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Back-analysis approach for the determination of hydraulic conductivity in rock caverns

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ABSTRACT: Water seepage related problem is often the major geological hazard in underground rock excavation. In order to reduce the risk associated with extensive seepage, a reliable hydro-geological model should be established based on the in-situ investigation data. One of the challenges for establishing a reliable hydro-geological model is on how to determine the hydraulic conductivities of the fractured rock masses using the limited in-situ investigation data. In this study, a back-analysis approach for the determination of the hydraulic conductivities along a rock cavern is presented. To take the advantages of both the analytical solutions and the numerical methods, this paper proposed a semi-analytical approach for prediction of the water inflow into caverns with the horseshoe section, and the semi-analytical solution is used for the back-analysis of the hydraulic conductivity around a rock cavern based on the in-situ monitoring data. The hydraulic conductivities are obtained by using the EXCEL spreadsheet’s build-in optimization routine SOLVER to minimize the error function. The computed water inflow into the cavern is compared with the in-situ measured data. The results indicate that the derived hydraulic conductivity is acceptable.

Keywords: rock cavern, hydraulic conductivity, seepage analysis, back analysis

1. Introduction

In the development of underground rock caverns for various uses, such as hydrocarbon storage (Kiyoyama, 1990) and hydropower projects (Li et al., 2008), the groundwater control during the excavation phase and in the operation phase plays a critical role in terms of construction cost and duration, and construction safety. As witnessed in many underground projects all over the world, water seepage related problems have been identified as the dominant geological hazards (Rebekka et al., 2003), which may potentially lead to: accidents during the construction, deterioration in working conditions which may threaten to worker’s safety, rock-falls, settlement of aboveground buildings, extended construction duration and a high cost (Schwarz et al., 2006).

In order to reduce the risks associated with the seepage, a reliable hydro-geological model should be established based on the in-situ investigation data. For many projects, the hydro-geological model is established simply based on the arithmetic average or linear interpolation of the borehole data with laboratory and in-situ tests (Sun and Zhao, 2010), which is often not reliable. Actual hydro-geological field is anisotropic and heterogeneous, but information on the hydro-geological setting is sparse. Limited boreholes are often used to get a profile of hydraulic conductivity versus depth at specific locations. In addition, geophysical survey can provide the thickness of weathered zone, the depth of the fresh rock mass and the location of fault zone or water bearing zone. The key challenging issue during site investigation phase is how to integrate limited geological and hydro-geological data to establish a reliable hydro-geological model.

The neural network (NN) is a flexible data mining tool to establish an input-output mapping. It has been successfully applied in a wide range of civil engineering applications, such as in fault detection (Jakubek and Strasser,
2004), and preliminary hydro-geological modelling (Sun et al., 2011), which can be used to calculate the total water inflow into the underground caverns, a critical factor for construction planning and construction cost evaluation.

Numerical modeling using finite element and finite difference schemes can be used in various complicated geological conditions and is suitable for various shapes of the cavern section. However, it is tedious and time-consuming. On the other hand, many analytical solutions (Lei, 1999; Tani, 2003; Park et al., 2008) on prediction of water flow into rock tunnel corresponding to specific conditions have been proposed which can be used easily to establish the relationship between the water inflow and the hydraulic conductivity around caverns. However, these analytical solutions are only applicable to caverns with simple geometrics, such as circular, elliptical or square cross-sections (Tani, 1999).

During the construction phase, there are more site data available around the excavation area, such as water pressure monitoring and seepage water measurement, but these data are often not fully utilized to analyze the mechanism of the seepage problem and to update the preliminary hydro-geological model. This paper continues our study on the determination of hydraulic conductivities of the rock masses (Sun and Zhao, 2010; Sun et al., 2011) for an underground rock cavern project. As the cavern is excavated, the data of water quantities into the underground rock cavern are collected together with the water pressure information at various locations measured through previously installed piezometers. This proposed model, named as “back-analysis approach” approach, considers adequately the data collected from the site measurements during excavation. It consists of minimizing the error function, which is employed to refine the preliminary hydro-geological model and to build a reliable hydro-geological model. Then, it could be used for the back-analysis of the hydraulic conductivities. In this paper, the main hydrogeological features of a real project are described, the analytical and numerical models of groundwater flow are shown, and details of the implementation of back-analysis are also provided. Finally, the model calibration and testing as well as a discussion of the main achievements are presented.

2. Preliminary semi-analytical solution for water inflow estimation

In order to back calculate hydraulic conductivity, one of the most popular approaches is the comparison of the measured inflow data with modeled inflows, and then the relationship between water inflow and the hydraulic conductivity can be established. Several researchers presented analytical solutions to establish the relationship between the hydraulic conductivity and water inflow for the circular tunnel (Lei, 1999; Tani, 2003). Park et al. (2008) revised and compared existing analytical solutions using a common notation and reference datum for the hydraulic head. Zhang and Franklin (1993) presented an analytical solution for the circular tunnel assuming a hydraulic conductivity gradient, which can be regarded as an extension of the solution of Goodman (1965). Most of these models assume two-dimensional flow in a plane perpendicular to the tunnel axis and the tunnel has a circular geometry. When the water level is higher than the ground surface (or upper boundary) and atmospheric pressure is effective inside the tunnel and at the tunnel perimeter (Fig. 1), the solution for the groundwater inflow \( Q \), which is the volume of water per unit tunnel length, into an unlined circular tunnel can be obtained as (Park et al. 2008).

\[
Q = k \frac{2\pi}{\ln\left(\frac{h}{r} + \sqrt{\frac{h^2}{r^2} - 1}\right)} (A + H)
\]
where \( h \) is the tunnel depth, \( r \) is the tunnel radius, \( k \) is the hydraulic conductivity, \( H \) is the water depth at the upper boundary, \( \alpha \) is a parameter defined as \( \alpha = (h - \sqrt{h^2 - r^2})/r \), and \( A \) is the other parameter defined as \( A = h(1 - \alpha^2)/(1 + \alpha^2) \).

In order to study more complicated scenarios, Tani (1999) derived formulae which permit the calculation of the water inflow into tunnels of elliptical or square cross-sections as

\[
Q = k \left[ \frac{2 \pi}{\ln \frac{4h}{a+b}} \left( h + H \right) \right]
\]

(2)

where \( a \) and \( b \) are horizontal and vertical semi-axis of the elliptical cross-section, respectively (Fig. 1).

\[
Q = k \left[ \frac{2 \pi}{6 - \pi} \frac{2\sqrt{4h}}{c} \left( h + H \right) \right]
\]

(3)

where \( c \) is the side length of the square cross-section (Fig. 1).

Fig. 1

It should be noted that it is common to use the analytical solution to estimate the water inflow of a cross-section if it is close to one of the mentioned standard cross-sections, which is not reliable as the differences of estimated hydrogeological parameters among tunnels with different cross-sections are noticeable. For example, the maximum water inflow rate corresponds to a tunnel with the square cross-section, while the minimum water inflow is for a circular tunnel. The difference between the maximum value and the minimum value is up to 30% (Li et al., 2010), and the difference increases with the increase of the cross-section size. Thus, for a horseshoe shaped rock cavern with a large diameter (i.e. more than 20 m), the existing analytical solutions are not suitable for prediction of water inflow due to the significant differences.

Over the last few decades, the numerical methods have been acknowledged as the most reasonable approach for estimating water flow related problems as many factors of geological conditions could be taken into account. However, a numerical method usually requires abundant skilled professional knowledge on geological and hydrogeological information and it is often considerable time-consuming. To take partial advantages of analytical solutions and numerical methods into account, this paper proposed a semi-analytical approach for preliminary hydrogeological modeling, which could be used to calculate the total water inflow around the cavern with horseshoe section.

Based on the existing analytical solutions and numerical method, we attempt to establish a relationship between water inflow and the corresponding boundary conditions. Interestingly, by making a comparison of existing analytical solutions (i.e. Eqs. (1) - (3)) and other empirical formulas (Li et al., 2010), it could be found that when the water level is higher than the upper boundary and atmospheric pressure is effective at the tunnel perimeter, the relationship between water inflow and the hydraulic conductivity for any cross-section can be expressed in the following general form:

\[
\frac{Q}{k} = S + CH
\]

(4)

where \( S \) and \( C \) are the two coefficients only related to the tunnel’s shape and depth. For a specific cavern, the size of the cross-section and the location are fixed, so the values of \( S \) and \( C \) are also constants. The equation (4) is very convenient
to use, if the values of $S$ and $C$ are given. However, the parameters of $S$ and $C$ should be recalculated if the cross-section and depth of rock caverns are changed, even though the caverns locate in the same construction site.

In order to determine $S$ and $C$ for a specific case, the code FLAC is adopted to model the groundwater flow into the caverns. In FLAC code, one value of $Q/k$ can be calculated if a certain water pressure $H$ is given. A group of $Q/k$ values will be obtained with changing the parameter of $H$ which ranges from 0 m to 120 m water column at the upper boundary. The results of $S$ and $C$ can be determined by regression analysis. Then, the preliminary semi-analytical solution for water inflow prediction formula would be successfully built, which could be used to compute the total water inflow into cavern and to back calculate the hydraulic conductivity along the cavern.

3. Back-analysis approach for determination of hydraulic conductivity

In underground engineering practices, some hydrogeological data could be obtained from geological mapping and probe hole testing. During excavation, water inflow rate and pressure and other related data could be continuously monitored. The valuable hydrogeological information is essential to upgrade the preliminary model and to establish a more reliable hydrogeological model.

With the help of EXCEL spreadsheet’s build-in optimization routine SOLVER, the back-analysis which mainly consists of minimizing the error function is introduced to determine hydraulic conductivity around the cavern on the basis of established preliminary model and in-situ hydrogeological observations during excavation. The error function is a nonlinear function of the unknown coefficients of hydraulic conductivity and its gradient cannot be determined analytically. As a consequence the minimization algorithm must handle general nonlinear functions and should not require the analytical evaluation of the function gradient. In this paper, the automated spreadsheet search algorithm (Zhang and Goh, 2012) is used to perform the minimization process to determine the hydraulic conductivity.

The back-analysis is based on the minimizing of the error function $E$ that represents the discrepancy between the measured water inflow into the cavern in the field and the corresponding computed results obtained from the semi-analytical solution, which in turn depend on the unknown coefficients of hydraulic conductivity in $n$ different hydrogeological units:

$$E = \sum_{i=1}^{\text{t}} \left| Q_i - \sum_{j=1}^{n} Q_{i,j} L_j \right|$$  \hspace{1cm} (5)

where $Q_i$ is the measured water inflow into tunnel/cavern on the given date $i$, $t$ is the total number of measuring days, $Q_{i,j}$ is the computed water inflow on the given date $i$ at the hydrogeological unit $j$ determined from geological structures, and $L_j$ is the length of the unit $j$. The error defined by Eq. (5) is a nonlinear function of the unknown parameters $k_j$ and its gradient cannot be determined analytically.

The unknown coefficients of hydraulic conductivity are obtained by using the EXCEL spreadsheet’s build-in optimization routine SOLVER add-in. The SOLVER add-in works with a group of cells that are related, either directly or indirectly, to the formula in one cell and uses techniques from the operation research to find optimal solutions. With the aids of SOLVER, minimization of the error function in Eq. (5) could be achieved by changing the $k_j$ values, under the constraints that all the hydraulic conductivities vary in the range obtained from previous hydrogeological investigation, and the hydraulic conductivity around the water bearing zone is larger than that at other locations. Prior to invoking the SOLVER search algorithm, the $n$ unknown coefficients of hydraulic conductivity are randomly set in the preliminary
range acquired from preliminary geological survey. Iterative numerical derivatives and directional search for these unknown coefficients of hydraulic conductivity are automatically carried out in the spreadsheet environment. Detailed implementation on the EXCEL spreadsheet can be found in Zhang and Goh’s paper (Zhang and Goh, 2012).

To quantitatively estimate the accuracy of the back-analysis solution, for every computed water inflow \( Q_{ij} \), we define an incorrect ratio \( a \) as:

\[
a_i = \frac{Q_i - \sum_{j=1, n} Q_{ij} L_i}{Q_i} \times 100
\]

which could be calculated for the whole selected duration. The incorrect ratio \( a \) represents the effectivity of back-analysis and whether the analysis results are reliable and precise enough for engineering applications.

4. Case study: determination of hydraulic conductivity along an underground oil storage rock cavern

4.1 Site investigation

An underground oil storage cavern project located at an offshore island is used here as a case study. This project comprises of 5 underground unlined cavern units, 9 km of operation tunnels, access tunnels, maintenance chambers and water curtain galleries built on two levels below the sea-bottom. The operation and access tunnels at upper level contain the pipes used for importing and exporting hydrocarbon product and for the pumping out of the water seeping into the rock caverns, while the rock caverns are located at the lower level. The caverns are located in sedimentary bedrocks where it is characterized by siltstone, sandstone, conglomerate and limestone and some intrusive rock dykes and veins. The crowns of caverns are located at -119 mACD, where ACD is the abbreviation of “Admiralty Chart Datum”. The shape of the cavern is horseshoe, with the size of 20 m in width and 27 m in height (Fig. 2). The Drill-Blast method is adopted in the excavation of the storage caverns. The caverns are excavated in 3 stages, viz top heading and 2 benches, each with a height of 9m. An advantage of this excavated method is that the geological/hydrogeological conditions and structural stability of the caverns can be assessed at the top heading level before moving forward with the two lower benches. The rock bolts and shotcrete are the major elements of structure support work. The hydraulic conductivity depends on the different geological features and jointing networks and is therefore highly heterogeneous. According to the site investigation report, the hydraulic conductivity varies between \( 10^{-10} \) m/s and \( 10^{-6} \) m/s. When faced with water inflow during the excavation works, additional activities such as drilling of probe holes, measuring the leak quantities, drilling of grout holes, and grouting will be carried out.

In order to have an in-depth understanding in the hydro-geological behavior of the underground rock caverns and associated underground tunnels, the water flow into the caverns and tunnels, water pressure, groundwater table and rainfall are monitored and collected. In this study, only the hydraulic conductivities around Cavern A are studied. The hydraulic heads at 8 control points (\( H1-H8 \)) in Tunnel A (Fig. 3) were monitored and the water flow into Cavern A was collected, as shown in Fig. 4. Data monitored during the 65 days after the completion of Cavern A and Tunnel A are used for model calibration and validation.
4.2 Preliminary semi-analytical solution for water inflow

Preliminary semi-analytical solution could be used to compute the total water inflow into Cavern A. It is essential to back calculate the hydraulic conductivity. As discussed, for a specific cavern, the values of $S$ and $C$ are constants while the relationship between water inflow and the hydraulic conductivity for any shape of cross-section is expressed by the general linear form, i.e. Eq. (4). In order to calculate the constants $S$ and $C$ for this specific rock Cavern A, the code FLAC 2D is employed to model the groundwater flow into the caverns. In this case, the numerical modeling obeys following assumptions:

1. Groundwater is assumed to obey Darcy’s law and is incompressible.

2. The dimensions of the model domain are chosen large enough to ensure that the boundaries will have little effect on the calculated results. The modeling area is 2000 m wide and 1000 m deep, and the mesh size is $2.5 \times 2.5 \text{ m}$. The representative cross section for the model is shown in Fig. 5.

3. Atmospheric pressure is effective inside the cavern and at its perimeter, thus the hydraulic head is not uniform but higher at the cavern crown than at the invert.

4. Groundwater flow is assumed to be steady.

5. Upper boundary is located at $-93 \text{ mACD}$, coinciding with the location of the water pressure monitoring holes in tunnel A. And the lateral and bottom boundaries are no-flow boundaries.

6. For the upper boundary, water pressure collected from the probe holes in water curtain gallery tunnel A varies from 0 m to 120 m water column, i.e the parameter of $H$.

A set of values of $\frac{Q}{k}$ are obtained with changing the parameter of $H$ which ranges from 0 m to 120 m water column. The results with regression analysis by EXCEL are shown in Fig. 6. The results show that the relationship between water inflow and the hydraulic conductivity is linear for this specific case, which means that the assumption of Eq. (4) is at least suitable for this project, and the parameters $S$ and $C$ are determined as 118.85 and 3.21, respectively. Thus, for this specific project, the semi-analytical solution for water inflow prediction is

$$\frac{Q}{k} = 118.85 + 3.21H$$

which will be conveniently used to predict the water inflow into cavern and to back analyze the hydraulic conductivity.

4.3 Back-analysis for hydraulic conductivity

In this project, there are two water bearing zones cross the Cavern A, and the hydraulic conductivity varies over many orders of magnitude along the length of the Cavern A. Eight control points were installed to monitor water pressure along Tunnel A, so the rock mass around the Cavern A is assumed to have 8 hydro-geological units, and each unit has a constant hydraulic conductivity. Data monitored during the first 50 days were used for model calibration, and the next 15 measured data after the first 50 days were used to test the validity of the model. Furthermore, according to previous geological investigation, all the hydraulic conductivities are larger than $10^{-10} \text{ m/s}$ and less than $10^{-6} \text{ m/s}$ and the hydraulic conductivity around the water bearing zone is larger than that at other locations.

The error function $E$ depends on the 8 unknown coefficients of hydraulic conductivity $k_j$. 

---

**Fig. 5**

**Fig. 6**
\[ E = \sum_{i=1,8} \left| Q_i - \sum_{j=1,8} Q_{ij} L_j \right| / Q_i \]  

(8)

The defined incorrect ratio \( a \) is

\[ a_i = \left| \frac{Q_i - \sum_{j=1,8} Q_{ij} L_j}{Q_i} \right| \times 100 \]  

(9)

where \( Q_i \) is the measured water inflow into Cavern A, \( Q_{ij} \) is the computed water inflow at the different units, and \( L_j \) is the length of each unit as shown in Fig. 3.

The 8 unknown coefficients of hydraulic conductivity are randomly set between \( 10^{-10} \) m/s and \( 10^{-6} \) m/s. Fig. 7 shows the water inflow time-history into the Cavern A. The results show that the back-analysis model reproduces the trend of the measured flow rates. The incorrect ratio \( a \) have been calculated for all days, as shown in Fig. 8. The results verify that the maximum incorrect ratio \( a \) is about 35%, and 92% of the incorrect ratios are less than 15% which is accurate enough for engineering applications.

Fig. 7

Fig. 8

The hydraulic conductivity distribution along the length of the Cavern A is shown in Fig.9. The result illustrates that the hydraulic conductivities are mainly in the order of \( 10^{-10} \) m/s except at two locations where the water bearing zones are intersected with the Cavern A. For this project, the grouting is used to reduce the hydraulic conductivity in the water bearing zone and some check holes are drilled to measure the water flow and the water pressure after grouting. If the water flow and the water pressure are less than the target values, the grouting will stop and the excavation will continue. The target hydraulic conductivity after grouting is in the order of \( 10^{-7} \) m/s. The result shows that the computed hydraulic conductivities at the two water bearing zones are in the order of \( 10^{-7} \) m/s, which means that the computed results are close to the real condition and can be acceptable to represent the hydro-geological condition around the Cavern A.

Fig. 9

5. Conclusions

There are two main problems with groundwater inflow estimates for the underground rock cavern. The first one is the lack of simple, realistic models that can be readily applied to underground caverns. The second difficulty is that the practical range of hydraulic conductivity in fractured rock may typically ranges over a few orders of magnitude, and this range typically repeats again and again over the lengths of long caverns. In this study, a new approach is proposed to back analyze the hydraulic conductivity along the length of the cavern. To take advantages of analytical solutions and numerical methods into account, this paper proposed a semi-analytical approach for assessing the water inflow into caverns of the horseshoe section. After that, the semi-analytical solution is used for the back-analysis of the hydraulic conductivity around cavern on the basis of the in-situ monitoring data. The hydraulic conductivities are obtained by using the EXCEL spreadsheet’s build-in optimization routine SOLVER to minimize the error function. The results show that the computed hydraulic conductivities are acceptable.

The above hydro-geological model is based on the equivalent properties of the fractured rock mass and is suitable to evaluate the total seepage influence. However, the problems on hydraulic properties of rock mass are always very
complicated. The influences of different rocks with different hydraulic properties, such as thickness and hydraulic conductivity for every rock, should be studied if more detailed data are available. As we known, the actual water flow is mainly along the rock joints or fracture zones and the water flow & pressure in the joints are the key information for water control, as well as the assessment of cavern stability. The influence caused by repeated blasting at nearby cavern/tunnel also should been discussed. Thus advanced numerical model suitable for seepage through joint models should be developed and the relationship between the continuum model and discrete model on the seepage analysis will be studied in the future.

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References

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**Fig. 4** Measured water heads at eight vertical manometer holes

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**Fig. 6** Determination of the two parameters S and C using FLAC software

**Fig. 7** Measured and computed water inflow rate into Cavern A

**Fig. 8** Quantitative differences of the measured and computed water inflow into the Cavern A

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