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<td>Emadzadeh, Adel; Chiew, Yee Meng; Afzalimehr, Hossein</td>
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Effect of Accelerating and decelerating flows on incipient motion in sand bed streams

Adel Emadzadeh\textsuperscript{a1}, Yee Meng Chiew\textsuperscript{b}, Hossein Afzalimehr\textsuperscript{c}

\textsuperscript{a} M.Sc. student in Water Engineering Department, Isfahan University of Technology, Isfahan, Iran, (Currently, Research Scholar, School of Civil and Environmental Engineering, Nanyang Technological University, Singapore).

\textsuperscript{b} Professor, School of Civil and Environmental Engineering, Nanyang Technological University, Singapore.

\textsuperscript{c} Associate Professor, Water Engineering Department, Isfahan University of Technology, Isfahan, Iran.

ABSTRACT

The effect of flow acceleration and deceleration on velocity, von Kármán constant, Reynolds and normal stress distributions under incipient motion were experimentally investigated in this study using eight positive and negative bed slopes (\(\pm 0.7\%), \(\pm 0.9\%), \(\pm 1.25\%), and \(\pm 1.5\)) and three uniform sediments with median grain sizes \(d_{50} = 1.8, 1.3 \text{ and } 0.8 \text{ mm} \). By using an Acoustic Doppler Velocimeter (ADV), the instantaneous velocities were measured at three cross sections 5, 7 and 9-m from the flume entrance giving a total of 72 measured profiles. The results showed that: (1) the shear and normal Reynolds stresses in decelerating flow are greater than those in accelerating flow; (2) the normal Reynolds stress is considerably higher than shear stress in all measured profiles; (3) The von Kármán constant associated with non-uniform flows has an average of \(\bar{K} = 0.26\), which is lower than the classical value of 0.4; (4) the critical shear stress and Shields parameter for incipient motion in accelerating flow are considerably higher than that in decelerating flow; and (5) acceleration and deceleration along with slope variation are key factors governing incipient motion in non-uniform flows.

Key Words: Accelerating flow, Decelerating flow, Incipient motion, Shields parameter, Sand bed, Reynolds stress.
1. INTRODUCTION

The initial motion of particles is of considerable interest to geomorphologists and river engineers. The Shields diagram is the most commonly used figure for the determination of the threshold condition for cohesionless bed sediments [7]. In this diagram the shear velocity, \( u_* \), is used to calculate the particle Reynolds number \( Re_* \) and Shields dimensionless parameter \( \tau_* \) as follows:

\[
Re_* = \frac{u_* d}{v} \tag{1}
\]

\[
\tau_* = \frac{\rho u_*^2}{(\rho_s - \rho)gd} \tag{2}
\]

in which \( d \) = grain size; \( v \) = kinematic viscosity; \( \rho \) = fluid density; \( \rho_s \) = sediment density; and \( g \) = gravitational acceleration. The Shields diagram was originally developed for the initiation of motion for uniform cohesionless sediments on a nearly horizontal bed where 2-dimensional uniform turbulent flow is present. Deviations from Shields’ results, as observed by several investigators [7], were mainly from the uncertainty in the evaluation of the average bed shear stress, and due to the bed geometry, shape, size, density, and precise definition of the threshold condition.

In rivers where sedimentation problems are important, the application of Shields’ diagram may yield erroneous results because of flow non-uniformity caused by the presence of bedforms, positive or negative longitudinal bed-
slopes and pressure or velocity gradients. In non-uniform flows, both the depth and cross-sectional area of the flow vary in the flow direction. If the flow’s cross sectional area decreases in the downstream direction, accelerating flow with a positive velocity gradient will form. On the other hand, decelerating flow will be generated. The positive and negative pressure gradients likely will affect the incipient motion.

A number of studies have been conducted in the past 40 years concerning the slope dependency of the critical Shields stress in longitudinal and transverse slopes [9, 10, 18]. The results of these experiments conducted on steep slopes approaching the angle of repose show that the critical shear stress is not only a function of the particle Reynolds number, but also the longitudinal bed slope. As the bed slope increases, the critical shear stress decreases because the positive slope enhances the gravitational component of forces acting on the sediment particles, resulting in sediment motion.

For mild slopes in which the bed slope is much smaller than the angle of repose, bed mobility is not significantly affected by gravity. Shvidchenko and Pender [25] stated that in a uniform flow with a particular bed shear stress and sediment size, the increase in bed slope and corresponding decrease in flow depth cause greater flow resistance and, accordingly a lower flow velocity and sediment
transport rate. As a result, a higher value of shear stress is required to produce
the same rate of transport [18].

Recently several studies examined the turbulence characteristics of non-uniform
flows. Notable amongst them is that by Song and Graf [28] and Kironoto and
Graf [16], who studied accelerating and decelerating flows over hydraulically
rough beds. They used an Acoustic Doppler Velocity Profiler (ADVP) to
measure the instantaneous velocity over a fixed gravel-bed under equilibrium
conditions. In this type of flow, its component distributions such as velocity,
Reynolds stresses and turbulence intensities are independent of their upstream
conditions, and are thus invariant along the main flow direction [30].

Therefore, in an equilibrium flow the velocity profiles and all the tensor stress
components distributions are self similar at different cross sections for each
measuring run. Afzalimehr and Anctil [1, 2] studied the behavior of shear
velocity along a gravel bed in decelerating and accelerating flows. They
examined different methods to estimate the shear velocity and proposed a new
method, which they called the boundary characteristics method. Song and
Chiew [27] studied the effect of non-uniformity on the mean velocity and
turbulence characteristics in accelerating and decelerating flows under
equilibrium conditions analytically and experimentally. More recently,
Afzalimehr and Rennie [3] determined the bed shear stress in a gravel bed river
having a non-uniform flow using the boundary layer displacement thickness and
the boundary layer momentum.

Dey and Lambert [11] and Yang et al. [33] studied the Reynolds stress and
velocity distribution in non-uniform flows analytically. They verified their
results with Song’s [26] data for non-uniform flows over fixed beds. Yang et
al.’s [33] analytical results showed that the non-zero vertical velocity in non-
uniform flows introduces an additional momentum to the flow that causes the
Reynolds stress distribution to have a convex form in decelerating and concave
form in accelerating flows. This form is different from the conventional linear
distribution in 2-dimensional uniform flows.

Despite these and other studies on the effect of non-uniformity on the velocity
distribution and turbulence characteristics of the flow, very few researchers
have to-date investigated the influence of flow non-uniformity on the initiation
of sediment motion. As far as the writers are aware, the only study on the
incipient motion of gravels in decelerating flows was that by Afzalimhr et al.
[4]; their results revealed that the Reynolds stress distribution over fixed and
mobile beds has a convex form. Moreover, the critical Shields parameter value
for their decelerating flow experiments was less than published results for
uniform flow experiments. Attaining knowledge on the incipient motion
process under this flow condition will undoubtedly enable researchers to gain
clearer insights into the erosion and sediment transport behavior in rivers.

The objectives of this research are to study (1) the incipient motion of
cohesionless particles in non-uniform accelerating and decelerating flows; and
(2) the normal and shear stresses distribution in non-uniform flows over
sediment bed under threshold condition.

2. EXPERIMENTAL SETUP AND PROCEDURE

The experiments were conducted in a slope adjustable flume (Fig. 1) that is 14m
long, 0.6m wide and 0.6m deep. An electromagnetic current meter was used for
discharge measurement with $3 \times 10^{-4}$ m$^3$/s precision and a mobile limn meter
with an accuracy of ±1 mm for flow depth measurements along the flume. The
oscillations in the head box were damped by using a grid to ensure a stabilized
flow condition in this study. Subsequently, the flow passed over a coarse bed
before reaching the test section. The flow eventually moved downstream into a
collection tank that was linked to the sump. The flow depth was controlled by
means of a tailgate at the outlet end of the flume.

Insert Fig. 1: Experimental setup: (a): Flume setup; (b): Accelerating flow (c)

Decelerating flow
There are three main techniques one may use to obtain non-uniform flow in laboratory open channel flumes: (1) By converging or diverging the flume walls; (2) By using negative or positive longitudinal bed slopes; or (3) By adjusting the flow rate or tailgate height to achieve desired accelerating and decelerating flow conditions. In this study, the longitudinal bed slope was altered to obtain flow non-uniformity.

The slope of the flume was first set to zero and by varying the thickness of the sediment layer along the flume, different longitudinal bed slopes could be obtained. Three different uniform sediments with median grain sizes $d_{50} = 1.8$ mm, 0.8 mm and 1.3 mm were used in the experiments. In order to prevent localized scour and achieve a fully developed flow, the first 4-m reach of the flume was covered with gravel. The physical characteristic and grain size distribution of the gravel and the uniform sediments are shown in Table 1 and Fig. 2, respectively.

Insert Fig. 2: Grain distribution of the employed sediment

Insert Table 1: Physical characteristics of the sediment

All measurements were carried out at three cross sections, i.e. at the locations 5, 7 and 9 m downstream from the flume entrance (See Fig. 1). These cross sections were located in the fully developed flow region and they were not affected by the presence of the tailgate.
A sudden change in the bed roughness at the end of the first 4 m reach of the flume may affect the measurements at the given cross section, especially that at cross section (5-m). The study of Chen and Chiew [8] revealed that the response of the roughness height, bed shear stress, turbulent intensity and Reynolds shear stress distribution to a sudden change in bed roughness occur gradually over a transitional length along the bed which is approximately 5-6 times the flow depth. Therefore, by maintaining the flow depth of the experiments to be less than 15 cm at the 4-m section, the measurements at the 5-m section should not be affected by the change in bed roughness.

In non-uniform flows, the mean velocity and bed shear stress vary along the flow. While bed particles may be at the threshold condition in one section, bed-load movement with bedform generation or no movement will take place on the other sections. Therefore, the cross section where incipient motion occurs must be selected so as to prevent local scouring and maintain the bed slope during the tests.

In accelerating flows where the velocity gradient is positive, the mean velocity and bed shear stress increase in the streamwise direction. Therefore, the section downstream of the main measurement section (at threshold condition) is subjected to sediment transport. In decelerating flows, on the other hand, the section upstream of the main measurement section is subjected to erosion. In
light of this, the 9-m and 5-m sections were assigned to be the main measurement sections where threshold condition prevails for both accelerating and decelerating flow experiments.

Additionally in accelerating flow experiments, the same sediment particles as those in the measuring reach were glued to a false floor in the 10-14 m reach (Fig. 1b). In this way, the bed slope was maintained and sediment particles were not transported; therefore no bedform was generated.

For decelerating flow experiments on the other hand, instead of filling the flume with gravel, only one layer of gravel was placed over the surface of a false floor in the first 4-m reach of the flume in order to ease slope adjustment and bed-leveling prior to each run (Fig. 1c). As a result, the gravel particles were not transported by the flow and they also would enhance boundary layer development. Moreover, in this way the bed slope upstream of the main (5-m) measurement section was maintained and particle movement and bedform generation was prevented.

Before each test was conducted, the bed was leveled and the flume filled with water from the downstream end in a way that the water depth was at least 10 cm above the highest bed elevation. The pump was then started and the flow gradually increased until the incipient motion was observed in the main measurement sections (9-m in accelerating and 5-m in decelerating flows).
Kramer’s [17] weak movement criterion was used to identify the incipient motion condition. Under this condition, a number of small grains at isolated spots with countable quantities are in motion. When the threshold condition was reached at the main measuring sections, velocity and turbulence measurements were carried out at all the three cross sections.

The instantaneous velocities of flow in three directions were measured using Sontek’s down-looking 3-dimensional MicroADV (Micro Acoustic Doppler Velocimeter) in which the flow velocity is determined based on the principle known as the Doppler effect. For each point the sampling frequency was set at 50 Hz and the total sampling time exceeded 4 minutes; therefore, more than 12000 instantaneous velocity data were recorded in each direction. MicroADV has a manufacturer reported precision of ±0.1 m/s and its sampling volume is located 5 cm below the probe [29], therefore velocity measurements at depths less than 5 cm below the free surface could not be obtained due to the limitation of the down-looking MicroADV probe.

MicroADV signals are affected by Doppler noise, or white noise, associated with the measurement process [20]. To remove possible aliasing effects, velocity time series were analyzed using WinADV, which is a windows based viewing and post-processing utility for ADV files. This software provides signal quality information in the form of a correlation coefficient (COR) and signal to
noise ratio (SNR). Moreover it has filters such as phase-space threshold despiking (first described by Goring and Nikora [14]) and acceleration spike filter. The manufacturer suggests that when (COR) does not exceed 70 %, and SNR is less than 5 db, the instantaneous velocity measurement is dominated by acoustic noise and, as a rule of thumb, these measurements should be discarded [29]. To remove noise effects in this study, the data with \( SNR < 5 \text{ db} \) and \( COR < 70\% \) were discarded and the velocity time series data was passed through phase-space threshold despiking and acceleration spike filters.

It should be noted that MicroADV cannot provide the required information in the roughness sublayer because this layer extends from \( 2h \) to \( 5h \) where \( h \) is the height of the roughness element [24]. If \( d_{50} = 1.3\text{mm} \) is considered for example, one needs to obtain at least six mean point velocities within a depth of approximately 6 mm from the bed to draw any significant conclusion for the roughness sublayer. Furthermore, the estimation of roughness sublayer depends on visual judgment and a quantitative approach to defining this sublayer does not seem to exist.

In order to estimate the computational error and uncertainties related to measurements and data analysis, this study was compared to Venditti [32] with similar hydraulic conditions and measuring instruments. Venditti expressed that the error associated with the flow depth measurement and the cross sectional
average velocity \((U = Q/bD)\) was less than 1 mm and \(\pm 4.5\) mm/s respectively. In addition Venditti reported the slope adjustment error to be \(\pm 2.8 \times 10^{-5}\). When these uncertainties in slope and depth were propagated through the total bulk shear stress calculation \((\tau = \gamma DS)\) the resultant error in the total boundary shear stress was in the order of \(\pm 0.15\) Pa [32]. Due to similarity between tests reported by Venditti [32] and those conducted in the present study, the error in the computed boundary shear stresses should be in the same order of magnitude.

For each profile at least 10 point measurements were carried out in the inner layer \((z/D \leq 0.2)\), for which \(z = \) distance from the bed and \(D =\) water depth, and an additional 10 points in the outer layer \((0.2 \leq z/D \leq 1)\).

In summary, 8 slopes at \(\pm 0.7\%\), \(\pm 0.9\%\), \(\pm 1.25\%\) and \(\pm 1.5\%\) and three sediment were tested. Measurements were carried out at 3 cross sections along the flume; therefore, a total of 72 velocity profiles were obtained.

Table 2 shows the experimental data obtained in this study. The test names shown in the first column of the table contain several parts, indicating the characteristics of each profile. In the first part, the letters \((A)\) and \((D)\) represent accelerating and decelerating flow, respectively; the Greek numbers I, II, and III indicate the bed sediment type, the numbers 5, 7 & 9 correspond to the cross section at which measurements were conducted. The range of discharge in this research was \(0.019 \leq Q \leq 0.050\) \((m^3/s)\); Reynolds number \(0.405 \times 10^5 \leq\)
$Re \leq 1.142 \times 10^5$; Froude number $0.102 \leq Fr \leq 0.37$. The aspect ratio $b/D$ in which $b$ is the flume width and $D$ the water depth at the location where the velocity profile was measured, is $2.32 \leq b/D \leq 4.11$. Aspect ratios $b/D$ less than 5 for all the tests indicate that the flow is three dimensional according to Graf and Altinakar [15].

Inset Table 2: The experimental data

3. THEORETICAL CONSIDERATIONS

The Navier-Stokes equation for a 3-dimensional, steady non-uniform flow in the stream-wise direction is as follows:

$$\rho \left[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right] = - \frac{\partial P}{\partial x} + \left[ \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} \right]$$  \hspace{1cm} (3)

In (3), $u$, $v$ and $w$ are velocities in the $x$ (streamwise), $y$ (transverse) and $z$ (vertical) directions, respectively; $\rho$ = water density; $P$ = pressure; $\tau_{xy}$ and $\tau_{xz}$ = Reynolds shear stresses and $\sigma_x$ = Reynolds normal stress, respectively.

On the bed, the mean flow velocities are zero, i. e., $u = v = w = 0$; so, the left hand side of (3) is zero. Since the evolution of the turbulent shear stress in the $x$ direction may be considered to be small when compared to that in the $z$ direction, the expressions $(\partial \sigma_x / \partial x)$ and $(\partial \tau_{xy} / \partial y)$ may be neglected when compared to the term $(\partial \tau_{xz} / \partial z)$. Consequently, (3) is simplified to:
\[
\frac{\partial p}{\partial x} = \frac{\partial \tau_{xz}}{\partial z}
\]  

(4)

By assuming the boundary conditions of \( z = 0 \Rightarrow \tau_{xz} = \tau_0 \) and \( z = D \Rightarrow \tau_{xz} = 0 \), integration of (4) yields

\[
\tau_{xz} = \tau_0 + \left( \frac{\partial p}{\partial x} \right) z
\]  

(5)

In accelerating flow in which the horizontal velocity \( u \) increases in the streamwise direction, the pressure gradient is negative \( (\partial p/\partial x < 0) \); so, according to (4), \( (\partial \tau_{xz}/\partial z < 0) \) and from (5) we obtain \( (\tau_{xz} < \tau_0) \). Moreover, the shear stress distribution reaches its maximum value at the bed and reduces in a non-linear concave form towards the free surface. The boundary shear stress \( \tau_0 \) in this kind of flow is smaller than that in uniform flow according to (5).

In decelerating flow where the pressure gradient is positive due to the negative velocity gradient, \( (\partial \tau_{xz}/\partial z > 0) \) and \( (\tau_{xz} > \tau_0) \) according to (4) & (5), respectively. Therefore, the boundary shear stress \( \tau_0 \) in decelerating flows is larger than that in uniform flow. In this kind of flow, the shear stress distribution has an increasing trend near the bed, and after reaching its maximum value over the bed, it decreases towards the free surface in a non-linear convex form which is consistent with previous studies done by Song [26]; Kironoto and Graf [16]; Graf and Altinakar [15]; Song and Chiew [27]; Afzalimhr et al. [4].
4. EFFECT OF FLOW NON-UNIFORMITY AND SLOPE VARIATION ON REYNOLDS STRESS DISTRIBUTION

Using the measured data, the Reynolds stresses \( \tau_{ij} (i, j = x, y, z) \) were calculated. The results show that the absolute values of \( \tau_{xy} \) and \( \tau_{yz} \) are very small when compared to \( \tau_{xz} \), similar to that reported by Song and Chiew [27]. Since the distribution of the primary shear stress \( \tau_{xz} = -\rho u'w' \) is the most predominant component for the evaluation of bed shear stresses [21], it is discussed in greater detail here.

Figs. 3(a) and 3(b) show two typical shear stress distributions for accelerating and decelerating flows, respectively. The results show that the Reynolds shear stress distribution \( \tau_{xz} \) has two distinct regions throughout the water depth. For accelerating flows, Fig. 3(a) shows that \( \tau_{xz} \) starts from a non-zero value close to the bed, which increases until a maximum value is reached at the boundary between Regions 1 & 2. In Region 2, \( \tau_{xz} \) decreases non-linearly towards the point of minimum shear stress near the water surface.

The results of this study show that the Reynolds shear stress distribution in Region 2 is similar to that in published literature e.g. [4, 16, 27]. However in
spite of this similarity, the behavior of the $\tau_{xz}$ distribution in Region 1 in accelerating flows is noticeably different. Region 1 in Fig. 3(a) is hardly visible in most of the profiles in Song and Graf [28] and Song and Chiew [27] but more recognizable in Afzalimehr and Anctil [2] on steeper slopes with higher relative roughness over cobble beds. All these researchers have reported that the maximum shear stress occurs on the bed, which is different from the results of this study.

The occurrence of the maximum shear stress above the bed in accelerating flow appears to contradict the inference one may deduce from (5). According to this equation the bed shear stress $\tau_0$ should be more than the Reynolds shear stress ($\tau_0 > \tau_{xz}$) due to the negative pressure gradient ($\partial P/\partial x < 0$); therefore the maximum shear stress should form on the bed with a concave $\tau_{xz}$ distribution near the bed.

Notwithstanding this inference, Bennett and Best [5] and Fedel and Garcia [12] showed that over the upstream slope of a dune where the flow depth decreases in the flow direction i. e., the flow is accelerating, the Reynolds stress distribution is convex. This means that the maximum shear stress does not occur on the bed. Fedel and Garcia [12] attributed this to the formation of an internal boundary layer in a weakly mobile sediment condition in which even if the flow
is accelerating, a convex distribution will form. Their results support the data in Fig. 3(a).

Fig. 3(b) shows a typical $\tau_{xz}$ distribution with decelerating flows. The distribution has a convex form with the maximum occurring well above the bed, at an average distance of $\bar{z}/D = 0.35$ over all the decelerating flow runs. The $\tau_{xz}$ distributions obtained in this study are consistent with previous works reported in the literature, i.e., Afzalimehr and Anctil [1] and Song and Chiew [27].

The measured Reynolds shear stress profiles associated with accelerating and decelerating flows with the same bed slope (same absolute value), measuring section and sediment particles were compared as $\tau_{xz}/\tau_0$ versus $z/D$ in Fig. 4.

In this figure $\tau_{xz} = -\rho u'w'$ and $\tau_0$ is computed from (6) where $u_*$ is computed by extrapolation of the $\tau_{xz}$ distribution towards the bed:

$$\tau_0 = \rho u_*^2$$

(6)

Insert Fig. 4: Dimensionless shear stress distributions in accelerating (●) and decelerating (□) flow experiments

Fig. 4 shows that $\tau_{xz}$ associated with decelerating flow are generally larger and Region 1 is thicker than its accelerating flow counterpart. This means that in decelerating flows the maximum shear stress is larger and it is located at a
higher elevation from the bed, with an average dimensionless depth at

\[ (z/D)_{\tau_{Max}} = 35.11\% \] compared to \(15.54\%\) for accelerating flows (see data in
the last column of Table 2).

Fig. 5 shows the effect of longitudinal bed slopes on non-dimensional shear stress distributions. In this figure, bed slope is the only varying parameter while all other variables are kept constant.

**Insert Fig. 5: Effect of slope variation on the dimensionless shear stress distribution**

In decelerating flows as the bed slope is steepened from +0.7\% to +1.5\% the Reynolds stress increases. This relationship is due to the dependency of the shear stress on the pressure gradient. In decelerating flows with positive pressure gradients, increasing the bed-slope results in larger pressure gradients and therefore higher shear stress. In accelerating flows, however, a negative pressure gradient is observed. Hence, steepening the slope from -0.7\% to -1.5\% will result in a more negative pressure gradient and therefore, lower Reynolds shear stress. The experimental data generally support this inference.
5. NORMAL REYNOLDS STRESS

Fig. 6 shows the measured normal Reynolds stresses in longitudinal direction in both accelerating and decelerating flows. The data in this figure and those in Fig. 7 contain discontinuities at around $z/D = 0.15$-$2.0$. This likely is related to near bed interference in which the signal reflected from the sediment bed interferes directly with the return signal from the measuring volume. This problem is particularly problematic with down-looking probes [19]. It occurs when the time taken for one pulse to transfer from the sampling volume to the bed and back to the sampling volume is the same as the time between pulses. This interference leads to a velocity discontinuity at certain elevations above the bed, where noise may mask the velocity signal. Notwithstanding this discontinuity, the general trend of the distributions is essentially good, showing that all the normal Reynolds stress profiles in the longitudinal direction is found to be larger than the normal-to-wall and vertical directions, i.e., $\sigma_x > \sigma_y > \sigma_z$.

Insert Fig. 6: Dimensionless $\sigma_x$ distribution in accelerating (●) and decelerating (□) flow

A closer look at Fig. 6 reveals that the measured normal Reynolds stress profiles show an increasing trend from the free surface to the near bed region. They reach their maximum value at $z/D \approx 0.15$, beyond which they decrease.
abruptly towards the bed. Their behavior in the near bed region is consistent with Nikora and Goring’s [22] work on the turbulence structure of flows that pass over weakly mobile beds.

In order to compare the bed shear stress $\tau_0$ with the bed normal stress $\sigma_0$, the $\sigma_x$ distribution was extrapolated to the bed. The result shows that $(\sigma_0/\tau_0)$ for all the measured profiles has an average value of approximately 10. This means that the bed normal stress is 10 times larger than the bed shear stress for both accelerating and decelerating flows.

Fig. 7 shows the influence of the longitudinal bed slopes on the normal Reynolds stress in both accelerating and decelerating flows. The data reveal that the normal Reynolds stress distribution has a similar behavior as the shear stress distribution with reference to acceleration, deceleration of flows and slope variation.

Insert Fig. 7: Effect of bed-slope variation on $\sigma_x$ distribution in accelerating and decelerating flows

6. VON KÁRMÁN CONSTANT DETERMINED FOR MEASURED FLOW FIELD

For more than five decades, extensive studies in sediment transport have revealed that the “constant” of von Kármán is neither universal nor constant but

20
depends on the size of the roughness elements and flow conditions, ranging from a value at around 0.4 to as low as 0.21 [31]. The method with which many of these researchers used to compute von Kármán’s constant, $\kappa$ is to first assume that the velocity in the streamwise direction follows the logarithmic law in the inner region ($z/D \leq 0.2$) of the velocity profile. The bed shear stress, which is obtained using a separate device, is then substituted into the logarithmic equation to compute $\kappa$. This approach is used not only to compute $\kappa$ in uniform flow with an immobile bed but also in uniform flows with weakly mobile beds Nikora and Goring, [22] and decelerating flows by Afzalimehr et al.[4].

In this study, the shear velocity obtained from the Reynolds shear stress distribution is similarly substituted into the logarithmic law and the so-computed $\kappa$ for each profile is tabulated in Table 2. The computed $\kappa$ -values for accelerating flows $0.216 \leq \kappa_{Acc} \leq 0.388$ and decelerating flows $0.161 \leq \kappa_{Dec} \leq 0.421$, giving an overall average of $\bar{\kappa} = 0.26$. This finding is in agreement with results of Bennett et al. [6] $\kappa = 0.33$, Nikora and Goring[22] $\kappa = 0.29 \pm 0.03$ and Afzalimhr et al. [4] $\kappa = 0.3$, and is less than the widely accepted value of $\kappa = 0.4$. Evidence obtained in recent years by experimental studies indicates that $\kappa$ probably does decrease with increasing intensities of turbulence [13]. Moreover, Oncley et al. [23] suggested that the variation could
be presented as an inverse function of particle Reynolds number \( (Re_\ast = u_\ast d/\nu) \).

The main reason of the variation of \( \kappa \) probably is due to the changing magnitude of the dissipation deficit in the turbulent kinetic energy (TKE) budget, especially in the non-equilibrium boundary layer condition. The TKE budget can be presented as a balance between the temporal variation of TKE (local variation) on the one hand and the shear production, dissipation, turbulent transport (velocity fluctuations) and pressure transport on the other. However, based on limited information on the turbulent and pressure transport, it is assumed that these are zero and the local TKE shear production is balanced by dissipation. Therefore, deviation of \( \kappa \) from the known value of 0.4 should be due to this assumption. In fact the shear production and the dissipation production do not balance each other due to the non-zero net transport. In addition, many investigators stated that the deviation of \( \kappa \) from the classical value is related to the limitation of existing measuring techniques. This is because the unstable data near the bed display the most scatter which illustrates the difficulty in getting very accurate evaluation of the von Kármán constant. More sophisticated instruments are needed to justify the assumption.

7. INCIPIENT MOTION OF COHESIONLESS SEDIMENT WITH NON-UNIFORM FLOWS
In order to compare the incipient motion of cohesionless sediments between uniform and non-uniform flows, the widely accepted Shields diagram is employed. To this end, the effect of two parameters, namely bed-slope and pressure gradient (acceleration and deceleration) on the critical Shields parameter $\tau_{*c}$ is investigated. This is because flow non-uniformity in this study is effected by varying the longitudinal bed slopes. As a result, modification of the bed slope not only causes a change in the pressure gradient, but also a gravitational component on sediment mobility, especially when the longitudinal bed slope is large. It is interesting to note that the longitudinal bed slope affects the incipient motion of the bed sediment in two opposing ways, which may be categorized as “pressure gradient” and “gravity” effects. Both these factors are discussed in the following sections.

To introduce flow acceleration in the study, the bed is steepened to form an adverse slope where a negative pressure gradient or positive velocity gradient is present. Consequently, one will expect easier movement for the bed sediments. In other words we expect to observe a lower critical Shields stress as the bed slope steepens ($S \uparrow \Rightarrow \tau_{*c} \downarrow$). This phenomenon is called the “pressure gradient” effect.

However, when the slope is steepened to produce an accelerating flow, the steeper negative longitudinal bed slope renders particle movement more
difficult since the particles now must move uphill against gravity [9]. As a result, one will expect to observe a higher critical Shields stress as the slope steepens ($S \uparrow \implies \tau_{sc} \uparrow$). This behavior is called the “gravity” effect. Clearly, the overall response of the bed sediment mobility to flow non-uniformity that is brought about by a change of the longitudinal bed slope is subjected to both these influences. In a similar manner, decelerating effect brought about by the introduction of a steep downward bed slope will likewise introduce both these effects on bed sediment mobility.

7.1. Accelerating Flows

Fig. 8 shows the overall response of the bed sediment to the combined “gravity” and “pressure gradient” effects with accelerating flows. The shear stress used in the figure is deduced from the 2nd order polynomial equation ($ax^2 + bx + c$) fitted to the measured Reynolds shear stress profiles. Subsequently, this equation was extrapolated towards the bed in order to estimate bed shear stress $\tau_0$.

Insert Fig. 8: Effect of slope variation on $\tau_{sc}$ in accelerating flows

The measured critical Shields stresses for accelerating runs are plotted against bed slope for all the three sediment groups in the figure. The data, which are tabulated in Table 3 and plotted in Fig. 8, generally shows a decreasing $\tau_{sc}$.
followed by an increasing $\tau_{*c}$ as the adverse bed slope steepens. The initial decreasing $\tau_{*c}$, which shows that the bed sediment is becoming easier to move, is a reflection of the dominant “pressure gradient” effect. The subsequent increasing $\tau_{*c}$, on the other hand, indicates a more dominant “gravity” effect, showing that the steeper adverse slope is making it harder for the bed sediment to move.

Insert Table 3: Critical Shields parameter

**7.2. Decelerating Flows**

In decelerating flows slope variation affects the initiation of particle motion in the same two opposing manners, viz. “pressure gradient” and “gravity” effects. These factors are clearly noticeable in the experimental results tabulated in Table 3 and plotted in Fig. 9.

Insert Fig. 9: Effect of slope variation on $\tau_{*c}$ in decelerating flow

The data in the figure show a general increasing $\tau_{*c}$ trend with an increase in downward bed slope. This increasing function reveals an overriding “pressure gradient” effect, indicating that the “gravity” effect appear less assertive when compared with the “pressure gradient” effect. In other words as the longitudinal bed slope increases, $\tau_{*c}$ increases or the bed sediment particles become less
mobile, opposing to what is generally believed that sediment particles move easier in a downward slope.

The only exception relates to one data point in Fig. 9(a) in which $\tau_{*c}$ reduces. It occurs with decelerating flow runs associated with sediment group (I) where the longitudinal bed slope increases from 1.25% to 1.5%. If the data were correct, the “gravity” effect becomes more dominant at this point, overshadowing its “pressure gradient” counterpart. Unfortunately, repeating this test run is now not possible, and therefore one can no longer confirm or refute the accuracy of the data. It is hope that future research could be conducted to verify the test result.

Finally in order to gain an overall perception on the incipient motion of cohesionless sediment associated with accelerating and decelerating flows, the computed critical Shields stress $\tau_{*c}$ is superimposed onto the Shields diagram in Fig.10 for comparison.

Insert Fig. 10: A comparison between Shields critical stress in accelerating and decelerating flows

The figure shows that in all cases, the critical Shields stress $\tau_{*c}$ associated with decelerating flows is less than that with accelerating flows. This phenomenon can be justified by the higher Reynolds shear stresses associated with
decelerating flows (see Fig. 4) and favorable positive bed slope in the flow
direction which enhances particle movement. In both instances the critical shear
stress is higher than those predicted using the Shields diagram for bed particles
subjected to uniform flows.

8. CONCLUSIONS

An experimental study was carried out to investigate the influence of
accelerating and decelerating flows on the incipient motion of three different
cohesionless particles over 8 bed-slopes by measuring 72 velocity profiles.
Based on the experimental results, the following conclusions are drawn:

1) The Reynolds shear stress distributions for both accelerating and
decelerating flows may be divided into two regions. The shear
stresses in Region 1 increase from the bed until a maximum value,
which constitutes the transition to Region 2. The shear stresses in
Region 2 decreases from the maximum value towards the water
surface. The same behavior is found in both accelerating and
decelerating flows.

2) The normal stress is larger than the shear stress in all profiles and $\tau_{xz}$
and $\sigma_z$ had greater values in decelerating flow when compared with
accelerating flow. The normal stress at the bed is found to be
approximately 10 times the bed shear stress for both accelerating and decelerating flows.

3) The average von Kármán constant is 0.26 for non-uniform flows and is 35% lower than the commonly used value of 0.4 reported for the fixed bed streams.

4) The critical Shields parameter $\tau_{sc}$ values for decelerating flows are lower than those for accelerating flows but both are higher than those associated with uniform flows.

5) The slope variation affects the incipient motion in two opposing ways: “pressure gradient” and “gravity” effects. Uphill slope in accelerating flow inhibits, and downhill slope in decelerating flow enhances bed sediment mobility. However, the corresponding flow accelerating or decelerating-induced “pressure gradient effect” has a completely opposite effect on the initiation of bed sediment motion.

It may be surmised from results of this study that flow acceleration and deceleration must be included as an important governing factor in affecting the incipient motion of cohesionless bed sediments in non-uniform flows.
Notations:

\( b/D \) = Aspect ratio

\( d \) = Grain size

\( D \) = Water depth

\( d_g \) = Geometric mean size, \( d_g = \left( d_{16}d_{84}\right)^{1/2} \)

\( d_{84}, d_{50} \) & \( d_{16} \) = Grain sizes for which 84%, 50% and 16% of grains are finer by weight respectively

\( Fr \) = Froude number, \( Fr = U/\sqrt{gD} \)

\( g \) = Gravitational acceleration

\( G_r \) = Gradation coefficient, \( G_r = 1/2 \left( d_{84}/d_{50} + d_{50}/d_{16}\right) \)

\( h \) = Height of the roughness element

\( P \) = Pressure

\( Q \) = Flow discharge

\( Re \) = Reynolds number, \( Re = uD/\nu \)

\( Re_\ast \) = Particle Reynolds number, \( Re_\ast = u\ast D/\nu \)

\( U \) = Cross sectional average velocity, \( U = Q/bD \)

\( u_\ast \) = Shear velocity

\( z \) = Distance from the bed
(z/D)_{\text{Max}} = \text{Non-dimensional depth at which maximum Reynolds shear stress } \tau_{xx} \text{ happens}

\kappa = \text{von Kármán constant}

\nu = \text{Kinematic viscosity of water}

\rho = \text{Fluid density}

\rho_s = \text{Sediment density}

\sigma = \text{Reynolds normal stress}

\sigma_g = \text{Geometric standard deviation.} \sigma_g = (d_{84}/d_{16})

\tau = \text{Reynolds shear stress}

\tau_\ast = \text{Shields dimensionless parameter.} \tau_\ast = \rho u_*^2 / (\rho_s - \rho) g d

\tau_0 = \text{Bed shear stress}
References


Figure Captions

Fig. 1: Experimental setup: (a): Flume setup; (b): Accelerating flow (c) Decelerating flow
Fig. 2: Grain distribution of the employed sediment
Fig. 3: Distribution of Reynolds stress $\tau_{xz}$ in accelerating and decelerating flows
Fig. 4: Dimensionless shear stress distributions in accelerating (●) and decelerating (□) flow experiments
Fig. 5: Effect of slope variation on the dimensionless shear stress distribution
Fig. 6: Dimensionless $\sigma_x$ distribution in accelerating (●) and decelerating (□) flow
Fig. 7: Effect of bed-slope variation on $\sigma_x$ distribution in accelerating and decelerating flows
Fig. 8: Effect of slope variation on $\tau_{xc}$ in accelerating flows
Fig. 9: Effect of slope variation on $\tau_{xc}$ in decelerating flow
Fig. 10: A comparison between Shields critical stress in accelerating and decelerating flows
**Table Captions**

Table 1: Physical characteristics of the sediment  
Table 2: The experimental data  
Table 3: Critical Shields parameter
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(b): Accelerating flow

(c) Decelerating flow
Figure 2: Grain distribution of the employed sediment

Click here to download Figure: Figure 2 Grain distribution of the employed sediment.pdf

Figure 2: Grain distribution of the employed sediment
Figure 3: Distribution of Reynolds stress $\tau_{xz}$ in accelerating and decelerating flows.

(a) Accelerating flow

(b) Decelerating flow
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Figure 6: dimensionless normal Reynolds stress distribution accelerating (●) and decelerating (□) flow experiments.
Figure 7: Effect of bed-slope variation on $\sigma_x$ distribution in accelerating and decelerating flows
Figure 8: Effect of slope variation on $c_\tau$ in accelerating flows
Figure 9: Effect of slope variation on $\tau_{ec}$ in decelerating flow
Figure 10: A comparison between Shields critical stress in accelerating and decelerating flows
Table 1: Physical characteristics of the sediment

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