Stochastic Simulation Based Reliability Analysis with Multiple Performance Objective Functions

SAHIL BANSAL[1], SAI HUNG CHEUNG[1]

[1] *Department of Civil and Environmental Engineering, Nanyang Technological University, Singapore*

Correspondence e-mail address: shcheung@ntu.edu.sg

ABSTRACT

In this paper, we propose a stochastic simulation approach to estimate the failure probability of multiple performance objectives as a function of various combinations of thresholds, each threshold corresponding to one performance objective. The proposed approach is robust with respect to the dimension of the failure probability integral, model complexity and nonlinearity. The effectiveness and efficiency of the proposed method are illustrated by a numerical example involving a structural dynamic system subjected to future earthquake excitations modeled as a stochastic process and the results are compared to those obtained using crude Monte Carlo simulation.

INTRODUCTION

Reliability of a dynamic system is concerned with the probability that the system will not reach some specific 'failure' states subjected to stochastic excitations. It involves calculating the reliability, or its complement the failure probability P_F, which requires the evaluation of a multi-dimensional integral of the form

$$P_F = \int_F p(\boldsymbol{\theta}) d\boldsymbol{\theta} \tag{1}$$

where $\boldsymbol{\theta} \subset \mathbb{R}^{n_\theta}$ is the parameter vector containing all the uncertain quantities of interest quantified by a joint PDF $p(\boldsymbol{\theta})$ and $F = \{\boldsymbol{\theta} : g(\boldsymbol{\theta}) > 0\}$ is the failure region with $g(\boldsymbol{\theta})$ as the performance function that separates the safe domain $g(\boldsymbol{\theta}) \leq 0$ and the failure domain $g(\boldsymbol{\theta}) > 0$. One example of a performance function is

$$g(\boldsymbol{\theta}) = u(\boldsymbol{\theta}) - c \tag{2}$$

where c is a threshold value and $u(\boldsymbol{\theta})$ is the response quantity of the model specified by $\boldsymbol{\theta}$. A reliability analysis with this performance function estimates the 'failure' probability that u exceeds the threshold c. For complex dynamic system problems involving high stochastic dimension, reliability analysis by state-of-the-art stochastic simulation based techniques has proved to be very efficient and reliable, especially for estimating small failure probabilities [1]. However, all these techniques focus on obtaining failure probabilities as a function of a single threshold. They assume a single performance function, or if there are multiple performance functions for a system, they are correlated to each another in some way to obtain a single performance functions, such as in Parallel Subset Simulation [2] where a principal variable is introduced which is correlated with all performance functions. For a problem with multiple performance functions, which involves estimation of failure probability as a function of multiple thresholds, the aforementioned techniques will require repetitive application that can be very inefficient (unless the combination of thresholds corresponding to which the failure probability is estimated is known a priori).

In this paper, a stochastic simulation approach is proposed to estimate the failure probabilities of multiple performance objectives as a function of various combinations of thresholds, each threshold corresponding to each performance objective. The approach adopts and modifies the Subset Simulation [3, 4] algorithm that can construct probability hypersurfaces at each conditional level in the Subset Simulation using conditional samples. Conditional samples are generated using samples belonging to a failure domain defined by a particular combination of thresholds. The proposed approach is robust with respect to the dimension of the failure probability integral, model complexity and nonlinearity and efficient in computing small failure probabilities.

For complex dynamic systems where the failure is defined by a union of multiple failure criteria, the failure surface can be expressed as:

$$\pi = \bigcup_{l=1}^{G} \{ g_l(\boldsymbol{\theta}) = 0 \} \tag{3}$$

Let vectors $\mathbf{U}(\boldsymbol{\theta})$ and $\mathbf{C}$ be the response quantity vector and the corresponding threshold vector in $\mathbb{R}^G$, respectively. The failure probability as a function of threshold vector $\mathbf{C}$ can then be estimated as:

$$P_F(\mathbf{C}) = P_F(\bigcup_{l=1}^{G} \{ u_l(\boldsymbol{\theta}) > c_l \}) = \int I_F(\mathbf{U}(\boldsymbol{\theta}), \mathbf{C}) p(\boldsymbol{\theta}) d\boldsymbol{\theta} \tag{4}$$

where I_F is an indicator function which is equal to 1 in case of failure and 0 otherwise. To efficiently evaluate the integral in (4) as a function of $\mathbf{C}$, the Subset Simulation based approach is adopted and modified to estimate the failure probabilities of multiple performance objectives as a function of various combinations of thresholds, each threshold corresponding to each performance objective and efficiently propagate the failure samples from one failure domain to the next failure domain. More details of the proposed method are presented in the journal version of this paper.

ILLUSTRATIVE EXAMPLE

The example used in the study is a hotel structure located in Van Nuys. This building is a seven story reinforced concrete structure that was severely damaged during the 1994 Northridge Earthquake. For illustration, failure is defined as an event where any inter-storey drift ratio (*IDR*) or floor acceleration (*FA*) exceeds specific threshold combination at any discrete time instant during the total duration.

$$P(F \mid c_{IDR}, c_{FA}) = \int P(F \mid \boldsymbol{\theta}, c_{IDR}, c_{FA})\, p(\boldsymbol{\theta})\, d\boldsymbol{\theta} \qquad (5)$$

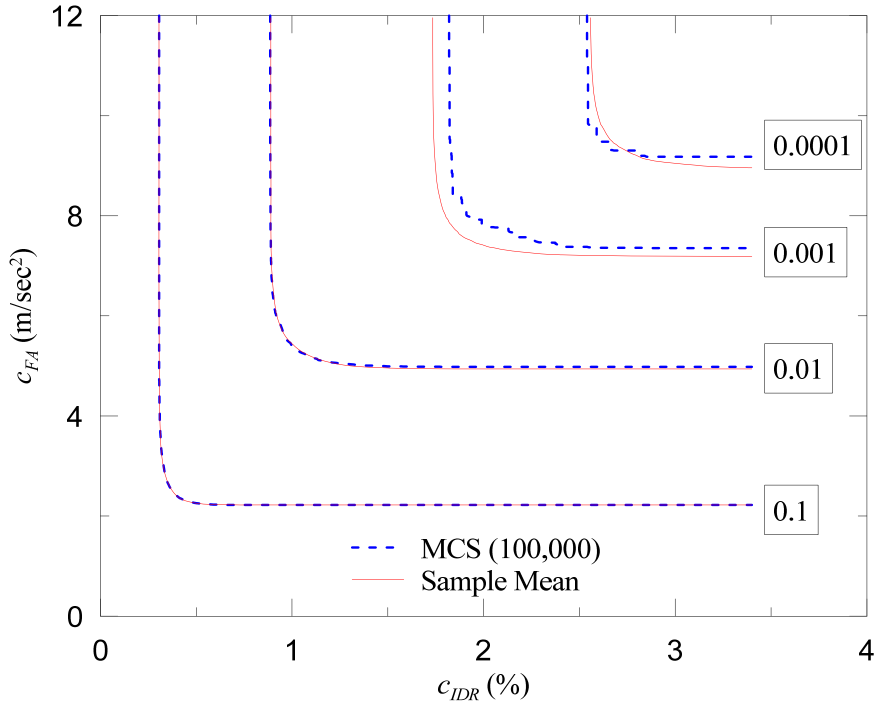

Figure 1. Sample mean of exceedance probability estimates from 60 independent simulation runs and exceedance probability estimates from MCS

The estimates of failure probability for different combinations of threshold of *IDR* and *FA* are presented using a contour diagram. Figure 1 shows the sample mean of exceedance probability estimates from 60 independent simulation runs and exceedance probability estimates from MCS computed using 100,000 independent samples for different combinations of threshold c_{IDR} and c_{FA}. It can be seen that the results using the proposed method and MCS agree well except for very small exceedance probabilities where the error in MCS is significant. To investigate the variability of the exceedance probability estimators, the sample c.o.v of the exceedance probability estimates over 60 independent simulation runs is computed and is shown in Figure 2. It can be observed, as expected, the c.o.v grows approximately in a linear fashion with the logarithm of decreasing exceedance probabilities, which is smaller than the c.o.v for MCS that grows exponentially with decreasing failure probabilities for the same number of samples used.

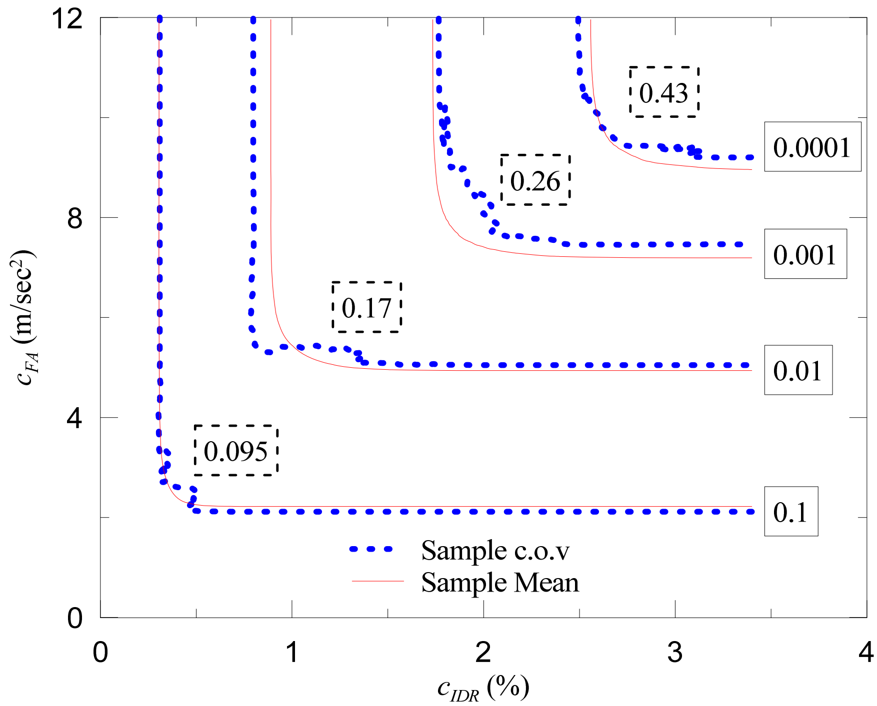

Figure 2. Sample mean and sample c.o.v of exceedance probability estimates from 60 independent simulation runs

CONCLUSION

A new stochastic simulation based approach is proposed for the estimation of failure probabilities of a dynamic system as a function of multiple thresholds of multiple performance objectives where failure is defined by a union of multiple performance objectives. The proposed approach allows for the simultaneous consideration of multiple performance objectives and the corresponding thresholds. An example is presented to show the application of the proposed approach. More discussion and results showing the merits of the proposed approach compared with crude Monte Carlo can be found in the journal version of this paper.

References

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