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LEO Satellite Formation for Space Solar Power: Energy and Doppler Analysis

Shu Ting Goh, Seyed A. (Reza) Zekavat, Senior Member, IEEE, and Ossama Abdelkhalik

Abstract—The space-based solar power (SBSP) concept was introduced during the 1970s. However, the technological challenges during that period stalled its development. Decaying natural energy resources, global warming, and geopolitical pressures have encouraged many countries and researchers to seek new reliable energy sources, such as SBSP. A solar power technology that comprises Low Earth Orbit (LEO) spacecraft formation has been proposed in the literature. All spacecraft in the LEO formation harvest the solar energy and transmit the power to a leader spacecraft or the ground station simultaneously. In the proposed LEO technique, the high and varying relative speed within the spacecraft formation, and also within the spacecraft and the earth, causes a non-homogeneous Doppler spread during solar power transmission. This Doppler spread reduces SBSP power transmission efficiency. In this paper, first, the transmission link budget from LEO and Geostationary orbits to the earth are compared, and the harvested energy by the ground station on the earth is investigated. This motivates low or mid earth orbit satellite usage for SBSP. Next, considering different spacecraft configurations, the impact of the distance between each spacecraft and the leader spacecraft on the variation of the Doppler frequency spread is investigated. In addition, the impact of the position of the leader spacecraft with respect to other spacecraft in the formation is studied. The results confirm that higher altitude or smaller formation size leads to lower Doppler spread effects. Launching the SBSP spacecraft into Geostationary Orbit (GEO) could eliminate the Doppler effect issue; however, huge spacecraft or the ground station experiences different Doppler effects. A communication link will be established when the spacecraft enters the ground station’s communication range. Then, the microwave transmission between spacecraft and ground station is initiated. A preliminary study in [18] shows that the rectenna efficiency of 80% or higher is achievable at 5.8GHz or lower transmission frequency bands, and the overall Direct Current (DC) to DC efficiency of the wireless power transmission system is 45%. In other words, the overall SBSP system efficiency is about 40% excluding the efficiency loss due to Doppler effect and path loss. Maintaining or improving the microwave transmission efficiency is crucial as the cost per kilowatt remains the biggest issue in the SBSP system [19], [20].

This paper considers options (1) and (2) for SBSP transmission. A communication link will be established when the spacecraft enters the ground station’s communication range. Then, the microwave transmission between spacecraft and ground station is initiated. A preliminary study in [18] shows that the rectenna efficiency of 80% or higher is achievable at 5.8GHz or lower transmission frequency bands, and the overall Direct Current (DC) to DC efficiency of the wireless power transmission system is 45%. In other words, the overall SBSP system efficiency is about 40% excluding the efficiency loss due to Doppler effect and path loss. Maintaining or improving the microwave transmission efficiency is crucial as the cost per kilowatt remains the biggest issue in the SBSP system [19], [20].

Although the Doppler spread for spacecraft application has been widely studied for Doppler spread estimation and tracking purposes [15], [21], [22]; however, most studies focus on either improving the spacecraft navigation [23], or the
spacecraft communication [24]. This study focuses on the Doppler effects for SBSP wireless transmission. The SBSP ground receiver uses a rectifier antenna. The resultant signal is the combination of multiple received signals from spacecraft in the formation, and each signal experiences different Doppler effects and have a slightly different distance from the ground station. This leads to different frequencies and phases that alternatively could create a destructive interference. Furthermore, the destructive interference is a function of the time varying effects of Doppler, and could greatly reduces the power transmission efficiency.

In this paper, the efficiency degradation due to the wireless transmission path loss from different altitudes, and the Doppler frequency spread within spacecraft formation and the ground station are studied. The total wireless transmission path loss of each spacecraft transmission to ground station is compared within different LEO altitudes and GEO. In addition, the Doppler frequency spread, which is defined as the difference of two Doppler spread between two transmitters and a receiver (leader spacecraft or ground station in Figure 1), is investigated. Two scenarios are considered: (1) two-spacecraft formation in LEO and Medium Earth Orbit (MEO); and (2) three-spacecraft formation in LEO where one spacecraft serves as the leader spacecraft. The Doppler frequency spread and its variation rate between two transmitters and a receiver is investigated in terms of parameters such as the distance between transmitter and receiver, and the position of the leader spacecraft. Finally, several techniques to improve the wireless power transmission efficiency is discussed. In addition, advantages and disadvantages of these methods are compared.

The paper is organized as follows: Section II presents the novel SBSP structures, and the transmission path loss analysis. Section III investigates the Doppler effect on wireless power transmission frequency. Sections IV and V-B simulate the Doppler effect in different spacecraft configurations, and discuss different techniques that improve the efficiency of microwave transmission due to Doppler effect. Section VI summarizes the paper conclusions.

II. PROPOSED NOVEL SBSP SATELLITE NETWORK

Ref. [9] and [16] have introduced an alternative novel LEO SBSP transmission method that utilizes the LEO satellite network. Here, a number of LEO spacecraft fly in a formation. Each spacecraft is equipped with a Solar Power Harvesting Unit Platform (SOPHU). The solar power is collected by the SOPHU platform, then, the spacecraft in the formation transmits the microwave energy to the ground station’s rectenna array when it is within the ground station’s field-of-view (FOV). The size of ground station’s rectenna array might be in the order of kilometers in diameter [2].

Two microwave transmission methods have been introduced in [9] and [16]: (1) A leader spacecraft is equipped with both rectenna and microwave transmitter. The leader spacecraft collects the power from all other spacecraft through microwave transmission, then, it transmits the collected power again via microwave energy to the ground station (see Figure 1(a)). (2) All spacecraft in formation directly transmit the harvested power to the ground station when they are within the ground station’s FOV (see Figure 1(b)).

The spacecraft in LEO has a higher orbital speed compared to the earth rotation. While in the GEO, the spacecraft can maintain a continuous communication to a specific ground station at all times. Therefore, a communication link will be initiated when the spacecraft enters the ground station’s FOV. Then, the spacecraft begins the microwave transmission to the ground station, as shown in Figure 2. Because rectenna has a limited FOV, in order to maximize the collected power, it is important to maximize the number of spacecraft that are simultaneously located within the ground station’s FOV at a given altitude, \( h \), (or FOV’s radius, \( d \)). It is also desirable to increase the total time interval for a spacecraft that remains within the ground station’s FOV. The rectenna array’s FOV
radius is approximated as:

\[ d \approx h \tan \phi \] (1)

where \( \phi \) is the FOV angle.

The rectenna has a typical FOV of 60 degrees [25], however, higher microwave transmission frequency decreases the effective rectenna’s FOV [26]. Furthermore, the incoming microwave energy direction (or directional-of-arrival) impacts the rectenna output efficiency [27]. Table I compares the total communication time between the spacecraft and the ground station (or the total time interval needed for a spacecraft to stay within ground station’s FOV) for different altitudes. Table I shows that lower altitudes lead to a lower total microwave transmission duration time. In addition, lower altitude allows lower number of spacecraft to simultaneously stay within ground station’s FOV. For example, the radius of FOV is 346km at the altitude of 200km, and 1372km at the altitude of 400km; thus, higher number of spacecraft can be accommodated in higher altitude. Taking the change in relative distance due to the earth gravitational field into account [28], and assuming the distance within neighboring spacecraft is 10km (collision avoidance purpose); then, at least 15 to 20 spacecraft could stay within ground station’s FOV at the altitude of 200km, if all the spacecraft are flying in a A-train formation [29].

As presented in Table I, the LEO spacecraft can transmit to the ground station only within few minutes. Therefore, multiple ground stations and spacecraft formation clusters would be required to maintain a continuous microwave transmission to any given ground station. A concept similar to mobile communication’s “handoff” process is considered in the proposed SBSP transmission method. Here, the spacecraft maintains continuous power transmission to the ground by switching the transmission from one ground station to another ground station that is within its FOV [16]. In addition, another spacecraft formation cluster would enter the previous ground station’s FOV to maintain a continuous transmission of microwave energy to a ground station, after one spacecraft formation leaves a particular ground station’s FOV. This requires a careful mission design to optimize the wireless power transmission and the “handoff” process. This technique not only allows spacecraft to maintain a continuous microwave transmission, but also allows each ground station to continuously receive microwave transmission from spacecraft.

Although Table I shows that higher altitude allows more spacecraft to simultaneously transmit the wireless power to the ground station for a longer microwave transmission time period; however, the wireless power path loss is required to be taken into account. Let the received power, \( S_r \), by the antenna at ground station correspond to [30]:

\[ S_r = \frac{S_t G_t G_r}{L} \] (2)

where \( S_t \) is the transmitted power, \( L \) is the path loss, \( G_t \) and \( G_r \) are the transmitter and receiver antenna gain respectively.

Given the diameter, \( D \), of an antenna, the antenna gain, \( G \), for both transmitter and receiver is [30]:

\[ G = \frac{\eta \pi D^2}{\lambda^2} \] (3)

where \( \lambda \) is the transmitted signal wavelength, \( \eta \) is the aperture efficiency, and the term \( \pi D^2/4 \) is the physical aperture area of antenna.

In addition, (2) can be expressed in term of decibel (db), which corresponds to:

\[ S_{r, dB} = S_{t, dB} + G_{t, dB} + G_{r, dB} - L_{dB} \] (4)

It should be noted that wireless power transmission path loss for LEO spacecraft is considerably lower than the wireless power transmission platform in higher altitudes, such as GEO [31]. Taking \( L_{dB} = 10 \log_{10} L \), the wireless transmission path loss from spacecraft to the ground station corresponds to [9]:

\[ L_{dB} = 32.45 + 10 n \log(d_{KM}) + 20 \log(f_{MHz}) + L_{Ion} + L_{Atm} + L_{Ecl} \] (5)

where \( n \) is the path loss exponent, which \( n = 2 \) for vacuum space [32], \( L_{Atm} \) is the atmospheric loss, \( f_{MHz} \) is the frequency measured in MHz, and \( d_{KM} \) is the altitude measured in km. \( L_{Ion} \) is the loss due to all ionosphere layers, and \( L_{Ecl} \) is the eclipse loss which is given as:

\[ L_{Ecl} = 10 \log \left( \frac{\pi}{\sin^2 \left( \frac{6370}{d_{KM} + 6370} \right)} \right) \] (6)

Table II summarizes the captured power by the ground station from the spacecraft at GEO and four different LEO

<table>
<thead>
<tr>
<th>Altitude (km)</th>
<th>FOV Interval (min)</th>
<th>Out-FOV Interval (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6878.15</td>
<td>≈4</td>
<td>≈90</td>
</tr>
<tr>
<td>7136.64</td>
<td>≈6</td>
<td>≈94</td>
</tr>
<tr>
<td>16763.39</td>
<td>≈109</td>
<td>≈251</td>
</tr>
<tr>
<td>26610.22</td>
<td>≈385</td>
<td>≈335</td>
</tr>
</tbody>
</table>

![Fig. 2. Two SBSP spacecraft transmit wireless power to rectenna array’s in the ground when they are within the field-of-view.](image-url)
TABLE II
THE CAPTURED POWER ON THE GROUND

<table>
<thead>
<tr>
<th>TX Antenna Radius</th>
<th>$F_1 (dB)$</th>
<th>Spacecraft altitude and corresponding captured power</th>
<th>GEO 1000km</th>
<th>700km</th>
<th>400km</th>
<th>200km</th>
</tr>
</thead>
<tbody>
<tr>
<td>10m</td>
<td>70 (10MW)</td>
<td>0.12 W 1.09 kW</td>
<td>2.41 kW</td>
<td>8.06 kW</td>
<td>34.3 kW</td>
<td></td>
</tr>
<tr>
<td></td>
<td>77 (50MW)</td>
<td>0.62 W 5.49 kW</td>
<td>12.1 kW</td>
<td>40.3 kW</td>
<td>174 kW</td>
<td></td>
</tr>
<tr>
<td></td>
<td>80 (100MW)</td>
<td>1.25 W 10.9 kW</td>
<td>24.1 kW</td>
<td>80.6 kW</td>
<td>348 kW</td>
<td></td>
</tr>
<tr>
<td></td>
<td>90 (1GW)</td>
<td>12.5W 109 kW</td>
<td>241 kW</td>
<td>806 kW</td>
<td>3.48 MW</td>
<td></td>
</tr>
<tr>
<td>50m</td>
<td>70 (10MW)</td>
<td>1.63 W 14.4 kW</td>
<td>34.7 kW</td>
<td>108 kW</td>
<td>357 kW</td>
<td></td>
</tr>
<tr>
<td></td>
<td>77 (50MW)</td>
<td>8.19 W 72.1 kW</td>
<td>158 kW</td>
<td>529 kW</td>
<td>2.29 kW</td>
<td></td>
</tr>
<tr>
<td></td>
<td>80 (100MW)</td>
<td>16.3 W 144 kW</td>
<td>317 kW</td>
<td>1.06 MW</td>
<td>4.57 kW</td>
<td></td>
</tr>
<tr>
<td></td>
<td>90 (1GW)</td>
<td>163 W 1.44MW</td>
<td>3.17 MW</td>
<td>10.6 MW</td>
<td>45.7 MW</td>
<td></td>
</tr>
<tr>
<td>100m</td>
<td>70 (10MW)</td>
<td>4.96 W 43.7 kW</td>
<td>96.0 kW</td>
<td>321 kW</td>
<td>1.38 MW</td>
<td></td>
</tr>
<tr>
<td></td>
<td>77 (50MW)</td>
<td>2.48 W 21.9 kW</td>
<td>480 kW</td>
<td>1.61 MW</td>
<td>6.93 MW</td>
<td></td>
</tr>
<tr>
<td></td>
<td>80 (100MW)</td>
<td>49.6 W 437 kW</td>
<td>960 kW</td>
<td>3.21 MW</td>
<td>13.8 MW</td>
<td></td>
</tr>
<tr>
<td></td>
<td>90 (1GW)</td>
<td>496 W 4.37 MW</td>
<td>9.60 MW</td>
<td>32.1 MW</td>
<td>138 MW</td>
<td></td>
</tr>
</tbody>
</table>

altitudes with respect to two receiver antenna (TX) diameters and four different transmission powers. The ionosphere loss is assumed to be 1dB, and the transmission frequency is 5GHz, and its associate atmospheric loss is given as 0.01 dB/nautical mile [33]. We assume the atmosphere attenuation occurs below K´arm´an Line, with the total atmospheric height of 54 nautical miles [34]. The antenna efficiency of the transmitter is assumed to be 80% [35], and the ground receiver rectifier antenna total radius is considered to be 1km with the efficiency of 95%, which results in $G_r = 89.66$ dBi.

Table II shows that with the same antenna size, the LEO spacecraft harvests a higher total energy compared to the GEO spacecraft. Although the GEO SBSP spacecraft may have a large harvesting unit area (kilometers square area), however, it suffers a higher wireless power transmission path loss. Comparing the performance of 10m antenna size for LEO spacecraft to the 100m antenna size for GEO spacecraft in Table II, the LEO spacecraft represents a higher transmission efficiency. These facts along with other features such as lower launch cost and ease of maintenance and replacement, suggest the LEO or middle earth orbit (MEO) power harvesting units as a better alternative compared to GEO. However, it should be noted that the LEO satellite at the altitude of 400km or lower suffers from the atmospheric drag, which requires additional propulsion system for drag compensation. On the other hand, the altitude of 700km, which is often used for sun synchronous orbit, provides a better choice due to the fact that the atmospheric drag is minimal. But additional 300km altitude could results a 70% of wireless power transmission loss as shown in Table II. Furthermore, it should be noted that other factors, such as the Doppler effect during wireless power transmission must be taken into account in SBSP system design. The impact of Doppler effect is investigated in the next section.

III. DOPPLER SPREAD ANALYSIS

Several space-based methods that incorporate a spacecraft formation as a power harvesting platform have been proposed in the literature [16], [36] that include: (1) All spacecraft in the formation transfer the solar energy to a specific ground station simultaneously; (2) A leader spacecraft collects the solar power from other spacecraft in the formation, then wirelessly transfers the solar power to the ground station. The implementation of these techniques assume a proper synchronization across multiple signals transmitted by multiple transmitters to a given receiver. This synchronization requires a fixed or predictable change in Doppler effect created due to a change in the relative distance between transmitter and receiver [37].

However, the spacecraft’s absolute speed in the orbit may vary from one position to another. The change of a spacecraft’s speed leads to rapid changes in the distance between the spacecraft and the ground station, and also, the distance between two spacecraft. These rapid changes in distance create a fast varying Doppler effect that impacts the phase of the received signal [38]. This problem impacts the phase tracking and synchronization across multiple transmitters and reduces the efficiency of the power transmission process [39]. Lack of synchronization across two transmitters may lead to destructive addition of the received power and, in turn, reduction in performance.

Figure 3 presents the resultant signal received by the re-
receiver from two transmitters. The signal amplitude of each transmitter is normalized to a maximum of 1W. In the figure, both transmitters transmit the signal at the frequency of \( f_r \). However, the receiver receives the signal at the frequencies of \( f_1 = f + \Delta f_1 \) and \( f_2 = f + \Delta f_2 \) from each transmitter respectively. Comparing the actual received signal (blue solid line) to the expected received signal (red-dotted line), the receiver experiences destructive effects over time due to the Doppler effect. The total received signal amplitude over the time period is much lower than the expected received signal amplitude. It is noted that Figure 3 assumes that the receiver observes the same receiving signal’s phase angle from the two transmitters. In practice, the difference in phase angle would further reduce the actual receiving signal amplitude. While synchronization can be considered to minimize the power loss due to the Doppler effect, the change of observed phase angle at the receiver. Therefore, the distance between the transmitter and receiver also results in the change of observed phase angle at the receiver. Therefore, the study of the rate of change of the Doppler effect is crucial.

When both transmitter and receiver are moving, the frequency of the signal received by the receiver, which is transmitted from the transmitter, corresponds to [40]:

\[
f_r = (1 - \frac{\dot{r}_{r/t}}{c}) f_c \tag{7}
\]

and

\[
\dot{r}_{r/t} = \frac{r_{r/T} T}{\| r_{r/t} \|} \tag{8}
\]

\[
r_{r/t} = r_t - r_r \tag{9}
\]

\[
v_{r/t} = v_t - v_r \tag{10}
\]

where \( r \) is the position vector, \( v \) is the velocity vector, subscript \( r \) and \( t \) denote receiver and transmitter respectively, \( \dot{r}_{r/t} \) is the relative speed between the transmitter and receiver, and \( f_c \) is frequency (or transmitter frequency).

It is noted that the relative speed between the receiver and transmitter is not equivalent to the magnitude of the relative velocity, \( v_{r/t} \), between transmitter and receiver, that is \( \dot{r}_{r/t} \neq \| v_{r/t} \| \). The relative speed is the time derivative of the relative distance between the transmitter and receiver, but the relative velocity is the velocity vector difference between transmitter and receiver with respect to a given reference frame, such as the Earth Center Inertial Frame. Equation (7) shows that the Doppler effect between the receiver and transmitter is a function of the relative speed. The negative sign shows that the received frequency increases when the transmitter and receiver are getting closer to each other (\( \dot{r}_{r/t} < 0 \)), and decreases when the transmitter and receiver move away from each other (\( \dot{r}_{r/t} > 0 \)). In addition, (7) can be expressed in terms of Doppler frequency, \( f_d \), which is:

\[
f_r = f_c + f_d \tag{11}
\]

where

\[
f_d = -\frac{\hat{r}_{r/t}}{c} f_c \tag{12}
\]

Literature studies have shown that the Doppler spread can be measured and estimated if the transmitter oscillator is stable [15]. However, the relative speed between each spacecraft, and between the spacecraft and ground station varies at all times. Thus, in this paper, the rate of change of Doppler spread, \( \dot{f} \) corresponding to (7) will be evaluated. If we assume that the transmitter frequency, \( f_c \), is always constant, then \( f_r \) is given as:

\[
f_r = f_d = -\frac{\hat{r}_{r/t}}{c} f_c \tag{13}
\]

First, the impact of two different receiving Doppler frequencies on the receiver end is presented. If a ground station or a leader spacecraft receives the power transmission from multiple spacecraft simultaneously, the received signal amplitude, \( P_R(t) \), is given as:

\[
P_R(t) = \sum_{k=1}^{M} \sin(2\pi f_{r,k} t + \psi_k) \tag{14}
\]

where \( M \) is total number of spacecraft in formation, \( \psi_k \) is the phase angle due to the signal traveling delay between each transmitter and receiver. In (14), we have assumed that the received power from all satellites in formation are approximately similar. The \( k^{th} \) transmitter’s phase angle, \( \psi_k \), is the function of distance and frequency, which corresponds to:

\[
\psi_k = 2\pi f_{r,k} \frac{\| r_{t,k} - r_r \|}{c} \tag{15}
\]

Equations (14) and (15) show that the changes of receiving frequency over the time to due the Doppler effects could lead to either a constructive or destructive effect on the \( P_R(t) \). For simplicity, we only consider two spacecraft (transmitter) and a receiver (ground station or a leader spacecraft) scenario, where \( P_R(t) \) in (14) becomes:

\[
P_R(t) = \sin(2\pi f_{r,1} t + \psi_1) + \sin(2\pi f_{r,2} t + \psi_2)
\]

\[
= 2 \cos(2\pi \frac{f_{r,2} - f_{r,1}}{2} t + \frac{\psi_1 - \psi_2}{2})
\]

\[
\times \sin(2\pi \frac{f_{r,1} + f_{r,2}}{2} t + \frac{\psi_1 + \psi_2}{2})
\]

The \( \frac{f_{r,1} - f_{r,2}}{2} \) in (16) is called the Doppler frequency difference, \( \Delta f_T \), that is a representation for Doppler frequency spread when two transmitters (or spacecraft) are available to the receiver. The term \( \cos(2\pi \frac{f_{r,1} - f_{r,2}}{2} t + \frac{\psi_1 - \psi_2}{2}) \) represents the change in the amplitude of the received signal due to the Doppler effect, which is typically shown in Figure 3. \( \Delta f_T \) and its variation rate are given as:

\[
\Delta f_T = \frac{f_{r,1} - f_{r,2}}{2} \tag{17}
\]

and respectively,

\[
\dot{\Delta} f_T = \frac{\dot{f}_{r,1} - \dot{f}_{r,2}}{2} \tag{18}
\]

In addition in (16), \( \sin(2\pi \frac{f_{r,1} + f_{r,2}}{2} t + \frac{\psi_1 + \psi_2}{2}) = \sin(2\pi f_{r,t} + \pi (f_{d,1} + f_{d,2}) t + \frac{\psi_1 + \psi_2}{2}) \), where either both
The spacecraft’s absolute position and velocity with respect to the earth center can be expressed in terms of six orbital elements, which are the semimajor axis, $a$, eccentricity, $e$, true anomaly, $\theta$, inclination, $\lambda$, Right Ascending of Ascension Node (RAAN), $\Omega$, and the argument of periapse, $\omega$. The conversion between these six orbital elements and the absolute position and velocity vector can be found in Ref. [41]. In general, the absolute distance of the spacecraft is often expressed with respect to the earth center, which is given as [42]:

$$ r = \frac{a(1 - e^2)}{1 + e \cos(\theta)} $$  \hfill (19)

However, we are interested in the rate of change of the distance between the spacecraft and ground station, as both (17) and (18) show that the relative distance and its variation rate between two transmitters and a receiver impact the Doppler frequency difference. Let the ground station’s absolute position and velocity be $\mathbf{r}_s$ and $\mathbf{v}_s$, respectively; then, the relative position and velocity between the spacecraft and the ground station can be written as $\mathbf{r}_{s/i}$ and $\mathbf{v}_{s/i}$. Here, by considering that the distance between the spacecraft and the ground station is, $r_{s/i} = ||\mathbf{r}_{s/i}||$, the rate of change of the distance between the spacecraft and the ground station is:

$$ \dot{r}_{s/i} = \frac{\mathbf{r}^T_{s/i} \mathbf{v}_{s/i}}{r_{s/i}} $$  \hfill (20)

Equation (20) shows that the rate of change of the relative distance between the spacecraft and the ground station is expressed in terms of the spacecraft’s position and velocity. Ref. [41] also shows that all six orbital elements impact spacecraft position and velocity at any time. Thus, the Doppler spread between the spacecraft and the ground station is a function of all orbital elements. It is possible that all spacecraft possess the same semimajor axis, eccentricity, and the true anomaly at any time; however, each spacecraft is required to have at least one different orbital element parameter to avoid collision. Therefore, the Doppler frequency, $f_d$, between the ground station and each spacecraft is always different.

Moreover, to evaluate $\dot{f}_r$ in (13), $\mathbf{r}_{s/i}$ should be evaluated to correspond to:

$$ \mathbf{r}_{s/i} = \frac{||\mathbf{v}_{s/i}||}{r_{s/i}} - \frac{\mathbf{r}^T_{s/j} \mathbf{a}_{s/j}}{r_{s/j}} - \frac{\mathbf{r}^2_{s/i}}{r_{s/j}} $$  \hfill (21)

where $||.||$ represents the magnitude of the vector, $\mathbf{v}_{s/i}$ is the relative velocity between ground station and spacecraft, $\mathbf{a}_{s/j}$ is the relative acceleration between ground station spacecraft:

$$ \mathbf{a}_{s/j} = \mathbf{\ddot{r}}_s - \mathbf{\ddot{r}}_i $$  \hfill (22)

Here, the motion of the ground station can be expressed in term of the earth’s angular velocity. The earth angular velocity vector is given as, $\omega_E = \begin{bmatrix} 0 & 0 & \omega_E \end{bmatrix}^T$, where $\omega_E = 7.292 \times 10^{-5} \text{rad/sec}$. Then, the ground station velocity with respect to the inertial frame is [42]:

$$ \mathbf{\dot{r}}_s = \mathbf{v}_s = \omega_E \times \mathbf{r}_s $$  \hfill (23)

The earth is rotating at a constant velocity. Hence, the velocity of the ground station, $\mathbf{\dot{r}}_s$ in the inertial frame, is always constant. Thus, $\mathbf{\dot{r}}_s$ in (22) is a zero vector. Moreover, the acceleration of $i^{th}$ spacecraft in the inertial frame is:

$$ \mathbf{\ddot{r}}_i = -\mu \frac{\mathbf{r}_i}{||\mathbf{r}_i||^3} $$  \hfill (24)

where $\mu$ is the earth gravitational constant. Thus, the relative acceleration vector, $\mathbf{a}_{s/j}$ in (22) becomes:

$$ \mathbf{a}_{s/j} = \frac{\mathbf{\ddot{r}}_s - \mathbf{\ddot{r}}_i}{r_{s/j}} $$  \hfill (25)

### B. Within the spacecraft formation

In LEO, each spacecraft travels at a very high speed, $v \approx 10km/s$, with respect to the earth center. For the non-circular orbit formation flying, the relative distance between each spacecraft changes at all times. The changes in relative distance creates a Doppler effect in the signal transmitted by a spacecraft in a formation to a leader spacecraft. As shown in Figure 3, if the Doppler effect is not carefully taken into consideration, the efficiency of the microwave transmission from the spacecraft in the formation to the leader spacecraft would be greatly reduced.

Here, similar to the previous subsection, the Doppler frequency, $f_r$, is studied through the rate of change of the relative distance between two spacecraft. The relative position between $i^{th}$ spacecraft and $j^{th}$ corresponds to:

$$ \mathbf{r}_{i/j} = \mathbf{r}_j - \mathbf{r}_i $$  \hfill (26)

The relative distance between the two spacecraft is $r_{i/j} = ||\mathbf{r}_{i/j}||$. Similarly, the rate of change of the relative distance between two spacecraft is not equivalent to the magnitude of the relative velocity between two spacecraft, that is $\dot{r}_{i/j} \neq ||\mathbf{v}_{i/j}||$. Therefore, the rate of change of the relative distance, $\dot{r}_{i/j}$ between two spacecraft corresponds to:

$$ \dot{r}_{i/j} = \frac{\mathbf{r}^T_{i/j} \mathbf{v}_{i/j}}{r_{i/j}} $$  \hfill (27)
As previously mentioned, the relative distance and the speed between two spacecraft varies over time. Therefore, the Doppler frequency, $f_d$, between two spacecraft never stays constant. The evaluation of $f_d$ is similar to (21), which is given as:

$$\tilde{r}_{i/j} = \frac{\|\mathbf{v}_{i/j}\|}{r_{i/j}} - \frac{r_{i/j}^T a_{i/j}}{r_{i/j}} - \frac{\tilde{r}_{i/j}^2}{r_{i/j}}$$  \hspace{1cm} \text{(28)}

where $\|\|\|$ represents the magnitude of the vector, $\mathbf{v}_{i/j}$ is the relative velocity between spacecraft $i$ and $j$, $a_{i/j}$ is the relative acceleration between spacecraft $i$ and $j$, which corresponds to:

$$a_{i/j} = \tilde{r}_j - \tilde{r}_i$$  \hspace{1cm} \text{(29)}

where both $\tilde{r}_i$ and $\tilde{r}_j$ can be calculated using (24).

### IV. Simulation

Simulations are conducted to study the Doppler frequency difference, $\Delta f_T$, and its variation rate, $\Delta f'_T$, in two scenarios: (1) between two spacecraft to the ground station, and (2) between two spacecraft to the leader spacecraft. These studies are investigated with respect to two simulation variables, which are the distance and its variation rate between the transmitter and receiver. In the first scenario, the impact of the spacecraft formation altitude, and its variation rate is considered as the simulation variable. In the second scenario, the relative distance and its variation rate between each spacecraft to the leader spacecraft are considered as the simulation variable.

In the first scenario, four different altitudes for the two-spacecraft formation are considered; the semimajor axis for these orbits are 7136.64km, 16763.39km, 26610.22km and 34869.26km. Here, we consider the spacecraft orbit of each given altitude is a circular orbit (or zero eccentricity). Other orbital elements, inclination ($\iota$), argument of perigee, ($\omega$), initial true anomaly ($\tau$), and right ascending of ascension ($\Omega$) of each spacecraft are given in Table III, where $\beta$ is a constant multiplier to determine the size of formation (maximum and minimum distance between two spacecraft). For the first scenario, the configuration of spacecraft 1 and spacecraft 2 in Table III is considered, with $\beta = 1$. In addition, the impact of field-of-view (FOV) of ground station on the $\Delta f_T$ is studied. The FOV will be studied in term of cosine angle, which is given as:

$$FOV = \cos(\phi) = \frac{r_i^T(r_i - r_s)}{\|r_i\|\|r_i - r_s\|}$$  \hspace{1cm} \text{(30)}

### Table III

<table>
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<tr>
<th>S/C no.</th>
<th>$\lambda$(deg)</th>
<th>$\Omega$(deg)</th>
<th>$\omega$(deg)</th>
<th>$\theta$(deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.05/5</td>
<td>0.0</td>
<td>0.0</td>
<td>0.13</td>
</tr>
<tr>
<td>2</td>
<td>0.15/5</td>
<td>0.0</td>
<td>0.0</td>
<td>-0.12</td>
</tr>
<tr>
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<td>0.05/7</td>
<td>0.0</td>
<td>0.0</td>
<td>0.03</td>
</tr>
<tr>
<td>4</td>
<td>0.05/7</td>
<td>0.0</td>
<td>0.0</td>
<td>0.08/7</td>
</tr>
</tbody>
</table>

In the second scenario, we consider a three-spacecraft formation, which orbits at the semimajor axis of 7136.64km. Here, all spacecraft are flying at an elliptic orbit, with the eccentricity of 0.05. Similar to the first scenario, all three spacecraft configurations are shown in Table III. Two different cases are considered in this scenario. First, the overall relative distance between the leader spacecraft and spacecraft 1 is similar to the overall relative distance between the leader spacecraft and spacecraft 2, $r_{31} \approx r_{32}$, which is considered a symmetric case (see Figure 4(a)). Here, spacecraft 3 in Table III is considered the leader spacecraft or collector, and spacecraft 1 and 2 are considered the transmitters. In the second case, spacecraft 4 in Table III is considered the leader spacecraft, and spacecraft 1 and 2 are considered the transmitters. The configuration of spacecraft 4 leads to a shorter overall relative distance between spacecraft 4 and spacecraft 1 compared to the overall relative distance between spacecraft 4 and spacecraft 2, $r_{41} \neq r_{42}$. The second case is considered a non-symmetric case (see Figure 4(b)).

The impact of relative distance and relative speed on the $\Delta f_T$ of each spacecraft with respect to spacecraft 3 (or spacecraft 4) is studied. Four different values of $\beta$ are considered, $\beta = 1, 5, 10$ and 20, which also correspond to 13km/24km, 108km/119km, 214km/236km and 428km/473km formation sizes, respectively. The 13km/24km represents that the minimum diameter of three-spacecraft formation at anytime is 13km, and the maximum diameter of the formation is 24km.

### V. Discussion

#### A. Simulation Results

Figure 5(a) compares the Doppler frequency difference, $\Delta f_T$ (see (17)) with respect to different semimajor axes (represented by $sma$ in Figure 5), also known as the altitude of two-spacecraft formation. The result shows that the spacecraft...
at a lower altitude has higher $\Delta f_T$. This is due to the higher relative speed of spacecraft with respect to ground at lower altitude. It is noted that the frequency difference of 100 Hz at the altitude 800 km results in a phase angle difference of approximately 96 degrees, observed by the receiver. Thus, the microwave transmission efficiency is greatly reduced. Figure 5(b) shows that the rate of change of $\Delta f_T$ is much higher at lower altitudes, when compared to higher altitudes. This also shows that $\Delta f_T$ is higher at lower altitude. Therefore, it is very important that either spacecraft formation control or wireless power transmission synchronization methods be implemented into LEO SBSP system to avoid the efficiency loss due to both high phase angle difference and high $\Delta f_T$. Although higher altitude creates low $\Delta f_T$ and its variation rate, however, they suffer from transmission loss, as shown in Table II.

In addition, both Figures 5(a) and 5(b) show that $\Delta f_T$ and its variation rate only experience large changes in certain time periods, e.g., between 0 to 0.1 orbital period. The reason is that the spacecraft speed is higher than the earth rotation at all times. The spacecraft requires less than two hours to complete an orbit (or returns to its original position in Inertial frame), but the ground station requires 24 hours to return to its original location in Inertial frame. Therefore, there is only a certain time period when the spacecraft is within the desired FOV ($\phi < 30 \text{deg}$ or $\cos \phi < 0.866$) of the ground station. Figure 6 compares the average of absolute $\Delta f_T$ and $\Delta f_T$ taken over observation samples (in log10 scale) during the period of all spacecraft are within the ground station’s FOV. $\Delta \bar{f}_T$ and $\Delta \hat{f}_T$ are given as:

$$\Delta \bar{f}_T = \sum_{i=1}^{N} | \Delta f_T(iT) |$$

$$\Delta \hat{f}_T = \sum_{i=1}^{N} | \Delta \hat{f}_T(iT) |$$

where $N$ is the total number of sampling taken during a FOV period, $T$ is the sample period, and $| \cdot |$ denotes the absolute value.

Figure 6 shows that at higher altitudes (e.g., beyond 5000 Km), the variation of Doppler frequency is insignificant (in the order of 10 Hz) compared to the lower altitudes, e.g., 500 Km that is in the order of 1000 Hz. A small increment of
$\Delta f_T$ is observed when the altitude changes from 24000km to 30000km. This is because the $\Delta f_T$ is very small (or insignificant, $\Delta f_T \leq 10^{-4}$) in very high altitude. In addition, Figure 7 shows that the spacecraft only stays within the ground station’s FOV for less than 200 seconds at the semimajor axis of 7136.63km (orbital period is 6000 seconds) before the spacecraft becomes farther away from the ground station. Thus, Figure 7 confirms that spacecraft has a very high relative speed to the ground at low altitude. Also, the rate of change of $\Delta f_T$ approaches zero when the distance between spacecraft and ground station increases: as shown in (21), the rate of change of $\Delta f_T$ is inversely proportional to distance.

Figures 8(a) and 8(b) compare the $\Delta f_T$ and its variation rate between follower and leader spacecraft with respect to different formation sizes. Here, the relative distance of each spacecraft to the leader spacecraft at any time is similar to each other. Figure 8(a) shows that smaller formation size results in lower $\Delta f_T$ and its variation rate. In addition, the phase angle difference observed by the leader spacecraft is much lower (less than one degrees for 13/24km and less than four degrees for 428/473km formation). The results in Figures 9(a) and 9(b) compare the $\Delta f_T$ and its variation rate between transmitter spacecraft and leader spacecraft with respect to different formation size. Here, spacecraft 4 in Table III is considered the leader spacecraft. The overall relative distance between the follower spacecraft and the leader spacecraft is closer when compared to the overall relative distance between spacecraft 2 and the leader spacecraft. In
addition, the rate of change of relative distance between spacecraft 1 and the leader spacecraft is higher than the rate of change of relative distance between spacecraft 2 and the leader spacecraft. The results in both Figures 9(a) and 9(b) confirm that the and its variation rate are much higher for the non-symmetric relative distance case, if compared to the results in Figures 8(a) and 8(b). Furthermore, the phase angle difference observed by the leader spacecraft is approximately 10 times higher than the symmetric relative distance case, which represents a higher transmission loss. Therefore, it can be concluded that the position of the leader spacecraft in the spacecraft formation plays a major role in the impact of Doppler frequency difference between each spacecraft to the leader spacecraft.

B. Improved Transmission Techniques that Address Doppler Effect

Previous section studied the impact of Doppler spread on the microwave transmission’s frequency within spacecraft formation, and between the spacecraft and the ground station. The observed transmission frequency difference varies within few to hundred Hertz, depending on the altitude (or speed), and formation size. This results in a large difference in observed phase angle at the ground receiver, which greatly reduces the overall microwave transmission efficiency. On the other hand, a symmetric formation flying suffers the least transmission loss due to the Doppler effect. But the DC to RF and RF to DC conversion, and also the possible orbit drift due to the perturbation could further impact the microwave transmission efficiency. Here, two transmission improvement techniques can be considered to overcome the efficiency loss due to the Doppler effect:

1. Microwave transmission using frequency and phase adjustment; and,
2. Multiple rectenna installation at ground station that are operating over certain frequency bands.

These approaches are analyzed in the following subsections.

1. Microwave transmission using frequency and phase adjustment

Ref. [48] has presented a synchronization method in which each spacecraft transmits the microwave power to the receiver at different frequencies and initial phase angles. The receiver would observe the same frequency and phase angle from each spacecraft after the received wireless signal experiences the Doppler effect. This study has shown that this method significantly improves the transmission efficiency. The proposed method in [48] requires a continuous change in both microwave transmission frequency and initial phase angle of the transmitter. This technique needs an extremely high sampling rate if the transmission frequency is high.

2. Multiple frequency rectenna at ground station

When one spacecraft enters the ground station’s FOV, a communication link is setup over a certain frequency band. Here, the rectennas which operate over different frequency bands are installed on the ground, and harvesting the energy transmitted by spacecraft at the assigned frequency band. Then, a ground of rectenna would only receive the microwave transmission from a specific spacecraft. The method avoids the efficiency loss due to the Doppler effect and different phase angle. In addition, the wideband cross dipole rectenna with a dual polarization in [49], which has a greater effective frequency range can be utilized as the rectenna in this method. However, it is noted that each rectenna is expected to be more than 1km in diameter. Therefore, a larger space area is required by the ground station if more spacecraft microwave transmissions are required to be simultaneously received.

Based on the presented two methods, a simplified wireless power transmission efficiency for each method corresponds to [50]:

$$P_{R1,TOTAL} = M \eta_{dop} \eta_t \frac{P_t G_t}{L}$$

$$P_{R2,TOTAL} = \eta_{com} \sum_{k=1}^{M} \eta_t \frac{P_{t,k} G_{t,k} G_{r,k}}{L}$$

where $M$ is given in (14), $\eta_{dop}$ is the doppler correction efficiency and $\eta_{com}$ is the multiphase power combine efficiency.

Now, consider the scenario that 19 LEO spacecraft at the altitude of 700km are within the ground station FOV. Similar to Section II, we assume $\eta_t = 95\%$, $\eta_t = 80\%$, $P_t = 90$dB and $G_t = 60$dBi for both methods. The ground rectenna for first method has the radius of 1km, which $G_r = 89.66$dB, and the ground rectenna for the second method, based on the circle packing method [51], has the radius of 518m, which $G_r = 84.23$dB. This allows the rectenna in both methods occupy similar total area on the ground. By assuming that $\eta_{dop} = 80\%$ and $\eta_{com} = 95\%$, we have $P_{R1,TOTAL} = 2.1$GW and $P_{R2,TOTAL} = 0.72$GW. Therefore, the first method would offer a higher transmission efficiency if the Doppler shift can be accurately estimated. If the second method is desirable due to the fact that the spacecraft positioning accuracy is lower than expected, then rectenna size should be increased up to 75% to achieve the same power at the ground station. The ground rectenna size should be increased by three time to receiver the same power.

Two different methods to overcome the microwave transmission efficiency loss due to Doppler effect have been briefly presented. Each method has its own advantages and disadvantage. The accuracy of the spacecraft positioning, the sampling rate of the oscillator, and the total available area occupied by the ground station are among the major factor to decide the optimum microwave transmission method. It is expected that with the accelerated rate of development of high speed processors and communication systems, the first method would offer a more realistic and efficient method.

VI. CONCLUSION

In this paper, simulations are conducted to investigate the impact of spacecraft altitude and relative distance between spacecraft on the Doppler frequency variation between spacecraft and ground station, and between two spacecraft. Studies
show that the SBSP wireless power transmission would experience a higher Doppler frequency variation at LEO, when the multiple spacecraft is within the ground station’s field of view. Although higher altitude leads to lower Doppler frequency variation and longer time period of field of view; however, high transmission path loss should be taken into account. It has been shown that the transmission path loss from LEO is much lower than the transmission from GEO, even though the GEO antenna size is at least 10 times larger than LEO antenna size.

Moreover, studies show that larger formation size results in higher Doppler frequency variation for the SBSP wireless transmission within the spacecraft. In addition, the difference of relative distance and its rate of change between each spacecraft and the leader spacecraft impacts the variation of Doppler frequency difference. If the relative distance and variation rate between each spacecraft and the leader spacecraft are similar, the Doppler frequency and the observed phase angle difference is lower. Therefore, the position of the leader spacecraft in the spacecraft formation plays an important role on the impact of Doppler frequency and observed phase angle difference.

In this paper, the Doppler frequency difference between two spacecraft and a ground station (or leader spacecraft) were investigated. This study can also be applied to three or more spacecraft scenarios. Here, the Doppler frequency of one spacecraft to the ground station (or leader spacecraft) is considered as a reference Doppler frequency. Then, the Doppler frequency difference of other spacecraft is investigated based on the reference Doppler frequency. Several improvement techniques, and their advantages and disadvantages have been discussed in the paper. The major concerns of these techniques include the accuracy of spacecraft position and velocity estimation, the oscillating sampling rate, and the total area required by the rectenna. In general, the results of this paper can be used for optimal orbit design and high performance transmission of power.

REFERENCES


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