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Error Probability Analysis of a Novel Adaptive Beamforming Receiver for Large-Scale MIMO Communication System

Tuong Xuan Tran and Kah Chan Teh

Abstract—A novel adaptive beamforming receiver is proposed for a multiple-input multiple-output (MIMO) communication system over multipath fading channels. By dividing a large number of receive antennas into many adaptive beamformers (ABFs), the proposed system reduces the array dimension in order to simplify the signal detection process and enjoys the benefit of beamforming techniques to improve the symbol-error rate (SER) performance. The effectiveness of the proposed system depends on the number of ABFs and the number of antennas per ABF. The statistical properties of complex Wishart matrices are applied to analyze the SER performance of the proposed system with both maximum-likelihood (ML) detection and zero-forcing (ZF) detection. Furthermore, we have derived an upper bound for SER of the ML detection based on the smallest eigenvalue distribution. Through our analysis, it has been shown that the proposed system using ML detection provides comparable SER performance, but with a lower complexity as compared to that of a conventional ML receiver due to reduced array dimensions.

Index Terms—Large-scale MIMO system, adaptive beamforming, spatial multiplexing, eigenvalue distribution.

I. INTRODUCTION

MULTI-antenna techniques provide significant performance improvement for wireless communication systems in terms of reliability as well as capacity [1], [2]. A multiple-input multiple-output (MIMO) system can use a large number of antenna arrays to reduce downlink and uplink transmit powers through coherent combining [3]. In [4], it has been shown that a base station (BS) equipped with 100-element antenna array needs to transmit only 1% of the power to achieve the same quality-of-service as compared to that of a single antenna. However, as the number of antennas at the BS grows, the computational complexity of signal processing modules for a large-scale MIMO system increases dramatically. In particular, when the maximum-likelihood (ML) detection is used to estimate the transmitted signal, the complexity grows exponentially with the number of transmit antennas and the size of constellation [5]. In order to reduce complexity, antenna selection methods [1] or near optimal (or a simplified) ML detection have been applied [5]. However, antenna selection techniques only use a subset of antennas to maximize the throughput of the system, thus they can not achieve optimal performance.

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The MIMO system performance can be improved through understanding the properties of fading channels. The channel state information (CSI) is estimated at the receiver by using a training sequence which is inserted at the beginning of each data stream, and it is fed back to the transmitter for the case of frequency division duplexing (FDD) mode. On the other hand, it is estimated at both links based on channel reciprocity for the case of time division duplexing (TDD) mode [6]. The CSI at transmitter (CSIT) can be applied for some transmission techniques such as power gains via water-filling, eigenmode transmission and rate adaptation [7]. However, the mobile channel is time variant and the feedback process would lead to the CSIT inaccuracy due to channel delay and quantization error [8]. It has been shown in [9] that the number of training symbols should be at least equal to the number of transmit antennas in order to have accurate channel estimation. Clearly, the estimation error is small if the power levels of the training symbols are sufficiently high, and the number of training symbols is large. The optimal number of training symbols can be defined by maximizing the achievable capacity which has been shown in [9] or by minimizing the channel estimation error which has been shown in [10]. In this paper, we consider a scenario for a MIMO communication system when the CSI at receiver (CSIR) is assumed to be available. Under the CSIR only assumption, the transmitter should allocate equal power to each transmit antenna and can not adapt to the fading channels.

From practical view points, large-scale multiple-antenna systems involve high computational complexities. Thus, the idea is to find a novel MIMO receiver with a reduced complexity, but it still uses all available antennas and it can provide comparable symbol-error-rate (SER) performance as compared to that of the conventional MIMO receivers. In particular, the proposed system is assumed to have tens of antennas. It combines adaptive beamforming (BF) and spatial multiplexing to improve the SER performance. Instead of considering a single adaptive BF receiver, the existing array antennas are divided into many blocks which are considered as many separate adaptive beamformers (ABFs). The main purpose is to decrease the array dimensions in order to simplify the signal detection process and enjoy the benefit of beamforming techniques. Based on the maximum signal-to-noise ratio (SNR) criterion [11], each of these ABFs focuses only on their strongest path of the transmitted signals in order to maximize the instantaneous received SNR at each ABF. It means that the SNR at each ABF is maximized when the strongest path of the transmitted signals is detected. Following

that, the effective channel matrix $\mathbf{H_e}$ is obtained. It is then applied to estimate the transmitted symbols from the outgoing signals of ABF by using the zero-forcing (ZF) detection or maximum-likelihood (ML) detection. Finally, the vector of the estimated symbols is combined for further demodulation by a multiplexer. Generally, the proposed system can achieve good performance because it provides an effective method to detect the desired signal for MIMO transmission by utilizing a set of strongest paths at the receiver.

In order to analyze the SER performance of the proposed system, a spatial channel model (SCM) adopted by the 3GPP is applied to generate spatial correlation fading channel [12]. The SCM is widely used by the 3GPP community to perform system-level performance evaluation of MIMO techniques. The main contributions of this paper are summarized as follows:

- A novel adaptive BF receiver for a MIMO spatial multiplexing system is proposed.
- An upper bound on SER for ML detection is derived based on the cumulative distribution function (cdf) of the smallest non-zero ordered eigenvalue of Wishart matrix.
- Analytical SER expressions of the proposed system with both ZF detection and ML detection are presented and validated by simulation. The effects of different system configurations on SER performance have been investigated.

Notations: The notations used in this paper follow usual conventions, vectors and matrices are denoted by symbols in boldface; $(.)^*$, $(.)^T$, $(.)^H$, and Tr(.) denote the complex conjugate, transpose, conjugate transpose and trace of (.), respectively. $\|.\|$ gives the Frobenius norm of the matrix argument.

II. CHANNEL MODEL AND SYSTEM DESCRIPTION

A. Channel Model

The performance of a MIMO system depends on the propagation environment and the properties of the antenna elements being used. To analyze the performance of the proposed MIMO system, the SCM is used to generate fading coefficients. We assume that the received signal at the BS consists of N time-delayed multipath replicas of the transmitted signal and each path consists of Q sub-multipaths [12]. Each path is characterized by its own spatial channel parameters (angle spread (σ_{AS}), angle of departure (AoD) or arrival (AoA) for

Fig. 1. The BS and MS angle parameters

the qth subpath of the nth path, power of the nth path (P_n), and lognormal shadow fading (σ_{SF})), as shown in Fig. 1.

For an S-element mobile station (MS) array and an U-element BS array, the channel coefficient $h_{u,s}$ of $\mathbf{H}_w$ is defined in equation (1). In (1), k is the wave number $2\pi/\lambda$ with λ denotes the carrier wavelength in meters, $G_{BS}(\theta_{n,q,AoD})$ and $G_{MS}(\theta_{n,q,AoA})$ are the antenna gains of each array element, d_s is the distance in meter from the sth antenna element of BS with respect to the reference ($s = 1$) antenna and d_u is the distance in meter from the uth antenna element of MS with respect to the reference ($u = 1$) antenna, $\Phi_{n,q}$ denotes the phase of the qth subpath of the nth path, θ_v is the angle of the MS velocity vector, and $\|v\|$ is the magnitude of the MS velocity vector. We consider a flat-fading communication system, the received signal is given by:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z}, \qquad (2)$$

where $\mathbf{x} \in \mathbb{C}^{S \times 1}$ is the transmitted signal vector, $\mathbf{y} \in \mathbb{C}^{U \times 1}$ is the received signal vector, $\mathbf{z} \in \mathbb{C}^{U \times 1}$ is the noise vector. $\mathbf{H} \in \mathbb{C}^{U \times S}$ is the channel matrix of a spatially correlated Rayleigh-fading channel which is expressed as $\mathbf{H} = \mathbf{R}_{BS}^{1/2}\mathbf{H}_w\mathbf{R}_{MS}^{1/2}$, where $\mathbf{R}_{BS}$ and $\mathbf{R}_{MS}$ are the BS and MS correlation matrices, respectively. Note that the effect of electromagnetic interaction between the receive antenna elements, which is normally known as mutual coupling, is not considered in this paper. Assuming that $G_{BS}(\theta_{n,q,AoD}) = G_{MS}(\theta_{n,q,AoA}) = 1$, the entries of $\mathbf{R}_{BS}$ and $\mathbf{R}_{MS}$ are given by [13]

$$\rho_{ij}^{BS}(\Delta d_s) = \frac{1}{Q}\sum_{q=1}^{Q} E\{\exp\left(jk\Delta d_s sin(\theta_{n,q,AoD})\right)\}, \quad (3a)$$

and

$$\rho_{ij}^{MS}(\Delta d_u) = \frac{1}{Q}\sum_{q=1}^{Q} E\{\exp\left(jk\Delta d_u sin(\theta_{n,q,AoA})\right)\}, \quad (3b)$$

respectively, where $\Delta d_s = |d_{s_i} - d_{s_j}|$ and $\Delta d_u = |d_{u_i} - d_{u_j}|$ denote the relative BS and MS antenna element spacings, respectively, and $\sigma_{h_{u_i,s_i,n}} = \sqrt{P_n}$ is the standard deviation of $h_{u_i,s_i,n}(t)$. We assume that the antennas at the BS can be placed sufficiently far apart which leads to zero correlation at the BS side. Thus, the effect of the spatial correlation in semi-correlated fading can be reflected as $\mathbf{H} = \mathbf{H}_w\mathbf{R}_{MS}^{1/2}$.

B. System Description

Figure 2 illustrates the structure of a novel adaptive MIMO receiver with U antenna elements. All the U antenna elements are divided into B blocks and each block consists of K antenna elements. The main purpose is to reduce the array dimension in order to simplify the signal detection process. Thus, each block is considered as an ABF. Performing as a spatial filter, each ABF can increase the output SNR by blocking most of the noise outside the maximum power direction. The separate optimal weight vector of each ABF is determined based on the SNR maximization by an adaptive processor.

In order to know the channel properties of a communication link, a training sequence is used to estimate the CSIR. In
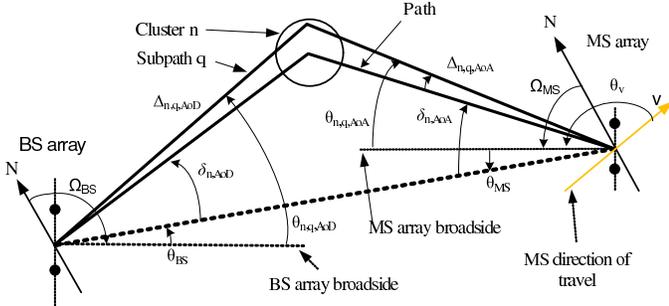
$$h_{u,s}(t) = \sqrt{\frac{P_n \sigma_{SF}}{Q}} \sum_{q=1}^{Q} \left(\sqrt{G_{BS}(\theta_{n,q,AoD)}} \exp\left(j[kd_s sin(\theta_{n,q,AoD}) + \Phi_{n,q}]\right) \times \right.$$

$$\left. \sqrt{G_{MS}(\theta_{n,q,AoA})} \exp\left(jkd_u sin(\theta_{n,q,AoA})\right) \exp\left(j\|v\|cos(\theta_{n,m,AoA} - \theta_v))\right) \right) \tag{1}$$

particular, the received training sequence is first used for determining the optimal weight vector of the ith ABF defined as $\mathbf{w}_{opt\_i} = [w_{i1}, \ldots, w_{iK}]^T = \frac{\mathbf{v}_1}{\|\mathbf{v}_1\|}$, where $\mathbf{v}_1$ is the eigenvector corresponding to the largest eigenvalue of $\mathbf{R}_i$. This operation is performed by using eigenvalue decomposition (EVD). The correlation matrix of $\mathbf{R}_i$ for the ith ABF is obtained as

$$\mathbf{R}_i = E\{\mathbf{y}_i(j)\mathbf{y}_i^H(j)\} = \frac{1}{T_t} \sum_{j=1}^{T_t} \mathbf{y}_i(j)\mathbf{y}_i^H(j), \tag{4}$$

where i=1,..., B. Note that $\mathbf{y}_i$ is the received training signals at each ABF, expressed as $\mathbf{y}_i = \mathbf{H}_i \mathbf{x} + \mathbf{n}_i$, where $\mathbf{H}_i \in \mathbb{C}^{K \times S}$ is a submatrix of the channel matrix $\mathbf{H}$ and T_t denotes the length of $\mathbf{y}_i$. After adaptation, a one-dimensional array can be determined in the output ABFs denoted as $\mathbf{y}_w = [y_{w1}, \ldots, y_{wB}]^T$, where $y_{wi} = \mathbf{w}_{opt\_i}^H \mathbf{y}_i$. Hence, the MIMO system channel as shown in (2) can be rewritten as

$$\mathbf{y}_w = \mathbf{H}_e \mathbf{x} + \mathbf{z}_e, \tag{5}$$

where the effective channel matrix, $\mathbf{H}_e \in \mathbb{C}^{B \times S}$, is defined as $\mathbf{H}_e = \mathbf{WH}$ and $\mathbf{z}_e = \mathbf{Wz}$. The matrix $\mathbf{W}$ is the optimal weight matrix with dimension $B \times U$ and $\mathbf{WW}^H = \mathbf{I}_B$. The entries of $\mathbf{W}$ are defined by

$$W_{i,j} = \begin{cases} w_j, \text{the } j\text{th optimal weight value} \\ \text{belongs to the } \mathbf{w}_{opt\_i} \text{ of } i\text{th ABF,} \\ 0, \text{otherwise.} \end{cases} \tag{6}$$

Finally, the effective channel matrix $\mathbf{H_e}$ can be determined by the least-square (LS) estimator which is obtained as $\mathbf{H}_e = (\mathbf{X}^H \mathbf{X})^{-1} \mathbf{X}^H \mathbf{Y}_w$, where $\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_{T_t}]$ and $\mathbf{Y} = [\mathbf{y}_{w_1}, \ldots, \mathbf{y}_{w_{T_t}}]$. This is summarized in Table I. It has been shown that channel estimation can affect the system performance significantly. In other words, a good estimate of the transmitted signal corresponds to a good channel estimation. The LS estimator can not exploit the statistics of the fading channel [14]. However, based on training sequences, LS estimator can achieve optimal performance for a Rayleigh fading channel [10]. Furthermore, channel estimation error depends on the length and the power of the training symbols. It has been shown in [9] that the optimal number of training symbols is equal to the number of transmit antennas if the power levels of the training symbols are different from the power levels of the data symbols. If those power levels are equal, the optimal number of training symbol can be larger than the number of transmit antennas.

We assume that the channel conditions vary slowly under the transmission of a sequence of information symbols, then the estimated $\mathbf{H}_e$ is reasonably accurate and can be used
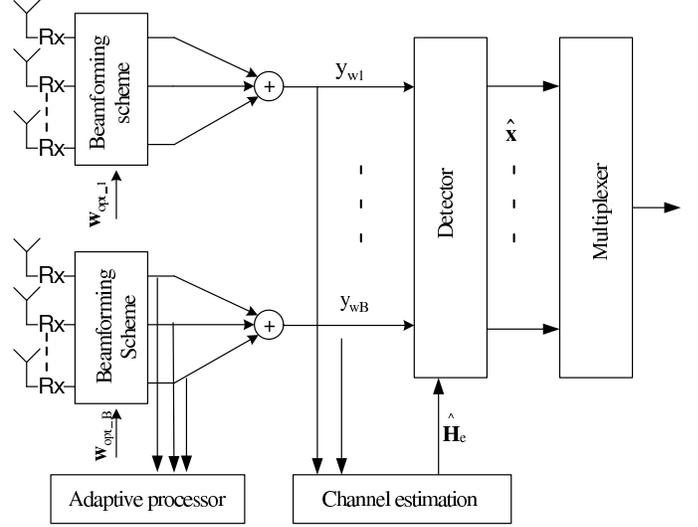

Fig. 2. Block diagram of a novel adaptive MIMO-BF receiver.

TABLE I
DETERMINE EFFECTIVE CHANNEL MATRIX $\mathbf{H_e}$

1. Input: The received training signals $\mathbf{y}_i$
2. Calculate the correlation matrix $\mathbf{R}_i$ for each ABF
3. By using eigenvalue decomposition, calculate the optimal weight vector $\mathbf{w}_{opt_i}$ for each ABF
4. Determine $\mathbf{y}_w = [y_{w1}, \ldots, y_{wB}]^T$, where $y_{wi} = \mathbf{w}_{opt\_i}^H \mathbf{y}_i$
5. By using the least-square estimator, calculate $\mathbf{H}_e = (\mathbf{X}^H \mathbf{X})^{-1} \mathbf{X}^H \mathbf{Y}_w$, where $\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_{T_t}]$ and $\mathbf{Y} = [\mathbf{y}_{w_1}, \ldots, \mathbf{y}_{w_{T_t}}]$
6. Output: The channel effective matrix $\mathbf{H_e}$

for adaptive reception before the next update is required. After performing the channel estimation based on the training symbols, the estimated $\mathbf{H_e}$ is used to estimate transmitted information symbols from the outgoing signals of ABF by using the ZF detection or ML detection. By using the ZF detection, the estimated symbols are given by

$$\hat{\mathbf{x}}_{ZF} = (\mathbf{H_e^H H_e})^{-1} \mathbf{H_e^H} \mathbf{y}_w. \tag{7}$$

By using the ML detection, the estimated symbols are given by

$$\hat{\mathbf{x}}_{ML} = \underset{\mathbf{x} \in \mathbf{X}}{\operatorname{argmin}} \|\mathbf{y}_w - \mathbf{H_e} \mathbf{x}\|, \tag{8}$$

where $\mathbf{X}$ is the set of all possible transmitted vector symbols. The vector of the estimated symbols is a multiple parallel data stream to be combined for further demodulation by a multiplexer.

Moreover, the diversity order of the proposed system with ZF detection is $B-S+1$ when $B > S$ [15]. In the case of ML detection, the proposed system can obtain a diversity gain on

the order of B [16]. Due to the fact that the system dimension is reduced, the diversity gain of the proposed system is less than that of the conventional ones ($U - S + 1$ for ZF receiver and U for ML receiver). At high SNR (γ) region, the average error probability of the proposed system with ZF detection and ML detection can be approximated as $P_e(\gamma) \approx (\gamma)^{-(B-S+1)}$ and $P_e(\gamma) \approx (\gamma)^{-B}$, respectively [16]. With a fixed value of $B \times K$, B is considered as the largest number of ABFs that can be obtained for a given $U \times S$ MIMO system when $K = 2$. Therefore, the proposed system with $K = 2$ can provide the best performance in term of error probability. It is similar to the case of a conventional MIMO system, if the number of receive antennas is increased, the performance is better.

1) Computational Complexity: The computational complexity of the proposed system is quantified by the number of floating-point operations (flop). Let the complexity order $O(.)$ represent the arithmetic order of flops. The optimal weight vector for each ABF is obtained from the correlation matrix in (4) by using EVD computation. Since the dimension of $\mathbf{R}_i$ is $K \times K$, the complexity of EVD computation is on the oder of $O(K^3)$ [17]. As the proposed system has B ABFs, the total complexity for determining the optimal weight vector is $O(BK^3)$.

Next, we consider the computational complexity of the detectors. If the proposed system uses ZF detection to estimate the transmitted symbols, the computational complexity of ZF detection is on the oder of $O(S^3)$. It is because the dimension of $\mathbf{H}_e$ is $B \times S$ (assuming $B \geq S$). Although ZF detection has low implementation complexity, it leads to noise amplification and will affect the detection performance significantly. On the other hand, the ML detection is an optimal one from the error probability point of view. The ML detection is performed through an exhaustive search over all possible symbol vectors. Thus, the total number of multiplications of ML detection is $4BSM + 2BM^S \sim O(BM^S)$ [5]. Note that M denotes the size of signal constellation, where the first and second term indicate the computations of $\mathbf{H_e x}$. Obviously, the computational complexity of the ML detection is excessively high, especially for a large number of transmit antennas or a higher-order modulation level. In summary, the computational complexities of the proposed systems using ZF detection and ML detection are $O(BK^3 + S^3)$ and $O(BK^3 + BM^S)$, respectively.

TABLE II
COMPLEXITY COMPARISONS OF THE PROPOSED SYSTEMS AND OTHER CONVENTIONAL RECEIVERS

Proposed system with ML detection	$O(BK^3 + BM^S)$
Proposed system with ZF detection	$O(BK^3 + S^3)$
Conventional ML	$O(UM^S)$
Conventional MMSE	$O(S^3)$

Table II shows the computational complexity comparisons for the proposed systems and two conventional receivers. We assume that the percentage of complexity reduction as compared to that of the conventional ML receiver, which has U receive antennas (without dividing antennas into subgroups), is obtained by $\xi = \dfrac{(UM^S - (BK^3 + BM^S))}{UM^S}$. Hence, for a

16×4 system with 4PSK modulated signal, the corresponding complexity of the proposed system with ML detection is reduced by 48.43% (when $B = 8, K = 2$), 68.75% (when $B = 4, K = 4$), and 62.5% (when $B = 2, K = 8$), respectively. It is because the dimension of the array is reduced from U to B, hence the number of computations is decreased. However, the proposed system is more complicated than the MMSE MIMO receiver due to the EVD computations.

2) Imperfect CSI: In a practical environment, the receiver is sensitive to the dynamic change of signal caused by multipath propagation effects. Thus, channel estimation based on a training sequence could be obtained with errors which give rise to imperfect channel information due to imperfect knowledge of the signal AoAs or uncertainty in $h_{u,s}(\theta_{n,q,AoA})$ (the array responses). The estimated channel state can be modeled as

$$\widehat{\mathbf{H}}_e = \mathbf{H}_e + \Delta, \tag{9}$$

where Δ is the estimation error matrix whose entries are assumed to be i.i.d. zero-mean complex Gaussian random variables with variance σ_e^2. The value of σ_e^2 reflects the quality of the channel estimation. It depends on the length T_t and the power P_t of the training sequence and it can be defined as $\sigma_e^2 = \dfrac{S\sigma_z^2}{T_t P_t}$ [13]. Clearly, the variance σ_e^2 is small if P_t is sufficiently high and T_t is large. Furthermore, in (9), $\mathbf{H}_e$ and Δ are considered to be statistically independent with each other.

III. SER PERFORMANCE ANALYSIS

In this section, we analyze the SER performance of our proposed system with both ZF detection and ML detection over Rayleigh fading channels. It is well known that MIMO performance is often dominated by the statistical eigenvalue properties of the instantaneous correlation matrix $\mathbf{G} = \mathbf{H}_e \mathbf{H}_e^H$ (or $\mathbf{H}_e^H \mathbf{H}_e$). Note that we have assumed that the correlation occurs only at the transmitter side. In this case, $\mathbf{G}$ is a complex semi-correlated central Wishart matrix which is expressed as $\mathbf{G} \sim CW(r, \boldsymbol{\Sigma})$, where $r = \min(B, S)$ and $b = \max(B, S)$ [18]. Therefore, the joint probability density function (pdf) of the ordered eigenvalues is given by [19]

$$f(\boldsymbol{\lambda}) = A|\mathbf{V}_1(\boldsymbol{\lambda})||\mathbf{E}(\boldsymbol{\lambda}, \boldsymbol{\sigma})| \prod_{l=1}^{r} \lambda_l^{b-r} \tag{10}$$

$$A = \prod_{i=1}^{r} \frac{1}{\sigma_i^b (b-1)!} \prod_{i<j}^{r} \frac{\sigma_i \sigma_j}{\sigma_j - \sigma_i},$$

where the vector $\boldsymbol{\lambda}$ denotes the eigenvalues of matrix $\mathbf{G}$ ($\lambda_1 \geq \cdots \geq \lambda_r > 0$), $\mathbf{V}_1(\boldsymbol{\lambda})$ is a Vandermode matrix with the (i, j)th entry denoted as λ_j^{i-1} and $\mathbf{E}(\boldsymbol{\lambda}, \boldsymbol{\sigma})$ has its (i, j)th entry denoted as $\exp(-\lambda_j/\sigma_i)$, with σ_i being the non-zero ordered eigenvalue of $\boldsymbol{\Sigma}$.

A. ZF Detection

When the proposed system uses ZF detection to estimate the transmitted symbols, based on the results from (7), the

instantaneous SNR for the lth data stream can be expressed as [13], [20], [21, Eq.(8.36)]

$$\gamma_l \simeq \frac{P/S}{(\sigma_{z_e}^2 + \sigma_e^2 P)[(\mathbf{H_e^H H_e})^{-1}]_{l,l}} = \gamma_0 \lambda_l, \qquad (11)$$

where $\gamma_0 = \dfrac{P/S}{(\sigma_{z_e}^2 + \sigma_e^2 P)}$, P is the transmit power and λ_l is the lth largest ordered channel eigenvalue of $\mathbf{G}$. In (11), the instantaneous SNR approximation is accurate if $\sigma_e^2 = 0$, and the approximation is considered very tight if $\sigma_e^2 \ll 1$. The analytical SER performance can be obtained by averaging the SER of the proposed system transmitting through the r strongest channel eigenvalues, which is given by [15, Eq.(50)]

$$P_s(\gamma) = \frac{1}{r} \sum_{l=1}^{r} \Big(\int_0^\infty \alpha_l Q(\sqrt{\beta_l \gamma_l}) f_{\lambda_l}(\lambda_l) d\lambda_l \Big), \qquad (12)$$

where α_l and β_l are the constellation parameters, $Q(.)$ is the Gaussian Q-function, and $f_{\lambda_l}(\lambda_l)$ is the pdf of the lth largest eigenvalue of $\mathbf{G}$. After some mathematical manipulations, the SER expression of the proposed system using ZF detection can be obtained as

$$
\begin{aligned}
P_{s,ZF} = \frac{1}{r} \sum_{l=1}^{r} \Big(& \frac{\alpha_l}{2} \sqrt{\frac{P\beta_l}{2\pi S(\sigma_{z_e}^2 + \sigma_e^2 P)}} \\
& \times \int_0^\infty \frac{e^{-\frac{P\beta_l \lambda_l}{2S(\sigma_{z_e}^2 + \sigma_e^2 P)}}}{\sqrt{\lambda_l}} F_{\lambda_l}(\lambda_l) d\lambda_l \Big),
\end{aligned}
\qquad (13)
$$

where F_{λ_l} is the cdf of the lth largest ordered eigenvalue λ_l. From (10) and using Theorem 3.2 of [19], the cdf of the lth largest eigenvalue is obtained as

$$F_{\lambda_l}(\eta) = A \sum_{i=1}^{r} \sum_{\mu \in D(i)} |\mathbf{F}(\mu, i; \eta)|, \qquad (14)$$

where $D(i)$ is the set of all permutations $\mu = (\mu_i, \ldots, \mu_r)$ of integers $(1, \ldots, r)$. The matrix $\mathbf{F}(\mu, i; \eta)$ is defined as

$$
[\mathbf{F}(\mu, i; \eta)]_{\varphi, v} =
\begin{cases}
\sigma_\varphi^{d(v+1)} \Gamma\left(d(v+1), \dfrac{\eta}{\sigma_\varphi}\right), \\
\quad \text{if } 1 \le \mu_v \le i, \\
\sigma_\varphi^{d(v+1)} \gamma\left(d(v+1), \dfrac{\eta}{\sigma_\varphi}\right), \\
\quad \text{if } 1 \le \mu_v \le r,
\end{cases}
\qquad (15)
$$

where $d(v) = b - r + v - 1, 1 \le \varphi, v \le r$. Note that, $\gamma(.)$ and $\Gamma(.)$ are the lower and upper incomplete gamma function, respectively, which are given by [22, Eq.(8.354)]

$$\gamma(x, a) = \sum_{i=0}^{\infty} \frac{(-1)^i}{(x+i)i!} a^{x+i}, \qquad (16)$$

and

$$\Gamma(x, a) = (a-1)! - \sum_{i=0}^{\infty} \frac{(-1)^i}{(x+i)i!} a^{x+i}. \qquad (17)$$

B. ML Detection

By dividing multiple receive antennas into many antenna groups, the complexity of the proposed system with ML detection is reduced significantly. To derive the corresponding analytical SER expression, we first consider conditional pairwise error probability (PEP), i.e., the ML detection in (8) decides erroneously in favor of x_j when x_i is transmitted. Thus, the PEP between x_j and x_i can be written as [23, Eq.(4.2-71)]

$$Pr(x_j \to x_i | \mathbf{H_e}) = Q\left(\sqrt{\frac{d_{i,j}^2}{2N_0}}\right), \qquad (18)$$

where $d_{i,j}^2 = \|\mathbf{H_e}(x_i - x_j)\|^2$ denotes the squared Euclidean distance between x_j and x_i. The well-known union bound of (18) is given by [23, Eq.(4.2-78)]

$$Pr(error | \mathbf{H_e}) \le (M-1) Q\left(\sqrt{\frac{\gamma_0 d_{min,rx}^2(\mathbf{H_e})}{2}}\right), \qquad (19)$$

where $d_{min,rx}^2(\mathbf{H_e})$ denotes the minimum squared Euclidean distance at the receiver which is defined by minimizing the difference over all candidate symbols. It is given by

$$d_{min,rx}^2(\mathbf{H_e}) := \min_{x,c \in \mathbf{X}, x \neq c} \|\mathbf{H_e}(\mathbf{x} - \mathbf{c})\|^2. \qquad (20)$$

Let $d_{min,tx}^2$ be the minimum squared Euclidean distance of the transmit constellation, which is only dependent on the size of constellation and the energy per bit. For example, $d_{min,tx}$ for the MPSK modulated signal can be defined as [23, Eq.(3.2-33)]

$$d_{min,tx} = 2\sqrt{\log_2 M \sin^2(\pi/M) E_b}. \qquad (21)$$

Note that, $d_{min,rx}^2$ is not a function of $d_{min,tx}^2$. This is because symbols are transmitted over a fading channel which does not preserve their distance properties.

By applying the Rayleigh-Ritz theorem [24], lower bounds on the minimum Euclidean distanse is given by [25, Eq.(4)]

$$\lambda_{min} \frac{d_{min,tx}^2}{S} \le d_{min,rx}^2(\mathbf{H_e}), \qquad (22)$$

where λ_{min} is the smallest eigenvalue of $\mathbf{G}$. The equality in (22) is achieved when there exists a non-zero error vector $\mathbf{x} - \mathbf{c}$ which is a scalar multiple of the right singular vector corresponding to λ_{min}.

By subtituting (22) into (19), and averaging (19) with respect to the statistics of λ_{min}, the analytical upper bound on SER for the proposed system using ML detection can be derived as

$$P_{s,ML} \le (M-1) \int_0^\infty Q\left(\sqrt{\frac{\gamma_0 d_{min,tx}^2 \lambda}{2S}}\right) f_{\lambda_{min}}(\lambda) d\lambda, \qquad (23)$$

where $f_{\lambda_{min}}(\lambda)$ is the pdf of the smallest non-zero ordered eigenvalue of $\mathbf{G}$. Now, applying integration by parts to (23), the analytical SER bound is defined as

$$P_{s,ML} \leq \frac{(M-1)}{4} \sqrt{\frac{Pd_{min,tx}^2}{\pi S^2(\sigma_{z_e}^2 + \sigma_e^2 P)}}$$

$$\times \int_0^\infty \frac{e^{-\frac{Pd_{min,tx}^2 \lambda}{4S^2(\sigma_{z_e}^2 + \sigma_e^2 P)}}}{\sqrt{\lambda}} F_{\lambda_{min}}(\lambda) d\lambda, \qquad (24)$$

where $F_{\lambda_{min}}(\lambda)$ is the cdf of the smallest eigenvalue. From (10) and using Corollary 2 of [26], the cdf of the smallest eigenvalue can be obtained as

$$F_{\lambda_{min}}(y) = 1 - \int_y^\infty \int_{x_{min}}^\infty \ldots \int_{x_2}^\infty f_{\boldsymbol{\lambda}}(x) dx_1 \ldots dx_{min-1} dx_{min}$$

$$= 1 - A|\mathbf{F}(y)|, \qquad (25)$$

where the (i,j)th entry of the matrix $\mathbf{F}(y)$ can be defined as

$$[\mathbf{F}(y)]_{i,j} = \int_y^\infty \lambda^{b-r+j-1} e^{\lambda/\sigma_i} d\lambda$$

$$= \sigma_i^{b-r+j} \Gamma(b-r+j, \frac{y}{\sigma_i}). \qquad (26)$$

Note that some existing methods have been presented to analyze the upper bound on SER of ML detection [27], [28]. In those studies, the SER bound is derived based on the PEP approximation by analyzing the pdf of the squared Euclidean distance. It has been shown that the pdf of $d_{i,j}^2$ is considered as a function of chi-square distribution which depends mostly on the correlation matrix at the transmitter and/or receiver. Obviously, from (24), the derived SER bound for ML detection is more sensitive to channel condition or the rank of the MIMO channel matrix than others as shown in [27] and [28]. This means that the SER bound becomes tighter when the signal is transmitted over a good channel.

IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section, both simulated and analytical SER results of the proposed MIMO systems over Rayleigh fading channels are presented. We assume that the received signal consists of 6 (each of them has 20 sub-paths) time-delayed multipath replicas of the transmitted signal and each path has a precise angle of departure which corresponds to an antenna gain at BS. The spatial parameters (such as lognormal shadowing, delay spread, path loss, mobile directions, angular spread (AS), AoDs, AoAs, and phases) of each path are obtained from [12] for the case of a suburban macrocell. The MS is assumed to be located at a place which is far away from the BS. It implies that the desired MS signal is subject to Rayleigh fading. Moreover, the BS and MS antenna heights are set as $h_{BS} = 32$ m and $h_{MS} = 1.5$ m, respectively, and the pathloss for the suburban environment is defined as $PL = 31.5 + 35\log(d)$, where d is the distance between the BS and MS in meters. A spatial correlation matrix in semi-correlated Rayleigh fading is then generated. A total of 1000 4PSK symbols ($\alpha = 2, \beta = 0.5$) are divided into 4 sub-streams and transmitted to the receiver over a Rayleigh fading channel.
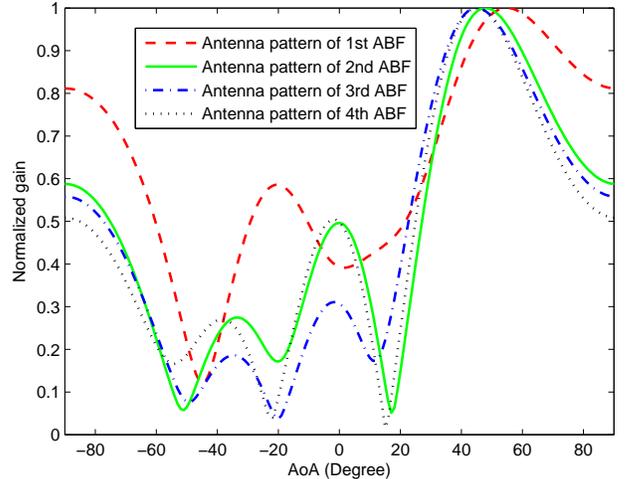

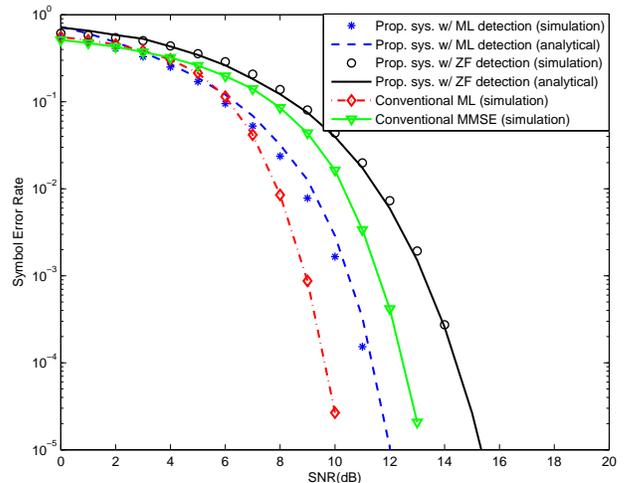Fig. 3. Antenna pattern of 4 ABFs in the proposed system for $K = 4$ and SNR = 12 dB.

Fig. 4. Analytical and simulated SER performance comparisons of the proposed MIMO systems ($S = 4, U = 16, K = 4$) using ML detection or ZF detection, and the conventional ML and MMSE receivers (16 receive antennas) for the case of perfect CSI.

Assuming that the MIMO system has $S = 4, U = 16, K = 4$ antenna elements, and inter-element antenna spacing is $d = 0.5\lambda$. Therefore, the proposed system has 4 ABFs which focus only on their strongest path of the transmitted signals. Figure 3 shows the antenna pattern of 4 ABFs in the proposed system. In this particular case, the AoA of the strongest paths reaching to different ABFs is varied in the range between 42 and 58 degrees. The adaptive beamforming can provide improvement in performance as it can effectively maximize SNR by utilizing a set of strongest paths, especially at low SNR region.

In Fig. 4, analytical and simulated SER results of the proposed MIMO system with ML detection and ZF detection, as well as simulated SER results of the conventional ML and MMSE receivers are presented. The covariance matrix $\boldsymbol{\Sigma}$ in (10) is defined by the Saturn formula, which is expressed as $\boldsymbol{\Sigma} = \mathbf{E}\{h_e(i)^H h_e(i)\}$, where $h_e(i)$ is the ith column of $\mathbf{H}_e$.

The analytical SER results of the proposed system using ZF detection and ML detection are obtained from (13) and (24), respectively. The simulation results are computed based on 10,000 independent channel realizations. It is observed that both the analytical and simulation results match well for the proposed system using ZF detection. For the proposed system with ML detection, the analytical SER bound is also close to the corresponding simulated results. We observe that the proposed MIMO system with ML detection improves significantly in SER performance, up to about two orders of magnitude as compared to that of using ZF detection when SNR=11dB. This is because the ML detection is considered as an optimal detector in terms of error probability. Furthermore, in Fig. 4, the curves labeled "Conventional ML" and "Conventional MMSE" refer to the simulated SER results corresponding to the ML and MMSE receivers, respectively, with 16 receive antennas (without dividing antennas into blocks). From Fig. 4, we observe that the SER performance of the proposed system using ML detection is better than that of the MIMO-MMSE receiver, but worse than that of conventional ML receiver. This is because the conventional ML receiver is the optimal one which has the highest computational complexity as shown in Table II. However, when SNR is less than 6 dB, the proposed system using ML detection performs better than the ML receiver because a set of strongest paths as shown in Fig. 3 is used for SNR maximization. It is one of the advantages of the proposed system. On the other hand, the SER performance of the proposed system with ZF detection is slightly worse by around 2-3 dB as compared to that of with ML detection because ZF detection is the simplest detector of all. Figure 5 presents the simulated SER performance comparisons of the proposed systems with different set of parameter values. The "Selection A" is set by considering 1000 4PSK symbols and 10,000 independent channel realizations, whereas the "Selection B" is set by considering 5000 4PSK symbols and 30,000 independent channel realizations. It is observed that the simulated SER results of both sets match perfectly for the proposed system with ZF detection and for the proposed system with ML detection. Generally, the proposed system using ML detection provides comparable SER performance, but with a lower complexity due to reduced array dimensions.

Figure 6 shows the effects of the number of ABFs (B) and the number of antennas (K) per ABF on the simulated SER performance of the proposed system using ML detection or ZF detection (assuming $S = 4$). We observe that the SER performance of the proposed system is improved by increasing the number of ABFs (from $B = 4$ to $B = 6$) or antennas per ABF (from $K = 3$ to $K = 6$). This is because a larger number of antennas per ABF gives narrower main beams, as well as narrower sidelobes. Note that the number of ABFs should be at least equal to the number of subchannels in order to achieve the full rank spatial multiplexing (the number of streams). However, as the number of antennas is increased, more spacing is required at BS which is often limited by the BS form factors and carrier frequencies. The solution for this practical limitation is to apply full-dimension MIMO which places a large number of antennas in a two-dimensional (2D)
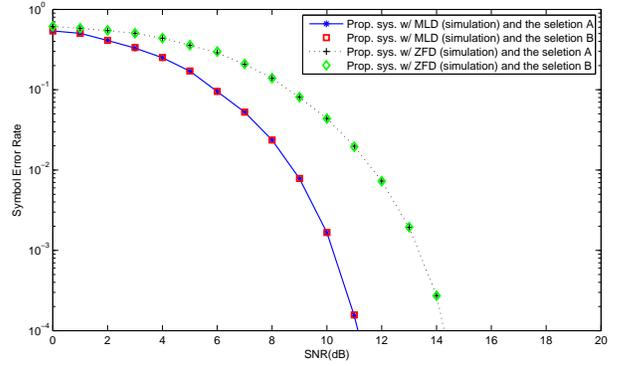

Fig. 5. Simulated SER performance comparisons of the proposed MIMO systems for different parameter settings (the number of 4PSK symbols and the number of independent channel realizations) for the case of perfect CSI.
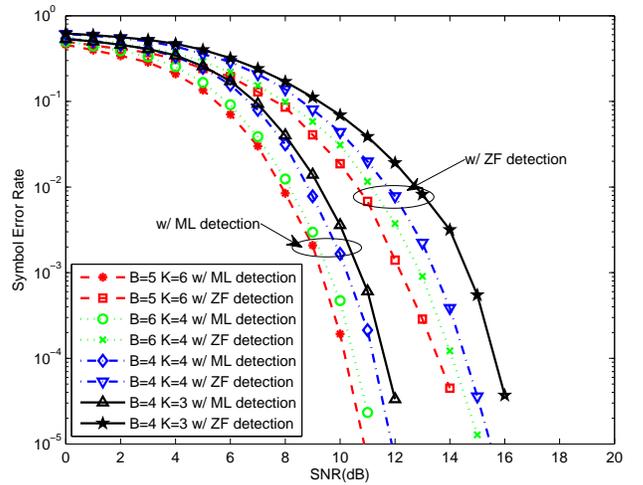

Fig. 6. Simulated SER performance comparisons of the proposed MIMO system using ML detection or ZF detection with different number of ABFs (B) and number of antennas (K) per ABF for the case of perfect CSI.
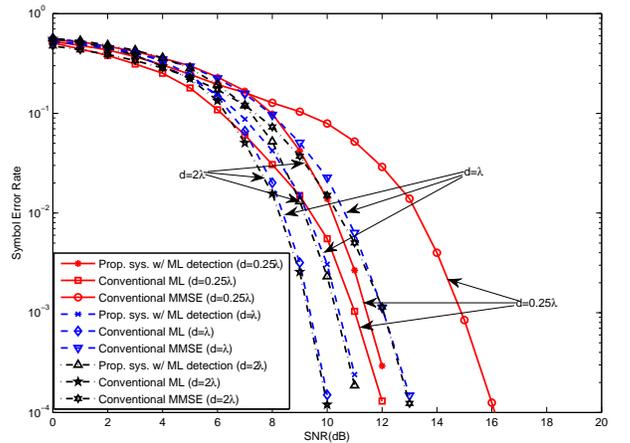

Fig. 7. Effect of spatial correlation with various values of inter-element antenna spacing ($d = 0.25\lambda, 1\lambda, 2\lambda$) on SER performance of the proposed MIMO system using ML detection, conventional ML and MMSE receivers ($S = 4, U = 16, K = 4$) for the case of perfect CSI.

grid at BS. It has been shown in [29] that it is possible to build a full-dimension MIMO system (a total of 32 port antennas) measured at about 0.5 m horizontally and about 1 m vertically at carrier frequency of 2.5 GHz. It means that the BS is configured by 8 antenna ports in the horizontal dimension and 4 antenna ports in the vertical dimension. Therefore, an extension of the spatial channel model needs to be investigated in order to analyze the SER performance of these full-dimension MIMO systems.

Figure 7 illustrates the effect of spatial correlation with various values of inter-element antenna spacing ($d = 0.25\lambda, 1\lambda, 2\lambda$) on SER performance of the proposed system using ML detection, and two conventional receivers. We observe that, when $d = 0.25\lambda$, the spatial correlation between two channel coefficients is strong, hence it gives negative impacts on SER performance of three systems up to a certain SNR level. It is observed that the proposed system performs well as compared with two conventional receivers. In particular, the SER results of the proposed system are close to those ML receiver at high SNR region ($d = 0.25\lambda$). This is because it can achieve a good link reliability which protects itself against spatial correlation. However, when antenna spacing is increased ($d = 1\lambda, 2\lambda$), the spatial correlation is decreased hence the effect of spatial correlation on SER performance is significantly decreased as shown in Fig. 7. In general, the proposed system is also an appropriate choice to achieve good performance in urban microcell propagation where spatial correlation is strong, such as in-building environments.

In Fig. 8, we investigate how channel estimation error affects the SER performance of the proposed system by setting different values of error variance ($\sigma_e^2 = 0$, $\sigma_e^2 = 0.02$ and $\sigma_e^2 = 0.05$). As observed from Fig. 8, the analytical SER expressions of the proposed system are also validated when channel estimation error occurs . The second observation is that the proposed system performance is degraded due to degradation of SNR when error variance is increased. However, the effect of the channel estimation error on the performance of the proposed ML detection is less than that of the ZF detection.
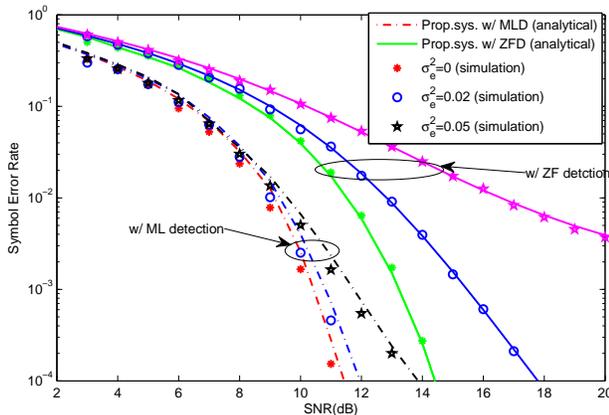

Fig. 8. Effect of channel estimation error with various values of error variance ($\sigma_e^2 = 0$, $\sigma_e^2 = 0.02$ and $\sigma_e^2 = 0.05$) on SER performance of the proposed systems with ML detection and ZF detection ($S = 4, U = 16, K = 4$).

V. CONCLUSION

In this paper, a novel adaptive beamforming receiver has been proposed for a MIMO system over multipath fading channels. The SER performance of the proposed MIMO system using ML detection and ZF detection has been analyzed. An upper bound on the SER for ML detection is derived based on the cdf of the smallest non-zero ordered eigenvalue of Wishart matrix. The close match between the analytical and simulated SER results validates our theoretical analysis. Based on the numerical results, the effects of different system configurations (such as number of ABFs, antennas per ABF) on the SER performance have been examined. It has been observed that its SER performance is improved significantly when the number of antennas per ABF or the number of ABFs is increased. To achieve the full rank spatial multiplexing, the number of ABFs should be at least equal to the number of the transmitting subchannels. In conclusion, the proposed MIMO system can achieve comparable performance, but with a reduction in complexity.

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