each component is effectively allocated only a small portion^[6].

To transmit a signal over a digital network, the signal must be sampled, encoded into a binary sequence, transmitted over the network and finally the data must be decoded at the receiver side. Variable network conditions such as congestion and channel quality may induce variable delays between sampling and decoding at the receiver due to the media access delays (the time the network takes to accept data) and the transmission delays (the time during which data is in transmit inside the network)^[7]. In feedback control systems delays are of primary concern.

Changes in the environment such as the random presence of large metallic objects will inevitably affect the propagation properties of communication channels, or even block the communication link. Data may be lost while in transit through the network, which is far more common in wireless than in wired networks. This happens in resource limited wireless sensor networks (WSN) where communications between devices are power constrained and therefore limited in range and reliability. Long transmission delay sometimes may amount to a packet loss if the receiver discards outdated arrival data. This essentially means that the reliable transmission protocols such as TCP are not appropriate for NCSs since the retransmission of the old data is generally not very useful in the real-time control.

Other issues arising from the network closing loops in NCSs include security, transmission errors, safety and so on. Those factors will inevitably lead to performance degradation or even stability loss of NCSs.

3 Brief history of control technology

As early as the 1950's, the control technology was shifted from the classical feedback control^[8] to digital control^[9] due to the use of digital computers as instrument for

feedback control. The main change in this transition is from continuous-time/continuous-state models to discrete-time/discretized state models. In comparison, the additional issues needed to be addressed in digital control involve the effect of sampling rates and finite word length, and compensation for phase lags, which is now maturely established^[10].

However, the use of spatially distributed components to control one or more dynamical systems has boosted the development of new control technology. Examples include chemical processes, airplanes, and power systems. In the past, the components of these systems were linked via wired point-to-point connections and the systems were designed by using all information from sensors to a centralized decision maker. This centralized point-to-point control system is not suitable to meet new requirements, such as modularity, decentralization of control, integrated diagnostics, quick and easy maintenance and low cost^[11]. With the development of sensing and communication technology, we are able to deploy low-cost microprocessors at remote locations and information can be transmitted via shared digital networks or even wireless channels. Today, systems with such an architecture is called networked control systems.

The research of NCSs is primarily fueled by the type of networks used in the feedback loop. After extensive research and development, several network protocols for industrial control have been established. For example, CAN was originally developed in 1983 by the German company Robert Bosch for use in car industries^[12]. Another example of industrial networks is Profibus^[13] developed by six German companies and five German institutes in 1987. Many other industrial network protocols including Foundation Fieldbus and DeviceNet were also developed during the same time. This architecture can improve the efficiency, flexibility and reliability of NCSs through reduced wiring and distributed intelligence, and so reduce the installation, reconfiguration and maintenance time and costs. As such, many cars manufactured in Europe include embedded systems integrated through CAN.

The recent trend toward integrating devices are through wireless rather than wired communication channels. When wireless networks are introduced, the reliability and the timely deliveries of packets become issues due to the less predictable properties of channels. The assumption that the data collected are accurate, timely, and lossless is no longer applicable for such networked systems. The shift from wired to wireless communication channels has highlighted important potential application advantages as well as several challenging problems for current research on NCSs.

Another important research interest in NCSs today is the emphasis on distributed control due to the availability of ample processing power at low cost, which allows sensor data to be processed locally. Generally, the aim of distributed control is to coordinate a group of subsystems by implementing control policies locally. It is natural to believe that distributed control can never be as good as centralized control. This holds only if there is no real-time limitation on the communication network, e.g. delay free and no packet losses. By accounting for the real network effect, distributed control may be better than centralized control in terms of robustness, scalability, security, and etc. A typical example of a distributed control system is to coordinate a group of unmanned air vehicles (UAVs) to be in a desired formation. Under this high-speed circumstance, the communication between UAVs becomes critical. The design procedure will impose a stringent requirement on simplify-

ing the computational complexity on finding a controller with a distributed architecture.

Overall, the shift of control technology to NCSs motivates us to consider control/estimation and communication in a unified way. In control, one is concerned with using feedback information to achieve some performance objective, and usually assumes that limitations in the communication links do not significantly affect performance. While in communication, the focus is on the reliable transmission from one point to another, regardless of the specific purpose of the transmitted message. Thus control and communication theories have traditionally evolved as separate subjects. The rising of NCSs has opened up new opportunity to incorporate ideas both from control and communication theories to study the tradeoff between the control and communication performance.

4 Control from information-theoretic approach

4.1 Control over noiseless digital channels

Current research on control with quantized state feedback in NCSs typically focuses on the structure shown in Fig. 2. Precisely, we consider an n -dimensional discrete-time system:

$$x_{k+1} = Ax_k + Bu_k, \quad (1)$$

where $x_k \in \mathbf{R}^n$ is the measurable state, $u_k \in \mathbf{R}^m$ is the control input and (A, B) is stabilizable.

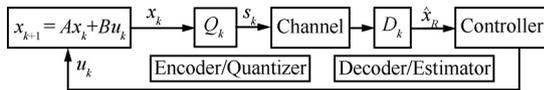


Fig. 2 Quantized feedback control

The distinct feature of an NCS is that the controller is connected to the plant via a communication network. Since only information with a finite of bits can be sent via a communication network per transmission, the continuous valued state x_k has to be quantized into a discrete valued version. To this purpose, a quantizer $Q_k : \mathbf{R}^n \rightarrow S_k$, which maps each point in the state space to an element of a countable set S_k , is deployed and the quantizer output is encoded into a binary sequence for transmission. On the other side, the decoder receives the channel output and recovers the sampled state. Finally, the controller generates an input signal using the recovered state and applies to the plant.

In the whole process, the quantizer converts a continuous state into a discretized state. This will inevitably introduce information loss due to the existence of quantization error, e.g., $x_k - \hat{x}_k \neq 0$ in general. It is clear that the quantizer is typically a nonlinear operator. The fundamentally interesting problem is how the quantization error will affect the quality of the system operation. Intuitively, the less number of bits used for quantization, the larger quantization error will be induced and vice versa. If the quantization error is too large, the controller may not be able to generate a stabilizing control input.

The research on control with quantized feedback is not a new topic and has been an important research area since as early as 1956^[14], in which the effect of quantization in a sampled data system is studied. The early work on quantized feedback control is mainly motivated by digital computers as instrument for implementing control systems. In

particular, Kalman^[14] pointed out that if a stabilizing controller is quantized using a finite-alphabet quantizer, the feedback system would exhibit limit cycles and chaotic behavior. Since the quantizer resolution induced by the finite word length of the microprocessor is relatively high, most of the early work in digital control concentrates on understanding and mitigation of quantization effects^[15–16]. Their common approach is to model quantization error as an additive stochastic noise and the standard solutions of stochastic control/estimation are applied.

However, this idea is challenged in the new environment where only very coarse information is allowed to propagate through the network due to the very limited network bandwidth or for the purpose of energy saving, e.g., in wireless sensor networks. The change of view on quantization can be traced back to [17] where the author treated the quantizer output as partial information of the quantized entity rather than its approximation from the information coder point of view, and demonstrated the significance of the historical values of the quantizer output. One of their remarkable findings is that if the linear system is open-loop unstable, there is a minimum rate for the coding of the feedback information to achieve stabilization. Since then, various methods for studying quantization feedback control have been developed.

Research on quantized feedback can be categorized depending on whether the quantizer is static or dynamic. A static quantizer is a memoryless nonlinear function with fixed quantization levels while a dynamic quantizer uses memory and is more complicated and potentially more powerful. Perhaps one of the most interesting static quantizers is the so-called logarithmic quantizer, whose quantization levels are linear in logarithmic scale and relative quantization error is bounded. It turns out that to achieve quadratic stabilization of single-input linear systems with quantized control feedback, the quantizer needs to be logarithmic^[18], where the coarsest quantization density ρ is given explicitly in terms of the system's unstable poles. Here the quantization density of a quantizer $Q(\cdot)$ is defined as follows:

$$\rho = \lim_{\epsilon \rightarrow 0^+} \inf \frac{\#Q[\epsilon]}{-\ln \epsilon},$$

where $\#Q[\epsilon]$ denotes the number of quantization levels that Q has in the interval $[\epsilon, 1/\epsilon]$. An interesting approach to deal with logarithmic quantization effects on control systems is called sector bound method^[19]. This idea is inspired by treating quantization error as uncertainty or nonlinearity and bounding it by a sector bound. Then, robustness analysis tools such as absolute stability theory^[20] can be applied to study the quantization effect. Due to the time-invariance nature, the memoryless quantizer requires an infinite number of quantization levels to achieve stabilization. When only a finite number of quantization levels is available, the so-called practical stability^[21] is obtained where the state of the close-loop system evolves within certain subsets of the state-space. One may consider to apply a dynamic scaling approach to make the subset arbitrarily small. This motivates the development of dynamic quantization using a finite number of quantization levels^[22–25].

Brockett et al.^[25] study a dynamic finite-level uniform quantizer for stabilization and point out that there exists a dynamic adjustment policy for the quantizer sensitivity and a quantized state feedback controller to asymptotically stabilize an unstable linear system. This raises a fundamental question: how much information needs to be communicated between the quantizer and the controller in order to

stabilize an unstable linear system? Various authors have addressed this problem under different scenarios^[24, 26–30]. To make it precise, let μ_k denote the number of quantizer outputs of Q_k , i.e., μ_k is the cardinality of S_k . It is obvious that it requires $\lceil \log_2 \mu_k \rceil$ bits to represent the quantizer output. The average data rate, denoted by R , of this quantization process is defined by

$$R = \lim_{k \rightarrow \infty} \inf \frac{1}{k} \sum_{i=1}^k \log_2 \mu_i \text{ bits/sample.}$$

The elegant data rate theorem states that the average data rate required for stabilization has to be strictly greater than a universal lower bound, i.e.,

$$R > \sum_{i=1}^n \max\{0, \log_2 |\lambda_i|\} = H(A), \quad (2)$$

where $\lambda_1, \dots, \lambda_n$ are all eigenvalues of the open-loop system matrix A . Conversely, if $R > H(A)$, one can design a dynamic quantizer and the corresponding control policy to achieve stabilization. The lower bound $H(A)$ only depends on the open-loop system matrix, regardless of how the information is encoded and decoded, and the control policy.

Obviously, this property bears close resemblance to the well-known Shannon's source coding theorem^[31], which states that to reliably (with arbitrarily small probability of error) transmit a given random process over a digital channel, the data rate of the channel has to be strictly greater than the entropy rate of the process^[32]. In the literature, $H(A)$ is always termed as the topological entropy^[33] of the system, which quantifies the uncertainty growth rate generated by the system. The data rate characterizes how fast the information received from the system can be processed to reduce the uncertainty. To asymptotically stabilize the system, the uncertainty reduction rate has to be greater than its growth rate. This is intuitively appealing. We also note that the larger the magnitude of the unstable poles, the larger the data rate through the feedback loop required for stabilization.

The above results establish the essential requirements for stability with the assumption that there is no channel uncertainty during the data packet transit inside the network. However, limited capacity channels in the classic communication theory are modeled in terms of not only quantization effects but also channel uncertainties and time delays. Many of the major results in this theory are developed on the ground of noisy channel models. So incorporating noisy digital channels into NCS problems seems to be unavoidable in the analysis and synthesis of NCSs. As an initial step, the issue of the minimum data rate for stabilizability of linear systems over noisy digital channels attracted good attention of researchers. The research results on this topic are not as fruitful as the case with noiseless digital channels due to that the optimal data rate assignment among unstable system state variables intertwines with the channel uncertainty process and also depends on the sense of stabilization notion^[34–41]. Nonetheless, some significant progresses have been recently achieved toward this topic.

4.2 Control over noisy digital channels

The framework in the previous subsection is generalized to noisy communication channels by many researchers. Due to the existence of channel uncertainties, the quantizer output s_k might not be exactly received by the decoder. This

will further induce information loss to the controller. To compensate for this uncertainty, a larger data rate R will be needed in comparison with the noiseless channels. We are particularly interested in the problem that how many additional bits will be needed to achieve stabilization to counter the effects of channel uncertainties. Although the problem was initiated by Tatikonda and Mitter^[34] in 2005, it is not fully understood for general vector linear systems to date. In [34], it is claimed that if the Shannon capacity of this channel is greater than $H(A)$, then the system with process disturbances can be almost surely stabilized with bounded error^[34]. This is shown to be incorrect in [40], which proves that to the contrary, any unstable linear system affected by arbitrarily and uniformly small external disturbances can never be almost surely stabilized via the erasure channel with nonzero erasure probability, irrespective of which algorithm of stabilization is employed. The almost sure stabilization is further investigated by the same authors^[41–43].

However, when we are concerned with the mean square stabilization of linear systems over noisy channels, the results are substantially different. We consider a Gilbert-Elliott channel model, where the packet loss process^[44] is modeled by a time homogeneous Markov process^[45]. Precisely, let the binary sequence γ_k denote whether the packet is received or not by the decoder at the sampling time k . If $\gamma_k = 1$, it means that the packet is successfully delivered to the decoder while for $\gamma_k = 0$, there is a packet loss. Assume that all the random variables in this survey are defined on a common probability space (Ω, F, \mathbb{P}) , where Ω is the space of elementary events, F is the underlying σ -field on Ω , and \mathbb{P} is a probability measure on F . The transition probability matrix of the Markov process γ_k is given by

$$(\mathbb{P}\{\gamma_{k+1} = j | \gamma_k = i\})_{i,j \in \mathbb{S}} = \begin{bmatrix} 1-q & q \\ p & 1-p \end{bmatrix}, \quad (3)$$

where $\mathbb{S} = \{0, 1\}$ is the state space of the Markov process. To avoid any trivial case, the failure rate p and recovery rate q are assumed to be strictly positive and less than one so that the Markov process γ_k is ergodic. Obviously, a smaller value of p and a larger value of q indicate a more reliable communication link.

We adopt a TCP-like network protocol^[46], which means that there exists a side feedback channel from the decoder to the encoder to notify whether the packet is received or not by the controller. We call the networked system mean square stabilizable if the close-loop system satisfies that

$$\lim_{k \rightarrow \infty} \mathbb{E}[\|x_k\|^2] = 0,$$

where the mathematical operator $\mathbb{E}[\cdot]$ is taken with respect to the probability measure \mathbb{P} under any initial state x_0 and γ_0 .

Theorem 1^[37–39]. For the special case that γ_k is an independent and identically distributed (i.i.d.) process, i.e., $p + q = 1$, a necessary and sufficient condition for mean square stabilization of a networked scalar system, e.g., $A = \lambda$, is that

$$\mathbb{E} \left[\frac{|\lambda|^2}{2^{2\gamma_k R}} \right] < 1. \quad (4)$$

The inequality (4) is amenable to the following intuitive interpretation. If the packet is lost during the transmission, the controller does not receive any information from the system. The mean square estimation error at the decoder

about the state of the system grows by $|\lambda|^2$. If the packet is successfully sent over the channel, it helps the decoder reduce this error by at most 2^{2R} , where R is the communication rate supported by the channel per transmission. If the average over the fluctuation of the rate $|\lambda|^2$ exceeds $2^{2\gamma_k R}$, the information sent over the channel cannot compensate (on the average sense) the dynamics of the system and it is not possible to stabilize the plant.

If there is no packet loss, this corresponds to that $\gamma_k = 1$. The inequality (4) reduces to

$$R > \log_2 |\lambda|,$$

which is consistent with the celebrated data rate theorem. Letting $R \rightarrow \infty$, we have that $p < |\lambda|^{-2}$. It recovers the well known result of the packet loss model studied from the network-theoretic approach^[47–50].

However, the extension of the above analysis to general vector linear systems entails the difficulty of the optimal rate allocation to the unstable modes with different magnitudes. By the entropy-power inequality^[51], one may modify (4) as the necessity for stability as

$$\mathbb{E} \left[\frac{|\det(A^u)|^{\frac{2}{n^u}}}{2^{2\gamma_k \frac{R}{n^u}}} \right] < 1, \quad (5)$$

where $A^u \in \mathbf{R}^{n^u \times n^u}$ corresponds to the unstable subspace of A due to that the stable part will automatically converge to the origin without using any control input. If the information sent across the channel is able to stabilize the whole unstable states, it is obvious that any invariant subset of the unstable states can be stabilized as well. Motivated by this observation, it is sensible to consider all invariant subsets of the unstable states to derive possibly stronger necessary conditions. Due to the interaction among the unstable states, we decouple the system into multiple unstable subsystems. Without loss of generality, assume that $A = \text{diag}\{J_1, \dots, J_u, A_s\}$, where A_s associates with the stable states and J_i is a real elementary Jordan block^[51] associated with the unstable eigenvalue λ_i . For instance, if λ_i is real, then

$$J_i = \begin{bmatrix} \lambda_i & 1 & & \\ & \lambda_i & & \\ & & \ddots & \\ & & & \lambda_i \end{bmatrix}. \quad (6)$$

If $\lambda_i = a_i + b_i i$ is complex, i.e., $b_i \neq 0$ and let $D(\lambda_i) = \begin{bmatrix} a_i & b_i \\ -b_i & a_i \end{bmatrix}$, then

$$J_i = \begin{bmatrix} D(\lambda_i) & I & & \\ & D(\lambda_i) & & \\ & & \ddots & \\ & & & D(\lambda_i) \end{bmatrix}. \quad (7)$$

For any subset $S = \{u_1, \dots, u_s\} \subset \{1, \dots, u\}$, let $J^{(S)} = \text{diag}\{J_{u_1}, \dots, J_{u_s}\}$. We obtain a corresponding decoupled subsystem as follows:

$$x_{k+1}^{(S)} = J^{(S)} x_k^{(S)} + B^{(S)} u_k^{(S)}, \quad (8)$$

where $x_k^{(S)}$ is the unstable state vector corresponding to $J^{(S)}$. Similar notations are made for $B^{(S)}$ and $u_k^{(S)}$. Recalling that if the state x_k is mean square stabilized by

the information sent over the channel, then $x_k^{(S)}$ can be stabilized by using the same amount of information. Let $n^S = \dim(x_k^{(S)})$, a stronger necessary condition is then obtained as follows:

$$\mathbb{E} \left[\frac{|\det(J^{(S)})|^{\frac{2}{n^S}}}{2^{2\gamma_k \frac{R}{n^S}}} \right] < 1, \forall S \subset \{1, \dots, u\}. \quad (9)$$

It can be easily verified that if there is no packet loss, i.e., $\gamma_k = 1$, (9) reduces to (2). Note that the coding strategy in the necessary condition does not consider the effect of packet loss and may not be sufficient. In fact, the optimal coding in the sense of using minimum data rate for stabilization should take into account the packet loss effect. Intuitively, in the optimal coding the more ‘‘important’’ state variables may require repeated transmissions to reduce the chance of loss over the channels. The effect of packet losses on the optimal coding is the main difficulty in comparison with the case over a noiseless digital channel. Unfortunately, this is still an open problem.

As for sufficiency, the design of a vector quantizer that dynamically adapts to the packet loss process is challenging. It is worth mentioning that the design of an optimal vector quantizer to minimize the mean quadratic quantization error of an arbitrary random vector is yet to be known. The current solution in the literature is to quantize each unstable state variable by one scalar quantizer^[37, 39, 52], which inevitably needs a higher data rate since this method does not consider the correlations among the state variables. The idea is to assign a sufficiently large data rate to each unstable state variable and treat its evolution as an independent scalar system.

Obviously, the above approach is not applicable to the case when the packet loss process γ_k is Markovian due to its temporal correlations. To circumvent this obstacle, we interpret the networked control problem under packet losses from a different point of view. In particular, if the packet at the sampling time k is lost during its transmission, this is somewhat equivalent to that there is no sampling at this time. From this perspective, we obtain a randomly down sampled system. To make it precise, we denote t_k the sampling time instants at which packets are successfully delivered via the channel. Mathematically, let

$$\begin{aligned} t_1 &= \inf\{k | k \geq 1, \gamma_k = 1\}, \\ t_2 &= \inf\{k | k > t_1, \gamma_k = 1\}, \\ &\vdots \\ t_j &= \inf\{k | k > t_{j-1}, \gamma_k = 1\}. \end{aligned} \quad (10)$$

One can easily verify that the k -th packet reception time t_k is a stopping time which is in general not equal to the sampling time k . Without loss of generality, let $\gamma_0 = 1$ and $t_0 = 0$. Define the sojourn time $\tau_k = t_k - t_{k-1}$, we obtain a randomly time-varying system as follows:

$$x_{t_{k+1}} = A^{\tau_{k+1}} x_{t_k} + \tilde{B} \tilde{U}_{t_k}, \quad (11)$$

where $\tilde{B} = [B \ AB \ \dots \ A^{\tau_{k+1}-1} B]$ and $\tilde{U}_k^T = [u_{t_{k+1}-1}^T \ \dots \ u_{t_k}^T]$. The study of the above randomly varying system often requires the statistics of τ_k .

Lemma 1^[53]. The sojourn time τ_k forms an i.i.d. process and the mass probability distribution is given by

$$\mathbb{P}\{\tau_1 = i\} = \begin{cases} 1 - p, & i = 1, \\ pq(1 - q)^{i-2}, & i > 1. \end{cases}$$

This lemma pinpoints a way for us to deal with the temporally correlated Markovian packet loss process.

Theorem 2^[39]. Under Markovian packet losses, a necessary and sufficient condition for mean square stabilization of a networked scalar system, e.g., $A = \lambda$, is that

$$\mathbb{E} \left[\frac{|\lambda|^{2\tau_k}}{2^{2R}} \right] < 1. \quad (12)$$

If there is no packet loss, i.e., $\tau_k = 1$, the inequality (12) reduces to (2). Similarly, by letting $p + q = 1$, it is not difficult to conclude that the inequality (12) reduces to (4). Intuitively, the above condition states that the data rate R should be large enough to compensate for the expansion of the state during the time interval in which packets are lost and thus no information can be accessed by the controller. For the down sampled system (11), the controller always receives R bits information at each down sampled time instant t_k . But the open loop matrix A^{T_k} is stochastically time-varying. However, when we look at the original discrete-time system, it is the data rate that is time-varying due to the possible packet loss. Again, the difficulty for the extension to general vector systems lies in the optimal rate assignment among the unstable states.

The problem on the minimum data rate required for stabilization over a noisy digital channel is further complicated by the fact that different data rate may be required under different notions of stabilizability. For instance, the necessary and sufficient condition on the almost sure stabilizability over an erasure channel for a certain class of linear systems turns out to be that the Shannon capacity of the channel should be strictly greater than the intrinsic entropy rate of the system^[34, 40–41], which unfortunately fails for the moment stabilizability^[54].

It is noted that the design of quantizer in the above works only focuses on data rate for stabilization. Recently, Como et al.^[55] propose two coding strategies for anytime reliable transmission of real-valued information through digital noisy channels by accounting for the trade-off between the convergence rate of the networked linear system and the computational complexity. In [56], an optimal quantizer in the sense of asymptotically achieving the minimum mean square error is designed by quantizing the innovation process. Clearly, quantizer and controller should be jointly designed so as to achieve the optimal performance for the overall system. Due to the nonlinearity of quantization, this problem is generally very challenging, not only because the quantizer and controller are inter-related but also the optimal quantizer-controller pair is performance dependent. For instance, different performance measures may require different optimal quantizer-controller. In [18–19], it is shown that the coarsest quantizer to quadratically stabilize an unstable single input linear system is a logarithmic quantizer, which is also optimal in the sense of approaching the minimal data rate for stabilization of a linear system^[22]. Although the logarithmic quantizer has been extended to address the LQR and H_∞ performance problems in [19], it is not optimal in general, at least, its optimality is not clear. The performance control with quantized feedback is another interesting and challenging problem^[57–62].

5 Control from network-theoretic approach

Different from the information-theoretic approach that was described in the previous section, the network-theoretic

approach has also been studied in the literature to model control/estimation over time-varying channels. In this case, the channel uncertainty is modeled as random packet losses. Packets are considered as single entities, and can be lost stochastically with some probability. There are two typical statistical processes to model the packet loss process of the observed data that are used to estimate the state of a stochastic system. One is an independent and identical binary process^[46, 50, 63–72], and the other is a Markov process^[47–48, 73–77]. The problem of interest is how the packet losses due to the unreliability of the network channels affect the estimate of the system state. To investigate this problem, we consider a discrete-time stochastic linear system:

$$\begin{cases} x_{k+1} = Ax_k + w_k, \\ y_k = Cx_k + v_k, \end{cases} \quad (13)$$

where $x_k \in \mathbf{R}^n$ and $y_k \in \mathbf{R}^m$ are state vector and measurement vector, respectively. $w_k \in \mathbf{R}^n$ and $v_k \in \mathbf{R}^m$ are white Gaussian noises with zero means and covariance matrices $Q > 0$ and $R > 0$, respectively. C is of full row rank, i.e., $\text{rank}(C) = m \leq n$. The initial state x_0 is a random Gaussian vector of mean \hat{x}_0 and the covariance matrix $P_0 > 0$. Moreover, w_k , v_k and x_0 are mutually independent.

The raw measurements of the system are transmitted to a remote estimator via an unreliable communication channel. Due to random fading and/or congestion of the communication channel, packets may be lost while in transit through the channel. See Fig. 3 for an illustration. Similarly, γ_k is used to indicate whether y_k is received by the estimator or not. At the estimator side, it has the knowledge of γ_k . This implies that the information obtained by the estimator at time k is given by

$$z_k = \{\gamma_k y_k, \gamma_k\}.$$

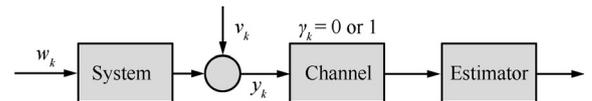


Fig. 3 Networked estimation configuration

Sinopoli et al.^[50] showed that the Kalman filter is still the minimum mean square error (MMSE) estimator in this setting. There is a slight change to the standard Kalman filter in that only the time update is performed when the measurement packets are dropped. Precisely, let $Z_k = \{z_1, \dots, z_k\}$ be the maximum information set accessed by the estimator at time k . Denote the MMSE state estimator and one-step predictor by $\hat{x}_{k|k} = \mathbb{E}[x_k | Z_k]$ and $\hat{x}_{k+1|k} = \mathbb{E}[x_{k+1} | Z_k]$, respectively. The associated estimation error covariance matrices are defined by $P_{k|k} = \mathbb{E}[(x_k - \hat{x}_{k|k})(x_k - \hat{x}_{k|k})^T | Z_k]$ and $P_{k+1|k} = \mathbb{E}[(x_{k+1} - \hat{x}_{k+1|k})(x_{k+1} - \hat{x}_{k+1|k})^T | Z_k]$.

By [50], it follows that:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + \gamma_k K_k (y_k - C\hat{x}_{k|k-1}), \quad (14)$$

$$P_{k|k} = P_{k|k-1} - \gamma_k K_k C P_{k|k-1}, \quad (15)$$

where the Kalman gain $K_k = P_{k|k-1} C^T (C P_{k|k-1} C^T + R)^{-1}$. In addition, the time update equations continue to hold: $\hat{x}_{k+1|k} = A\hat{x}_{k|k}$, $P_{k+1|k} = A P_{k|k} A^T + Q$ and $\hat{x}_{0|-1} = \bar{x}_0$, $P_{0|-1} = P_0$.

Due to the randomness of the lossy network, the error covariance matrix becomes stochastic, which is different from

the standard Kalman filter where the error covariance matrix is a deterministic quantity. Let $P_k = P_{k|k-1}$, the update recursion for P_k can be characterized by

$$P_{k+1} = AP_k A^T + Q - \gamma_k AP_k C^T (CP_k C^T + R)^{-1} CP_k A^T. \quad (16)$$

To reveal how the random packet losses affect the estimate, the stochastic behavior of P_k is extensively examined in the literature. Since the recursion in (16) is nonlinear in P_k , it is usually difficult to achieve this goal. In [75], they study the performance of Kalman filtering by considering the metric $\mathbb{P}(P_k \leq M)$, i.e., the probability that the one-step prediction error covariance matrix P_k is bounded by a given positive definite matrix M , which is related to finding the cumulative distribution of P_k . This probability could be exactly computed for scalar systems and only has lower and upper bounds for vector systems^[75], which is consistent with the spirit in [77]. Another performance metric called the stochastic boundedness is introduced in [72] for an i.i.d. packet loss model. It is worth pointing out that under different metrics or scenarios, the effects of random packet losses on performance would be substantially different.

In this section, we focus on the mean stability of P_k , i.e.,

$$\sup_{k \in \mathbf{N}} \mathbb{E}[P_k] < \infty,^1$$

where the mathematical expectation is taken with respect to the packet loss process γ_k . For the standard Kalman filter^[78], e.g., $\gamma_k = 1$ for all $k \in \mathbf{N}$, it is known that the detectability of (C, A) is sufficient to achieve stability of the error covariance matrix. Since γ_k randomly switches between the two states, the recursion in (16) is a randomly time-varying system, which makes the derivation of a necessary and sufficient condition on γ_k to achieve stability generically challenging. When $\gamma_k = 0$, it becomes an open-loop update. If A is unstable, the open-loop update will drive the error covariance matrix to expand. However, when it switches to $\gamma_k = 1$, the error covariance matrix tends to be reduced if (C, A) is detectable. To achieve stability of P_k , it is expected that there might exist a cutoff probability for the two states of γ_k . In view of this, it is helpful to consider a simpler case at first, where γ_k is an i.i.d. process with the packet loss rate $p = 1 - \mathbb{E}[\gamma_k]$. Intuitively, a larger loss rate will result in more severe information loss and a higher chance of losing stability. Does there exist a critical loss rate above which the error covariance matrix will diverge? The answer is positive as follows.

Theorem 3^[46,50]. If $(A, Q^{1/2})$ is controllable, (C, A) is detectable, and A is unstable, then there exists a $p_c \in (0, 1)$ such that

$$\begin{aligned} \lim_{k \rightarrow \infty} \mathbb{E}[P_k] = \infty, & \text{ for } p_c \leq p \leq 1 \text{ and } \exists P_0 \geq 0, \\ \sup_{k \in \mathbf{N}} \mathbb{E}[P_k] \leq M_{P_0}, & \text{ for } 0 \leq p < p_c \text{ and } \forall P_0 \geq 0, \end{aligned}$$

where $M_{P_0} > 0$ depends on the initial condition $P_0 \geq 0$. Moreover, let

$$\bar{p} = \frac{1}{\max_i \{|\lambda_i|^2\}} \text{ and } \underline{p} = \frac{1}{\prod_{i=1}^n \max\{1, |\lambda_i|^2\}}.$$

Then, it follows that $\underline{p} \leq p_c \leq \bar{p}$. In particular, $p_c = \bar{p}$ if C is of full column rank and $p_c = \underline{p}$ if C is of column rank one.

¹There exists a positive definite matrix \bar{P} such that $\mathbb{E}[P_k] \leq \bar{P}$ for all $k \in \mathbf{N}$.

The existence of a critical value p_c is based on the fact that the quantity $\mathbb{E}[P_k]$ is monotonically increasing with p . In fact, if $p = 1$, then $\exists P_0 \geq 0$ such that $\lim_{k \rightarrow \infty} \mathbb{E}[P_k] = \infty$ while if $p = 0$, then $\forall P_0 \geq 0, \sup_{k \in \mathbf{N}} \mathbb{E}[P_k] < \infty$. Combine the above, it is not difficult to understand the existence of p_c . The upper bound of p_c is obvious since $P_{k+1} \geq (1 - \gamma_k)AP_k A^T + Q$, which directly implies that $p_c \leq \bar{p}$. To derive the lower bound of p_c , an upper bound of $\mathbb{E}[P_k]$ is found. By taking expectation on both sides of (16) and using Jensen's inequality^[79], it follows that

$$\begin{aligned} \mathbb{E}[P_{k+1}] &\leq A\mathbb{E}[P_k]A^T + Q - \\ &(1-p)A\mathbb{E}[P_k]C^T(CE[P_k]C^T + R)^{-1}CE[P_k]A^T. \end{aligned}$$

Thus, it is not difficult to check that $\mathbb{E}[P_k] \leq M_k$ for all $k \in \mathbf{N}$, where M_k is computed recursively via a modified Riccati update

$$\begin{aligned} M_{k+1} &= AM_k A^T + Q - \\ &(1-p)AM_k C^T (CM_k C^T + R)^{-1} CM_k A^T \quad (17) \end{aligned}$$

with initial condition $M_0 = P_0$.

In comparison with (16), the recursion in the above is deterministic and the techniques in linear matrix inequalities (LMIs)^[80] can be applied to (17), from which the inequality $\underline{p} \leq p_c$ is established.

Since p_c characterizes the minimum packet loss rate requirement on such a lossy network, a large amount of research effort has been made toward finding the critical value^[50,67-68,81]. For instance, it is shown that $p_c = \bar{p}$ if C is invertible on the observable subspace^[67] or the system is non-degenerate^[68]. Recently, there are concrete examples showing that $\underline{p} < p_c < \bar{p}$ ^[47]. Then, a natural question is how to exactly quantify p_c ? By a close look at the above method, the conservativeness mainly lies in the use of two different bounds for $\mathbb{E}[P_k]$. This result also indicates that the value of p_c may depend on the system structure, e.g., the structure of the observation matrix C .

To fully exploit this, a new method needs to be developed. Since the main difficulty to study the stability of P_k lies in the complexity of the recursion in (16), perhaps it is necessary to find a new method to avoid directly analyzing (16). In addition, the use of the upper bound is only valid for the i.i.d. case of γ_k , which cannot be generalized to other packet loss models. We note that P_k is the error covariance matrix of an MMSE estimator, and under Gaussian noise of (13), the MMSE estimator coincides with the least square (LS) estimator. In fact, the use of technique in LS allows us to derive the necessary and sufficient condition on the network to achieve stability of P_k .

Let us focus on a Markovian packet loss model. In particular, γ_k is a Markov process with a transition probability matrix (3). Similarly, it is reasonable to use Lemma 1 to deal with the temporal corrections of a Markov process. At the stopping time t_k , it is clear that measurements y_{t_1}, \dots, y_{t_k} are received by the remote estimator. By the iterations of system (13), one can write the following equations²

$$\begin{aligned} y_{t_k} &= Cx_{t_k} + \tilde{v}_{t_k}, \\ y_{t_{k-1}} &= CA^{t_{k-1}-t_k}x_{t_k} + \tilde{v}_{t_{k-1}}, \\ &\vdots \\ y_{t_1} &= CA^{t_1-t_k}x_{t_k} + \tilde{v}_{t_1}, \end{aligned}$$

²We have implicitly assumed that A is invertible.

where \tilde{v}_{t_i} is a linear function of v_{t_i} and $w_{t_i}, w_{t_i+1}, \dots, w_{t_k}$. Let $Y_{t_k} = [y_{t_k}, \dots, y_{t_1}]$, $V_{t_k} = [\tilde{v}_{t_k}, \dots, \tilde{v}_{t_1}]$, and

$$M_{t_k} = \begin{bmatrix} C \\ CA^{t_k-1-t_k} \\ \vdots \\ CA^{t_0-t_k} \end{bmatrix}. \quad (18)$$

We write the above in a compact form $Y_{t_k} = M_{t_k}x_{t_k} + V_{t_k}$, which suits for the use of the LS technique to estimate x_{t_k} . Particularly, the LS estimator of x_{t_k} is given by $\hat{x}_{t_k} = (M_{t_k}^T M_{t_k})^{-1} M_{t_k}^T Y_{t_k}$. Since the LS and MMSE estimator are the same, then $P_{t_k|t_k} = E[(\hat{x}_{t_k} - x_{t_k})(\hat{x}_{t_k} - x_{t_k})^T | Z_{t_k}] = (M_{t_k}^T M_{t_k})^{-1} M_{t_k}^T \text{cov}(V_{t_k}) M_{t_k} (M_{t_k}^T M_{t_k})^{-1}$, where $\text{cov}(V_{t_k})$ denotes the covariance matrix of V_{t_k} . One can easily check that there exists a positive α such that $\text{cov}(V_{t_k}) \geq \alpha I$. On the other hand, if all the eigenvalues of A lies outside the unit circle, there exists a positive β such that $\text{cov}(V_{t_k}) \leq \beta I$ for all $k \geq 1$ ^[82]. The good news is that the stable part of A does not affect the stability condition since we can do a coordinate transformation to decompose the stable states, and the stable states will automatically become mean square stable. For the critical stable part of A , e.g., the eigenvalues lie on the unit circle, it can be slightly perturbed into unstable one^[82]. This essentially illustrates that there is no loss of generality to make the following assumption.

Assumption 1. All the eigenvalues of A lie outside the unit circle.

Under the above assumption, it follows that

$$\alpha(M_{t_k}^T M_{t_k})^{-1} \leq P_{t_k|t_k} \leq \beta(M_{t_k}^T M_{t_k})^{-1}. \quad (19)$$

Then, the stability of $P_{t_k|t_k}$ is equivalent to that of $(M_{t_k}^T M_{t_k})^{-1}$. Since we are primarily interested in the stability of P_k , we need to verify whether it is equivalent to that of $P_{t_k|t_k}$. The answer turns out to be positive by [47]. Thus, it is sufficient to focus on the mean stability of $(M_{t_k}^T M_{t_k})^{-1}$. Let $\tilde{M}_{t_k} = M_{t_k} A^{-t_k}$, then $\tilde{M}_{t_k}^T \tilde{M}_{t_k}$ is exactly a stochastically time-varying observability matrix^[83], and its stability condition is expected to be determined by the observability property of the system. For instance, if C is nonsingular, then $M_{t_k}^T M_{t_k} \geq ((A^{t_0-t_k})^T C^T C A^{t_0-t_k})^{-1} \geq \lambda_m(C^T C)(A^{-t_k})^T A^{-t_k}$, where $\lambda_m(C^T C) > 0$ is the minimum eigenvalue of $C^T C$. This implies that $E[(M_{t_k}^T M_{t_k})^{-1}] \leq \lambda_m(C^T C)^{-1} E[(A^{t_k})^T A^{t_k}] < \infty$ if $|\lambda_M|^2(1-q) < 1$ by Lemma 1, where λ_M denotes the spectral radius of A . Similarly, it can be shown that $|\lambda_M|^2(1-q) < 1$ is necessary as well. It should be noted that if $p+q=1$, which corresponds to the i.i.d. packet loss model, the critical packet loss rate $p_c = 1/|\lambda_M|^2$. This is consistent with Theorem 3.

For a general C , i.e., $\text{rank}(C) = m < n$, the result is much more involved. We use a second order system to illustrate this point. Clearly, the open-loop matrix A of a second-order system has the following cases.

- 1) $A = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$;
- 2) $A = \text{diag}\{\lambda_1, \lambda_2\}$ with $|\lambda_1| \neq |\lambda_2|$;
- 3) $A = \text{diag}\{\lambda_1, \lambda_2\}$ with $\lambda_1 = \lambda_2 \exp(2\pi\psi i)$, where ψ is irrational and $i^2 = -1$;
- 4) $A = \text{diag}(\lambda_1, \lambda_2)$ with $\lambda_1 = \lambda_2 \exp(2\pi\psi i)$, where $\psi = r/d$ and $d > r \geq 1$ are irreducible;
- 5) $A = \text{diag}\{\lambda, \lambda\}$.

The reason for the above classification is to explore how the observability of the system will be affected under ran-

dom packet losses. Actually, if (C, A) is observable, then (C, A^k) continues to be observable for all $k \in \mathbf{N}$ under Cases 1)~3). This property does not hold for Case 4), where (C, A^{kd}) is not observable for all $k \geq 1$. This means that the measurements $\{y_{kd}\}_{k \in \mathbf{N}}$ is only able to observe one mode of the system. To observe both modes of the system, we must resort to the measurement not belonging to $\{y_{kd}\}_{k \in \mathbf{N}}$. There is no such a problem for Cases 1)~3). This intuitively implies that the condition for stability of the error covariance matrix for Case 4) is stronger than that of Cases 1)~3). It should be noted that (C, A) is not detectable if $|\lambda| \geq 1$ for Case 5) where the standard Kalman filter will diverge^[78].

Theorem 4^[47]. Consider the second-order networked system (13) satisfying Assumption 1 and (C, A) is observable. The packet loss process is governed by a Markov process with transition probability matrix (3). Then

1) If A satisfies Case (d), a necessary and sufficient condition for $\sup_{k \in \mathbf{N}} E[P_k] < \infty$ is that $(1 + \frac{pq}{(1-q)^2})(|\lambda_M|^2(1-q))^d < 1$;

2) otherwise, a necessary and sufficient condition for $\sup_{k \in \mathbf{N}} E[P_k] < \infty$ is that $|\lambda_M|^2(1-q) < 1$.

For the i.i.d. packet loss model, i.e., $p+q=1$, the necessary and sufficient condition in 1) of Theorem 4 is reduced to that $p < |\lambda_M|^{-2d/(d-1)}$, which implies that the critical loss rate should be $p_c = |\lambda_M|^{-2d/(d-1)}$. Thus, the bounds given in Theorem 3 are not tight.

In comparison with Theorem 3, the new approach fully exploits the system structure, including the structure of observation matrix C and the presence form of each eigenvalue of A . We also observe from the second-order system that the necessary and sufficient network condition for stability of P_k significantly depends on the system structure. This also indicates that the extension of Theorem 4 to higher-order systems is challenging. We refer the reader to [47, 77, 84-85] for more details.

6 Control from graph-theoretic approach

Multi-agent systems, which consist of a large number of nodes or agents connected by links or edges, have become a key approach to understanding systems of interacting agents, and are being studied across many fields of science^[86]. Examples of such systems are numerous in nature, engineering applications and society, e.g., the wide range of interactions between genes, proteins and metabolites in a cell, and a network of people connected by various relationships. Distributed control of such systems has attracted considerable interest in various scientific communities due to broad applications in many areas including formation control^[87-88], distributed sensor networks^[89-90], flocking^[91-92], distributed computation^[93], and synchronization of coupled chaotic oscillators^[94-96]. Their common property is that each individual agent lacks global knowledge of the whole system and can only interact with its neighbors to achieve certain global behaviors.

A set of common and important research problems for multi-agent systems focus on how the agent dynamics and the interacting network topology affect their behaviors. Recently, the emergence of NCSs has stimulated the research interest on multi-agent systems. One of the interesting problems is the consensus of multi-agent systems, which requires all networked agents to reach an agreement on quantity of common interest using the shared data through local

communications. Toward this objective, a key step is to design a network based control protocol such that as time goes on, all the agents asymptotically reach consensus.

Distributed computation of multi-agent systems was initiated in the pioneering work of Borkar et al. in 1982^[97], where each agent updates its estimate of the same random variable whenever it makes a new observation or receives the estimate from another agent such that their estimates asymptotically agree. The asynchronous asymptotic agreement problem for distributed decision makers is studied in [98–99]. Actually, a wide variety of consensus problems for parallel computing have been pursued in computer science, see e.g. [98–102].

Those early works lay a foundation for more recent applications in distributed control and emergent behavior in networks of multiple dynamical agents. In [103], Vicsek et al. demonstrated simulation results for a discrete-time model of n autonomous agents using a local rule based on the average of its own heading and those of its “neighbors”. It is found that the nearest neighbor rule they are studying can drive all agents to eventually move in the same direction, even without a centralized coordination. This work attracted the interest of researchers in system and control. In [104], Jadbabaie et al. provided a theoretical explanation for this observed behavior and convergence results are derived for several similarly inspired models. Since this influential work, consensus control of networked multi-agent systems have been widely studied in the community to derive the network topology condition required for reaching consensus, see recent survey papers^[105–106]. For example, to achieve an average consensus which requires the states of all agents to asymptotically converge to the average of their initial values, the communication graph must contain a spanning tree for a fixed topology^[107–108] while for a switching topology, the union of the communication graphs should contain a spanning tree frequently enough as the system evolves^[104, 108–109]. In addition, the convergence rate to consensus directly relies on the second smallest eigenvalue of the graph Laplacian matrix^[105, 107].

Research on the consensus problem can be roughly categorized depending on whether the agent dynamics is continuous or discrete. The machinery for studying the continuous-time average consensus is based on the graph Laplacian matrix theory while for the discrete-time case, the Perron matrix theory becomes a convenient tool. Noticeable works focusing on the first-order integrator networks include [87, 107–108, 110–111] for the continuous-time consensus and [104, 108–109, 112] for the discrete-time consensus. The higher-order consensus problem has been considered in [113–116].

We use a discrete-time multi-agent system with N agents for an illustration. The dynamical equation for each agent dynamics is expressed as

$$\dot{x}_{k+1}^i = x_k^i + h \cdot u_k^i, \quad (20)$$

where $x_k^i \in \mathbf{R}$ is the state of agent i at time k , and h is the step size. The distributed control input u_k^i is given by

$$u_k^i = \sum_{j \in N_i} a_{ij} (x_k^j - x_k^i), \quad (21)$$

where N_i denotes the set of the neighbors of agent i , and $a_{ij} > 0$ if $j \in N_i$, otherwise $a_{ij} = 0$. The goal of consensus is to make the states of all networked agents asymptotically

agree, i.e.,

$$\lim_{k \rightarrow \infty} x_k^i - x_k^j = 0. \quad (22)$$

Furthermore, if $\lim_{k \rightarrow \infty} x_k^i = 1/N \cdot \sum_{i=1}^N x_k^i$, it is called average consensus.

To model interactions among agents, it is convenient to adopt the tool of graph theory^[117]. For instance, define a graph $G = \{V, E\}$ with $V = \{1, \dots, N\}$ denoting the indices of all interacting agents. The links among agents are represented by the edge set E . If there exists a link from agent j to agent i , then the link $(j, i) \in E$. Note that we devoid the self-loop, i.e., $(i, i) \notin E$. The neighbor set of agent i is given by $N_i = \{j | (j, i) \in E\}$. Let the weighed adjacency matrix be $A = \{a_{ij}\}$, the corresponding degree matrix is defined as $D = \text{diag}\{d_1, \dots, d_N\}$, where $d_i = \sum_{j=1}^N a_{ij}$. The Laplacian matrix is then given by $L = D - A$. Let $x_k = [x_k^1, \dots, x_k^N]^T$, it follows that

$$x_{k+1} = (I - hL)x_k. \quad (23)$$

Thus, the consensus problem is reduced to the analysis of the Perron matrix $P = I - hL$ with step size $h \in (0, 1/\max_i d_i)$, where the well-known Perron-Frobenius theorem^[51] plays an essential role, see [105] for more details. In [108], it is shown that if G contains a spanning tree, i.e., there is an agent that can be reached via links in E from any other agent, P has algebraic multiplicity equal to one for eigenvalue $\lambda = 1$ and all the other eigenvalues strictly lie in the unit circle. Thus, there exists a nonnegative vector v such that $\lim_{k \rightarrow \infty} P^k = \mathbf{1}v^T$ where $v^T \mathbf{1} = 1$ and $P^T v = v$. This implies that $\lim_{k \rightarrow \infty} x_k = \lim_{k \rightarrow \infty} P^k x_0 = \mathbf{1}(v^T x_0)$. For time-varying interacting topologies, the product $\prod_{i=1}^k P_i$ is to be studied using the ergodic property^[104]. Overall, the consensus problem of discrete-time multi-agent systems with first-order agent dynamics is conveniently converted to the analysis of the Perron matrix.

A very natural extension is to consider agents of linear vector dynamics, e.g.,

$$\dot{x}_{k+1}^i = Ax_k^i + Bu_k^i, \quad (24)$$

under the common control protocol

$$u_k^i = K \sum_{j \in N_i} a_{ij} (x_k^j - x_k^i), \quad (25)$$

where $x_k^i \in \mathbf{R}^n$, and the gain matrix K is time-invariant and independent of agent index.

The above control protocol is designed to drive the agent to move toward a common value while an unstable open-loop matrix A will make agents deviate from each other. The problem of interest is whether there exists a gain matrix K to achieve consensus. Such a fundamental problem has received an increasing interest in the recent literature^[118–127].

Inserting the control input to the agent dynamics, we obtain that

$$x_{k+1} = (I \otimes A - L \otimes BK)x_k, \quad (26)$$

where “ \otimes ” denotes Kronecker product^[51]. Different from the case of the first-order agent dynamics, the tool of perron matrix is no longer applicable.

In this section, a new method of integrating system control and graph theories will be presented. In comparison with the stabilization problem, which requires the system

states asymptotically converge to zero, the design of the gain matrix in consensus problem is to make the state of (26) converge to a manifold. Note that for the first-order agent dynamics case, the state of each agent converges to a fixed point $v^T x_0$.

It is obvious that $L\mathbf{1} = 0$, then there exists a left eigenvector u corresponding to the zero eigenvalue such that $u^T L = 0$ and $u^T \mathbf{1} = 1$. Multiply $u^T \otimes I$ on both sides of (26) and let $\bar{x}_k = (u^T \otimes I)x_k$, it follows that

$$\bar{x}_{k+1} = A\bar{x}_k. \quad (27)$$

Then, we conclude that the state of each agent does not converge to a stationary point if A is unstable. It is not difficult to show that the multi-agent systems reach consensus if and only if $\lim_{k \rightarrow \infty} x_k^i - \bar{x}_k = 0, \forall i \in V$. Define the displacement vector $\delta_k^i = x_k^i - \bar{x}_k$, it evolves as follows:

$$\delta_{k+1}^i = A\delta_k^i + BK \sum_{j \in N_i} a_{ij}(\delta_k^j - \delta_k^i). \quad (28)$$

Let $\delta_k = [\delta_k^1, \dots, \delta_k^{N^T}]^T$, we similarly obtain that

$$\delta_{k+1} = (I \otimes A - L \otimes BK)\delta_k. \quad (29)$$

Since $(u^T \otimes I)\delta_k = (u^T \otimes I)x_k - (u^T \otimes I)(\mathbf{1} \otimes I)\bar{x}_k = 0$, it is preferable to decouple this stable subspace before analyzing the stability of dynamical equation (29). To be precise, we use a transformation matrix $T \in \mathbf{R}^{N \times N}$ such that $TLLT^{-1} = \text{diag}\{J_1(\mu_1), J_2(\mu_2), \dots, J_s(\mu_s)\}$ to decompose the system (29) into several subsystems, where $J_i(\mu_i)$ is composed by Jordan block(s) associated with eigenvalue μ_i of L . Let $\tilde{\delta}_k = (T \otimes I)\delta_k$ and partition $\tilde{\delta}_k^T = [\tilde{\delta}_k^{1T}, \dots, \tilde{\delta}_k^{sT}]^T$ by taking into account the form of $TLLT^{-1}$, we yield several decoupled subsystems

$$\tilde{\delta}_{k+1}^i = (I \otimes A - J_i(\mu_i) \otimes BK)\tilde{\delta}_k^i, \forall i \in \{1, \dots, s\}. \quad (30)$$

Since $L\mathbf{1} = 0$, one of μ_i must be zero. For simplicity, let $\mu_1 = 0$. Then, a necessary condition for $\lim_{k \rightarrow \infty} \tilde{\delta}_k = 0$ is $J_1(0) = 0$. This essentially implies that G contains a spanning tree since the zero eigenvalue has algebraic multiplicity one. Note that $L^T u = 0$, then $\tilde{\delta}_k^1 = (u^T \otimes I)\delta_k = 0$. Thus, we have the following result.

Theorem 5. A necessary and sufficient condition for reaching consensus of multi-agent systems (24) under control protocol (25) is that G contains a spanning tree, and there exists a gain matrix K such that all the eigenvalues of $A - \mu_i BK, \forall i \in \{2, \dots, s\}$ strictly lie in the unit circle.

The design of a gain matrix K to achieve consensus is thus converted into a simultaneous stabilization problem^[128], which is usually difficult to be solved. The good news is that the above simultaneous stabilization problem bears a special structure and can be solved by using a Riccati design method.

Lemma 2^[127]. Given a positive $\eta \in \mathbf{R}$, consider a modified Riccati inequality as follows:

$$P > A^T P A - (1 - \eta^2) A^T P B (B^T P B)^{-1} B^T P A. \quad (31)$$

Assume that A is unstable and (A, B) is stabilizable, then there is a critical value $\eta_c \in (0, 1)$ such that for any positive $\eta < \eta_c$, there always exists a positive definite matrix solution P to (31). Moreover,

$$\prod_{i=1}^n (\max\{1, |\lambda_i|\})^{-1} \leq \eta_c \leq \left(\max_i |\lambda_i| \right)^{-1}.$$

If B is invertible, $\eta_c = (\max_i |\lambda_i|)^{-1}$. If B is of rank one, $\eta_c = (\prod_{i=1}^n \max\{1, |\lambda_i|\})^{-1}$. Otherwise, η_c is solved by LMIs.

If A is stable, it is obvious that there always exists a positive definite matrix P to solve (31). Then, let $\eta_c = 1$.

Theorem 6^[127]. If G contains a spanning tree and the following conditions hold.

- 1) (A, B) is stabilizable;
- 2) There exists an $\omega \in \mathbf{R}$ such that

$$\eta(\omega) = \max_{i \in \{2, \dots, s\}} |1 - \omega \mu_i| < \eta_c, \quad (32)$$

where μ_2, \dots, μ_s are all nonzero eigenvalues of the graph Laplacian matrix.

Then, the control gain $K = \omega(B^T P B)^{-1} B^T P A$ solves the consensus problem, where P is a positive definite solution to the modified algebraic Riccati inequality (31) with $\eta = \eta(\omega)$.

In fact, let $\eta_i = 1 - \omega \mu_j$. Then, $|\eta_i| \leq \eta(\omega)$ for all $i \in \{2, \dots, s\}$. Using the proposed gain matrix K , we obtain that

$$\begin{aligned} & (A - \mu_i BK)^H P (A - \mu_i BK) - P = \\ & A^T P A - (1 - |\eta_i|^2) \frac{A^T P B B^T P A}{B^T P B} - P \leq \\ & A^T P A - (1 - \eta^2) \frac{A^T P B B^T P A}{B^T P B} - P < 0, \end{aligned}$$

for all $i \in \{2, \dots, s\}$, which implies that all the eigenvalues of $A - \mu_i BK$ strictly lie in the unit circle.

We remark that (35) can be verified as follows.

Lemma 3. Let $\mu_j = r_j \exp(\theta_j i)$, then the inequality (35) holds if and only if the intersection

$$\bigcap_{j=2}^s \left(\frac{\cos \theta_j - \sqrt{\eta_c^2 - \sin^2 \theta_j}}{r_j}, \frac{\cos \theta_j + \sqrt{\eta_c^2 - \sin^2 \theta_j}}{r_j} \right) \quad (33)$$

is not empty, which is equivalent to that

$$\frac{1 - \eta_c^2}{\min_{j \in \{2, \dots, s\}} r_j f(\theta_j)} < \min_{j \in \{2, \dots, s\}} \frac{f(\theta_j)}{r_j}. \quad (34)$$

Here $f(\theta) = \cos \theta + \sqrt{\eta_c^2 - \sin^2 \theta}$ is decreasing in $\theta \in (0, \arcsin(\eta_c))$, where $\arcsin(x)$ is the inverse sine function of x .

Moreover, the sufficient condition in Theorem 7 is also necessary for a certain class of agent dynamics.

Theorem 7^[118]. If A does not contain any stable eigenvalue and B is of rank one, a necessary condition for reaching consensus of multi-agent systems (24) under control protocol (25) is that

- 1) (A, B) is controllable;
- 2) G contains a spanning tree and there exists an $\omega \in \mathbf{R}$ such that

$$\eta(\omega) = \max_{i \in \{2, \dots, s\}} |1 - \omega \mu_i| < \eta_c. \quad (35)$$

For an undirected graph, all the eigenvalues of L are real and nonnegative, i.e., $\mu_i \geq 0$. By writing the eigenvalues in an ascending order, e.g., $0 \leq \mu_2 \leq \dots \leq \mu_s$, the condition for the existence of $\omega \in \mathbf{R}$ to satisfy (35) is analytically expressed as

$$\frac{1 - \frac{\mu_2}{\mu_s}}{1 + \frac{\mu_2}{\mu_s}} < \eta_c. \quad (36)$$

Then, the effects of agent dynamics and interaction topology on the consensus problem are decoupled. In particular, the effect of graph is quantified by μ_2/μ_s , which denotes the network synchronizability^[94–95], while the effect of agent dynamics is characterized by η_c .

By Lemmas A.1 and A.2^[129], an upper bound of the eigenratio is immediately obtained, i.e.,

$$\frac{\mu_2}{\mu_s} \leq \frac{\min_i d_i}{\max_i d_i}.$$

The convergence rate of the average consensus over an undirected graph is determined by μ_2 ^[105,107]. By the Courant-Weyl interlacing inequalities^[51], adding an undirected edge to an undirected incomplete graph G will never decrease μ_2 , suggesting that the consensus performance will not deteriorate. However, adding an undirected edge to a graph may lead to a smaller network synchronizability. For example, consider the following two graph Laplacian matrices:

$$L_{G_1} = \begin{bmatrix} 3 & -1 & 0 & 0 & -1 & -1 \\ -1 & 3 & -1 & -1 & 0 & 0 \\ 0 & -1 & 3 & -1 & 0 & -1 \\ 0 & -1 & -1 & 3 & -1 & 0 \\ -1 & 0 & 0 & -1 & 3 & -1 \\ -1 & 0 & -1 & 0 & -1 & 3 \end{bmatrix},$$

$$L_{G_2} = \begin{bmatrix} 4 & -1 & -1 & 0 & -1 & -1 \\ -1 & 3 & -1 & -1 & 0 & 0 \\ -1 & -1 & 4 & -1 & 0 & -1 \\ 0 & -1 & -1 & 3 & -1 & 0 \\ -1 & 0 & 0 & -1 & 3 & -1 \\ -1 & 0 & -1 & 0 & -1 & 3 \end{bmatrix}.$$

It is clear that G_2 with network synchronizability 0.3970 is formed by adding an undirected edge to G_1 , which has network synchronizability 0.4. Thus, it is possible to lose consensus of the multi-agent systems (24) under the control protocol (25) by adding an edge. It appears to be counter-intuitive since the communication graph with a “better” connectivity may result in a worse consensus capability. Whether the network synchronizability will increase or decrease by adding an edge is not conclusive, see [95] for more details.

For a continuous-time system under a sufficiently small sampling period, the unstable eigenvalues of the discretized system (24) can be made arbitrarily close to one. This implies that η_c will be arbitrarily close to one as well. Hence, the inequality (35) will be eventually satisfied for any graph containing a spanning tree. This is consistent with the result in [130]. In fact, for a continuous-time system, information can be transmitted arbitrarily fast so that the network synchronizability of the communication graph becomes less important for achieving consensus.

Motivated by the fact that real communication networks are often operating in uncertain environments, stochastic consensus of multi-agent systems with single-integrator dynamics when the communication topologies are randomly switching has been extensively studied in the literature^[131–136]. In [132,136], a necessary and sufficient condition for achieving consensus of agents with single-integrator dynamics is that the mean topology with respect to the stationary distribution must be connected. This result is extended to Markovian switching topologies in [131] for both the discrete and continuous time consensus of single-integrator agent dynamics. The consensus condition is that the union of topologies corresponding to the

states of the Markov process is strongly connected. Similar conclusion is made for the discrete multi-agent systems of double integrator in [137]. However, the consensus problem of multi-agent systems (24) under time-varying topologies is much more involved, see [138–141].

7 Control from sampling-theoretic approach

If there are no uncertainties in the initial state and the system dynamics, and no process disturbances, one can achieve most control objectives by an open loop controller. While uncertainty is inevitable in any real system, a close-loop controller using feedback can more robustly deal with these uncertainties. Ramaprasad^[142] defines feedback generally as “information about the gap between the actual level and the reference level of a system parameter which is used to alter the gap in some way”. From this perspective, a good sampling mechanism should be designed to reduce the aforementioned uncertainties as much as possible. The traditional way is to sample the system periodically in time. A nice feature of this approach lies in that analysis and design become very simple. In particular, the periodically sampled systems from linear time invariant processes continue to be linear, and are described by difference equations with constant coefficients. To date, there exists a vast literature about the periodic implementations of control laws.

However, it seems “inefficient” to take samples regardless of what happens in the system. One alternative possibility is to sample the system only when something “significant” happens, and control is executed unless it is required. This sampling scheme is called event-based approach. In contrast to the time-based approach, the analysis and design of event-based control technology is usually complicated. The research on this control implementation is not new, and much work was done in the period 1960~1980 in many of early feedback systems, see [143] for a comprehensive discussion.

Recently, event-based control over networks has regained research interest in NCSs since it creates a better balance between this control performance and other aspects in NCSs (communication load, computational load, system cost etc)^[144]. It is shown in [143] that the output variance can be significantly reduced by using the event-based control in comparison to time-based control for the first-order stochastic system. This shows that event-based control has a great potential for decreasing the bandwidth requirements for the network^[145], where the limited communication resources put hard restrictions on the number of measurement and control input that can be transmitted. This motivates the development of various event-based sampling/communication protocols^[146–155], most of which aim to reduce the number of transmissions to satisfy the communication constraints.

A key problem in event-based control is how the events should be “optimally” generated. It is apparent that events cannot be generated arbitrarily often. That is, the generation of events should exclude Zeno behavior, i.e., there is no trajectory of the system with an infinite number of events in a finite time^[156]. Intuitively, the less the number of events are generated, the less the feedback information is available to the controller. When defining events, these two competing factors should be jointly considered. The problem is further complicated by the fact that the generation of the optimal events is performance dependent. Moreover, analysis of systems with event-based sampling are related

to general work on discontinuous and time-varying systems.

In this section, we use an event-based scheme for cooperative control of multi-agent systems over networks as a design example^[154]. Each agent dynamics is expressed by a single integrator as follows

$$\frac{dx_t^i}{dt} = u_t^i, t \geq 0 \text{ and } i \in V. \quad (37)$$

The controller of agent i monitors its own state x_t^i continuously. Based on local information, it decides when to broadcast its current state over the networks, which is different from the traditional method of either continuously or periodically broadcasting its states. Let the latest broadcast state of agent i be given by $\hat{x}_t^i = x_{t_k^i}^i$, $t \in [t_k^i, t_{k+1}^i)$, where t_0^i, t_1^i, \dots is the sequence of event times of agent i .

Motivated by [107], the following event-based control protocol is adopted^[154].

$$u_t^i = - \sum_{j \in N_i} (\hat{x}_t^i - \hat{x}_t^j). \quad (38)$$

It is interesting to consider self-triggered events. That is, events are generated only based on the true and previously broadcast states of agent i . Precisely, an event for agent i is triggered once the triggering condition $f^i(t, x_t^i, \hat{x}_t^i) > 0$ is satisfied. Consequently, the event times are given iteratively by

$$t_{k+1}^i = \inf\{t : t > t_k^i, f^i(t, x_t^i, \hat{x}_t^i) > 0\}.$$

An intuitive idea is that once a significant event is detected, i.e., the measurement error between the broadcast and the true states of agent i is larger than a threshold, it is better to inform its neighbors so that the neighbors can keep a good estimate of the true state of agent i . Thus, it is reasonable to define $f^i(t, x_t^i, \hat{x}_t^i) = |x_t^i - \hat{x}_t^i| - h^i(t)$, where $h^i(t) > 0$ is the triggering threshold. For example, $h^i(t) = c_0 + c_1 \exp(-\alpha t)$, where c_i and α are nonnegative. Certainly, there are numerous ways to measure the "significance" of an event.

Under a connected and undirected graph, it is proved in [154] that by choosing $c_0 + c_1 > 0$ and $0 < \alpha < \mu_2$, the multi-agent systems with event-based control do not exhibit Zeno behavior, i.e., there exists a universal lower bound $\tau > 0$ such that $\inf_{k \in \mathbf{N}, i \in V} |t_k^i - t_{k+1}^i| \geq \tau$, and the displacement vector converges exponentially to a ball centered at the origin with radius

$$r = \|\mathcal{L}\| \sqrt{N} \frac{c_0}{\mu_2}.$$

This result is appealing since the size of the final displacement vector is adjustable by tuning the parameter c_0 . The fact that the radius scales with c_0 is due to the persistent existence of measurement error, which can not be eliminated by the above generated events if $c_0 > 0$. Obviously, one can achieve exact consensus by letting $c_0 = 0$. Additionally, α can not be too large. Otherwise, the states of agents will converge faster than that the threshold decreases, which may lead to the Zeno behavior. Note that μ_2 determines the convergence rate of the states of multi-agent systems with continuous state feedback^[107].

Another important problem is how to quantify the benefit of using event-based control, i.e., what is the number of transmissions per unit time, and how the convergence speed of consensus is affected by this quantity? Unfortunately, this is a challenging problem and remains open in the literature.

8 Conclusion

This survey has reviewed some recent developments and provided insights to some important results in networked control. The reader can find in-depth discussions on any issue of the survey by referring to the corresponding literature. It is our hope that readers will find the survey useful for their endeavour in this fascinating area. Due to page limitation, it should be pointed out that not all aspects of current research in networked control have been covered, and we extend apology to anyone who feels that some important topics have been missed.

We suggest several potential research directions to conclude this paper.

1) Information transmission theory for networked control

As mentioned, one of the most interesting developments in NCS over the past decade is an information-theoretic approach for networked control where the studies have been centered around the issue of the minimum data-rate/bandwidth for stabilizing unstable systems. Various results relating unstable open-loop poles to minimum data rate have been obtained for linear systems with or without disturbance inputs over ideal or noisy channels. The challenges mainly come from uncertainties in disturbances as well as communication channels. It has been revealed that the commonly used channel capacities in information theory such as the Shannon capacity may not be sufficient to characterize the trade-off between communication and information rate production of a dynamical system with disturbances. For example, in the Shannon capacity, message is encoded and communicated reliably only asymptotically with respect to the length of codeword. It remains important to further understand the operational mechanism of feedback control loops over data rate constrained communication channels and develop a rigorous theory of information transmission for control systems that takes into consideration the trade-off between transmission delay and information reliability.

2) Performance control

Existing results on data rate are mostly concerned with stabilization. It is imperative to study performance control in networked systems. Clearly, control system performance relies on both the communication networks and controllers which cannot be designed separately. In particular, it is of both theoretical and practical significance to study the trade-off between communication rate and achievable performance as well as develop joint design methods for computationally efficient and practically implementable encoding-decoding schemes and controllers. The issue of communication and control co-design deserves further investigation.

3) Network topology and data rate for multi-agent systems

The existing information-theoretic approach is mainly concerned with single-loop control. For networked systems, an important feature is that multiple systems/loops share common communication resources. Thus, it is important to study encoding-decoding schemes and network protocols for efficient and effective cooperation among multiple loops or multi-agent systems. In [118], we have shown that reaching consensus with perfect state feedback implies that with quantized state feedback, provided that data rate of each link in the communication graph is not less than an explicitly determined lower bound. Although this result is meaningful, its conservativeness is still unclear. What is of particular interest is to derive results that link consensus to both network topology and data rates of communications links. A combined information-theoretic approach

and graph-theoretic approach may be exploited.

4) Cooperative control over uncertain large-scale networks

There are certain limitations in existing studies of multi-agent systems. First, the assumption that the communication link is perfect and an agent has some global knowledge on network topology is somehow restrictive. Note that changes in the operating environment, such as the random presence of large metallic objects between agents will inevitably affect the propagation properties of the channels. Thus, it is more interesting to consider the scenario that the communication channel is time-varying and unreliable. The investigation of consensus over time-varying graphs may have far-reaching consequences on the understanding and engineering of networked multi-agent systems. With fixed graphs, consensus of multi-agent systems under a common control protocol is converted into a simultaneous stabilization problem. However, this key property does not hold in the case of time-varying graphs. Perhaps a completely new method needs to be developed. On the other hand, in networked systems, there may be the case that different kinds of agents join and leave the network from time to time, it is unrealistic to assume that an agent has perfect knowledge of other agents' dynamics. How each agent will optimize its utility while minimizing its interferences to others requires some new thinking.

5) Cyber-security and fault-tolerant control

Needless to mention that cyber-security is of paramount importance. There have been many recent interests in this area based on analysis of system structure properties such as observability. Other issues that deserve further research include robust control and fault-tolerant control over networks.

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