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Efficient Estimation of a Sequence of Frequencies for M -ary CPFSK Demodulation

Saman S. Abeysekera

School of Electrical and Electronic Engineering,
Nanyang Technological University,
50, Nanyang Avenue, SINGAPORE 639798.
esabeysekera@ntu.edu.sg

Abstract— A Discrete Fourier transform (DFT) based method for estimating a sequence of frequencies is presented. The method is computationally efficient and also meets the Cramer-Rao bound. The method is then used for symbol detection in a Continuous Phase Modulation receiver. The performance of detection has been analyzed and simulation results are provided for comparison against other existing algorithms. It is shown that the proposed DFT based method is advantageous as its implementation is independent of the modulation index and the bit error performance can be predetermined.

Keywords—frequency estimation; DFT; Cramer-Rao bound; CPM; CPFSK.

I. INTRODUCTION

Frequency estimation from a constant amplitude signal is necessary in many signal processing applications [1]. With sampled signals, the peak location of the Discrete Fourier Transform (DFT) can be used for efficient estimation, which also achieves the Cramer-Rao bound (CRB) [2]. Pioneering work related to frequency estimation based on the DFT appears in [3], and since then many other algorithms, in particular for efficient implementations have been reported. These can be classified into the following: (a) Interpolating the Discrete Fourier Transform [4–5], (b) Use of Autocorrelation [6], (c) Phase-Vocoder [7], (d) Sub-space based [2], and (e) Iterative filtering [8]. A review on frequency estimation techniques appear in [9], which shows that the efficient use of DFT can be used for optimal frequency estimation. In this paper we use the DFT approach for estimating a sequence of frequencies and investigate CRB for estimation. Based on this we propose an efficient sequential frequency estimation algorithm.

In the later part of the paper, the proposed algorithm is used for continuous phase modulation (CPM) communication signal demodulation [10]. The attractiveness of the CPM is resulting from its bandwidth and power efficiency, and it is widely used in GSM based mobile communications as well as in Bluetooth and DECT transmitter standards [11]. The optimum decoding of CPM often utilizes a maximum-likelihood sequence estimation (MLSE) by means of the Viterbi algorithm, which is rather complicated in implementation [10]. In the literature, several suboptimal demodulation methods have been proposed to reduce this complexity. These are: the use of matched filters [12], pulse amplitude modulated (PAM) decomposition [13], differential sequence detection [14]. For binary transmissions

these methods achieve good performances at a low complexity. However, CPM receivers are usually complicated in the implementation when non-rational modulation indices are used especially with quaternary or octal transmissions.

Some work on the use of frequency estimation for CPM demodulation, without detailed performance comparisons, appears in the literature [15]. Possibility of the use of frequency estimation in CPM has been proposed in [16], but only for a restrictive selection of the modulation index through pre-coding. In here, we show that using the proposed frequency estimation method a low complexity CPM receiver can be implemented in an efficient manner, operable at any modulation index. Since the method is based on frequency estimation, it is robust against channels having large phase instability, e.g. in underwater communications. The method is also independent of the modulation index, hence insensitive to modulation index variations. (Even a small change in the modulation index has a large impact on error rates in CPM demodulation [16]). We provide a detailed performance analysis and compare over the matched filter implementation. The derivation of the CRB for estimating a sequence of frequencies, the proposal of an efficient algorithm, and its use in CPM demodulation are the novelty of work reported here.

II. BOUNDS OF FREQUENCY AND PHASE ESTIMATION

Consider a constant amplitude frequency modulated noisy signal $z(t)$ in the presence of additive complex white Gaussian noise $\nu(t)$ i.e.,

$$z(t) = Ae^{j\phi(t)} + \nu(t) \quad , \quad (1)$$

where $\phi(t)$ is the instantaneous phase of the signal and t is the time variable. The real constant A is the signal amplitude. The SNR in equation (1) is defined as $SNR = A^2/\sigma^2$, where σ^2 is the noise variance. The instantaneous phase can be related to the signal instantaneous frequency $f(t)$, via the following expression.

$$f(t) = (1/2\pi)(d\phi(t)/dt) \quad . \quad (2)$$

Let the signal be represented in P segments each of duration T , i.e. the total signal duration is PT . We consider the case where the instantaneous frequency is piece-wise constant within a segment, and these frequencies are given by the set

$\{f_1, f_2, \dots, f_P\}$ with an initial phase of $\phi_0 = \phi(0)$. Such a signal is shown in Fig. 1 using a CPM communication transmission. It is well known that an unbiased estimate of frequency, \hat{f}_i , can be obtained by evaluating the peak position of the magnitude of the Fourier Transform of $z(t)$ evaluated within the i^{th} segment [1]. For sampled signals, the most efficient estimation for \hat{f}_i would be the use of Discrete Fourier Transform (DFT). As shown in [3] the DFT based estimates are maximum-likelihood estimates and reach the Cramer-Rao bound for estimation.

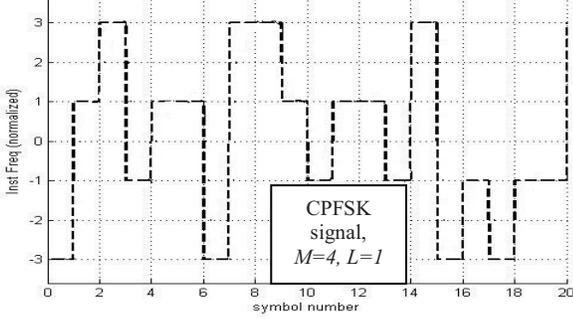


Fig.1. Frequency variation of a quaternary CPM signal. Four frequency levels are present.

In here, we investigate the joint estimation of the frequency set and first examine the CRB for such estimation. The unknown parameters are the set of $(P+2)$ values given by $\{\alpha_i\} = \{A, \phi_0, f_1, f_2, \dots, f_P\}$. The $(i, j)^{\text{th}}$ element of the Fisher information matrix (FIM) associated with the data in (1) are given by [1]

$$F_{i,j} = \text{Re}\left\{\left(\frac{\partial z}{\partial \alpha_i}\right)\left(\frac{\partial z}{\partial \alpha_j}\right)^*\right\}. \quad (3)$$

Assuming that each segment consists of N samples (i.e. the sampling frequency is N/T), and after some algebraic manipulation, the result in (4) can be obtained for the FIM.

Note: The values in (4) are asymptotic i.e. the number of samples within a segment, N is assumed to be large (e.g. $N > 8$). For simplicity of presentation the terms involving A and N are not shown in (4). Their effect would be incorporated later in (5).

$$FIM = \begin{pmatrix} P & 0 & 0 & 0 & 0 & 0 \\ 0 & P & P - \frac{1}{2} & \dots & 2\frac{1}{2} & 1\frac{1}{2} & \frac{1}{2} \\ 0 & P - \frac{1}{2} & P - \frac{2}{3} & \dots & 2\frac{1}{2} & 1\frac{1}{2} & \frac{1}{2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 2\frac{1}{2} & 2\frac{1}{2} & \dots & 2\frac{1}{3} & 1\frac{1}{2} & \frac{1}{2} \\ 0 & 1\frac{1}{2} & 1\frac{1}{2} & \dots & 1\frac{1}{2} & 1\frac{1}{3} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} & \dots & \frac{1}{2} & \frac{1}{2} & \frac{1}{3} \end{pmatrix} \quad (4)$$

Using (4), the CRB for estimate variances can be expressed as,

$$\text{Var}\{\hat{\alpha}_i\} = \frac{\sigma^2}{A^2} \left\{ \frac{A^2 (FIM)_{1,1}^{-1}}{2N}, \frac{(FIM)_{2,2}^{-1}}{2N}, \frac{(FIM)_{3,3}^{-1}}{8\pi^2 N^3}, \dots \right\}. \quad (5)$$

Following observations are made from the evaluation of (5).

- Always, $\text{Var}\{\hat{A}\} = \sigma^2 / 2NP$,
- If frequencies are known, $\text{Var}\{\hat{\phi}_0\} = 2\sigma^2 / (A^2 NP)$,
- Phase estimate variance at the centre of segments is half the initial phase estimate variance.
- If the initial phase is unknown, the minimum frequency variance is for the centre segment frequency,
- If the initial phase is known, the minimum frequency variance is for the first segment frequency,

We further investigate the CRB, using Fig. 2, which shows the variance variation against the number of segments used in the estimation.

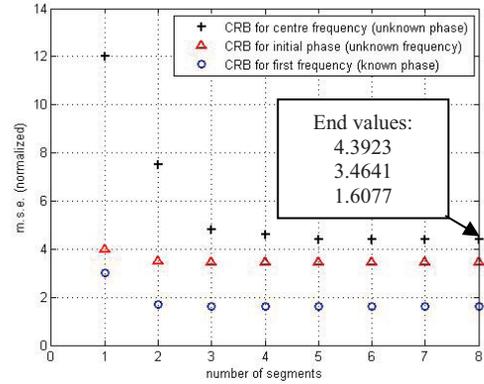


Fig.2. CRB (normalized) for frequency and phase estimates.

Fig. 2 shows that estimations have a floor effect with respect to P , and the following can be noted.

- If the initial phase is unknown, the minimum frequency variance is $\text{Var}\{\hat{f}_i\} = 4.39\sigma^2 / (8\pi^2 A^2 N^3)$, and $P=5$ is sufficient to achieve this;
- If the initial phase is known, the minimum frequency variance is $\text{Var}\{\hat{f}_i\} = 1.60\sigma^2 / (8\pi^2 A^2 N^3)$, achieved when $P=2$;
- The minimum initial phase variance is, $\text{Var}\{\hat{\phi}_0\} = 3.46\sigma^2 / (2A^2 N)$, achieved with $P=2$.

The above observations can be used to devise an algorithm for efficient estimation of frequency from a sequence of segments.

III. EFFICIENT SEQUENTIAL ESTIMATION

As seen in Fig. 2 using 5 segments, frequency can be estimated optimally. Thus, we consider a data segment with

following frequencies $\{f_1, f_2, \dots, f_5\}$. It is easy to show that optimal maximum-likelihood estimation requires the joint estimation of 5 frequencies which necessitates an exhaustive 5-dimensional search. Here we propose a sequential, computationally efficient algorithm and investigates its impact on estimation. Let the common phase between i^{th} and $(i+1)^{\text{th}}$ segments be $\phi_{i(i+1)}$. The algorithm works by estimating the phase term $e^{j\phi_{i(i+1)}}$, and the initial estimation is done using single segments.

Estimation with $P=1$:

The set of frequencies $\{f_1, f_2, \dots, f_5\}$ is initially estimated with $P=1$. That is for the data in the i^{th} segment

$$\hat{f}_i = \max_{\arg f} \left| \sum_{n \in i} z(n) e^{-j2\pi f n} \right| \quad (1 \leq i \leq 5) \quad (6)$$

Estimation with $P=2$:

Using, \hat{f}_i and $\hat{f}_{(i+1)}$, estimated from (6) the following can be obtained

$$e^{j\phi_{i(i+1)}} = 0.5 \left(e^{j2\pi \hat{f}_i N} \sum_{n \in i} z(n) e^{-j2\pi \hat{f}_i n} + \sum_{n \in (i+1)} z(n) e^{-j2\pi \hat{f}_{i+1} n} \right) \quad (7)$$

Using (7) the single segment frequency estimates of (6) can be improved as follows to obtain 2 segment frequency estimates.

$$\hat{f}_i = \max_{\arg f} \operatorname{Re} \left(e^{j\hat{\phi}_{i(i+1)}} \sum_{n \in i} z^*(N-n) e^{-j2\pi f n} \right) \quad (8)$$

$$\hat{f}_{i+1} = \max_{\arg f} \operatorname{Re} \left(e^{-j\hat{\phi}_{i(i+1)}} \sum_{n \in (i+1)} z(n) e^{-j2\pi f n} \right) \quad (9)$$

Estimation with $P=4$:

Using the 2 segment frequency estimates of \hat{f}_i , and $\hat{f}_{(i-1)}$ (estimated from previous segments), the following 4 segment estimate can be derived.

$$e^{j\phi_{(i-1)i}} = 0.5 \left(e^{j2\pi \hat{f}_{i-1} N} \sum_{n \in (i-1)} z(n) e^{-j2\pi \hat{f}_{i-1} n} + \sum_{n \in i} z(n) e^{-j2\pi \hat{f}_i n} \right) \quad (10)$$

Estimation with $P=5$:

Using the 4 segment estimations in (10), a final 5 segment frequency estimate is obtained as,

$$\hat{f}_i = \max_{\arg f} \operatorname{Re} \left(\begin{array}{l} 0.5 e^{-j\hat{\phi}_{(i-1)i}} \sum_{n \in i} z(n) e^{-j2\pi f n} \\ + 0.5 e^{j\hat{\phi}_{i(i+1)}} \sum_{n \in i} z^*(N-n) e^{-j2\pi f n} \end{array} \right) \quad (11)$$

Note: The improvements in (8) and (9) are due to use of real parts (via phase term) instead of absolute value as in (6). As all frequency estimations in (6)-(11) are performed within a single segment, existing efficient DFT peak search algorithms [9] can be used for very efficient implementation of the algorithm.

The efficiency of the above proposed sequential frequency estimation is demonstrated using a simulation where N is selected as 16. Frequency estimation results are shown in Fig.3 where symbol 'o' denotes the m.s.e. of estimation using single segments. The symbol 'Δ' shows results from sequential 5 segment estimation. The results have a SNR thresholding effect (which is typical in DFT search algorithms) around 0dB [3]. For comparison, 2-segment joint estimation results are also shown in '+' symbols. (Joint estimation produces a marginal improvement in SNR thresholding effect but is extremely computational intensive.) The estimation compared with the shown CRBs demonstrates the effectiveness of the proposed sequential algorithm.

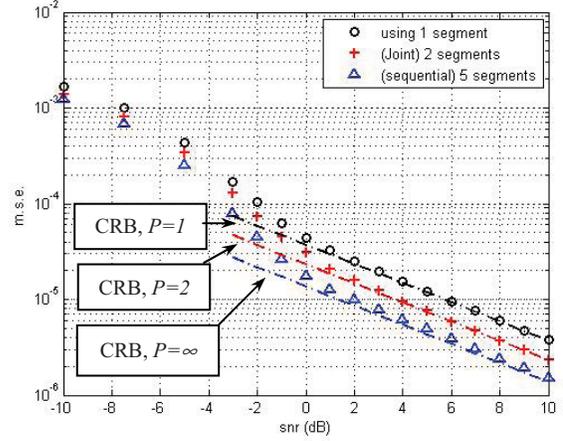


Fig.3. M.S.E. for frequency estimates. (Corresponding CRBs obtained from Fig.2 are shown as dashed lines.)

IV. CONTINUOUS PHASE MODULATION AND CPFASK

Similar to the signal in (1), a CPM signal received in an additive, white, Gaussian (AWGN) channel can be expressed as

$$r(t) = s(t) + v(t) \quad ; \quad s(t) = \sqrt{E_s/T} e^{j\phi(t)} \quad (12)$$

where T is the symbol duration and E_s is the energy per symbol (or per bit for binary transmission). The transmitted information is contained in the signal phase as given by

$$\phi(t) = 2\pi h \sum_n a_n q(t-nT) \quad ; \quad q(t) = \int_{-\infty}^t g(\tau) d\tau \quad (13)$$

where h is the modulation index, $q(t)$ is the phase response, $g(t)$ is the (instantaneous) frequency pulse and a_n is an M -ary data symbol sequence selected from the integer set $\{\pm 1, \pm 3, \dots, \pm(M-1)\}$. In here we consider a frequency pulse of rectangular shape having LT duration. If $L=1$ the transmission is a full response system, otherwise it is a partial response system (see [10] for details). The use of rectangular pulse results in CPFASK transmission.

Fig. 1 has shown the instantaneous frequency variation of a quaternary CPFSK transmission. The 4 symbols $\{\pm 1, \pm 3\}$ can be detected using the frequency value within a symbol period (segment), and the 5-segment sequential algorithm presented in section III can be effectively used. As noted earlier, the DFT based frequency estimation deviate largely from the CRB when $SNR \ll N/24$, and therefore, it is important to avoid this limit on SNR for proper symbol estimation [9]. As shown in [16] the signal SNR can be related to energy of transmission per bit, E_b , and noise power spectral density N_0 , hence the following SNR threshold for DFT based detection can be obtained.

$$(E_b/N_0)_{TH} \cong 12/(\log_2 M) \quad (14)$$

For binary transmission, the above threshold is 12 dB , while for quaternary and octal transmission it is 9 dB and 7 dB , respectively. As M increases, the threshold decreases, thus, detection is more suitable when $M \geq 4$ and in particular when $M \gg 4$ even very low energy signals could be detected. It is also noted here that when M increases other methods of symbol detection, e.g. matched filter (MF) or Viterbi algorithm based (MLSE), requires heavy computational loads as the number of trellis states exponentially increase with M , but the complexity of DFT based frequency estimation is independent of M .

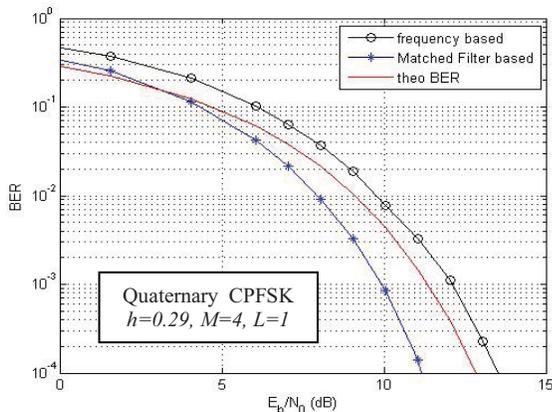


Fig. 4. BER for Full Response Quaternary ($M=4$) CPFSK transmission.

V. BER SIMULATIONS

Fig. 4 shows the bit error rates (BER) for a full response CPFSK transmission with $h=0.29$, evaluated using 10^6 symbols. As the MLSE performance is similar to the use of matched filters, we compare the frequency based receiver with a MF receiver of 5 symbol durations. The results of Fig. 4 demonstrate that the frequency based estimation performance compares favorably with MF performances, with a 2 dB loss in (E_b/N_0) . Theoretical BER error rate is also shown in Fig. 4. These are obtained using the limiting frequency CRB value of Fig. 2 in conjunction with M -ary signal constellations, and is given by,

$$BER = (1 - 1/M) \text{erfc} \left(\pi h \sqrt{(E_b/N_0) (\log_2 M / 4.3923)} \right) \quad (15)$$

Note that the proposed frequency estimation works with any h and any M with the same complexity in estimation procedure. All other methods require different algorithms to handle any change in h and/or M . The method used here therefore, is computationally efficient.

VI. CONCLUSIONS

A new sequential, 5-symbol period based frequency estimation algorithm is proposed for implementing CPFSK demodulators. The method can be efficiently implemented using existing DFT based frequency estimation algorithms. The performance of the method has been analyzed and simulation results are provided for comparison. Following advantages of the proposed method are noted: (i) useful when $M > 2$ as the complexity does not increase as in other methods, (ii) independent of h , hence, blind and computationally favorable when an irrational h is selected, (iii) BER performance can be predicted, hence ease in design with varying M and h , for a given (E_b/N_0) environment (iv) works with or without pre-coding, hence good in phase unstable channels.

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