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<th>Novel frequency-domain oversampling receiver for CP MC-CDMA systems</th>
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Abstract—The multi-carrier code division multiple access (MC-CDMA) technique is a promising technology because it incorporates the benefits of the orthogonal frequency division multiplexing (OFDM) and the direct sequence code division multiple access (DS-CDMA) techniques. The bit error rate (BER) of the MC-CDMA system reduces more quickly than that of the OFDM system when increasing the transmission power. In this paper, we propose a novel frequency-domain oversampling (FDO) minimum mean square error (MMSE) receiver for the cyclic prefix (CP) MC-CDMA. Simulation results show that the proposed FDO based receiver for the CP MC-CDMA systems can outperform the traditional zero forcing (ZF) and MMSE equalizers in terms of BER performance for all the spreading factors (SF).

Index Terms—Multi-carrier CDMA, frequency domain oversampling, MMSE detection.

I. INTRODUCTION

The multi-carrier code division multiple access (MC-CDMA) technique has been first proposed in [1]-[2], then studied by many researchers. It has inherited the merits of the orthogonal frequency division multiplexing (OFDM) and code division multiple access (CDMA) techniques by combining the direct sequence (DS) CDMA and OFDM, and is becoming one of the candidate technologies for the 4th generation wireless communication systems [3]. There are several kinds of combining equalizer that can be used for the MC-CDMA systems including the minimum mean square error (MMSE) combining, the equal gain combining (EGC), the maximum ratio combining (MRC), the controlled equalization (CE), the orthogonal restoring combining (ORC) and the partial combining (PC) [4]-[7]. It has been shown that the MMSE combining can provide a better performance than other combining techniques.

The frequency-domain oversampling technique for OFDM systems has been studied in some literatures. In [9], the authors have adopted the frequency-domain oversampling (FDO) method for the zero padded (ZP) OFDM in underwater acoustic communications. They have assumed that the data subcarriers are separated from the pilot subcarriers by at least two null subcarriers [8]. Based on it, they have proposed the FDO based MMSE and the ZF receiver to improve the system performance over an underwater acoustic channel with a large Doppler spread. In [9], they have combined the FDO method with the interference subtraction to remove the inter-block interference caused by asynchronous multiuser transmissions.

The works in [10]-[11] have introduced the MMSE equalization for the ZP OFDM systems by using the complete OFDM symbols including the guard interval. The authors in [12] have proposed a FDO based MMSE equalizer for ZP MC-CDMA systems. Although these FDO based MMSE equalizers have better BER performance than the conventional equalizer, they are only suitable for the ZP based multi-carrier modulation (MCM) systems such as the ZP OFDM and the ZP MC-CDMA systems. As far as we know, the FDO MMSE equalizer is never studied for CP based MCM systems. In this paper, we propose a novel MMSE based receiver for CP MC-CDMA systems by using the FDO and the symbol reconstruction techniques. The proposed receiver consists of two stages. At the first stage, the received OFDM symbols are equalized by the conventional receiver. At the second stage, the CP and the tail parts of the current OFDM symbols will be reconstructed by eliminating the inter-symbol interference (ISI) from the previous and the subsequent symbols. The re-constructed OFDM symbols, including the CP and the tail parts, will be oversampled in the frequency domain and equalized by using the proposed MMSE equalizer. Simulations show that the proposed FDO with MMSE equalizer can achieve a better performance than the traditional ZF and MMSE receivers. To the best of our knowledge, this is the first piece of work in the world to combine the OFDM symbol reconstruction with the FDO scheme to be used in the CP MC-CDMA systems. This approach enables the MMSE equalizer to more efficiently exploit the whole OFDM symbol including the CP part. Hence, the system BER performance over the frequency selective channels could be much improved.

The rest of this paper is organized as follows. Section II reviews the structure of traditional CP MC-CDMA system with conventional equalization. Section III describes the proposed two-stage MC-CDMA receiver. Simulation results in Section IV demonstrate the performance of the proposed FDO based receivers. We draw our conclusions in Section V.

II. SYSTEM MODEL FOR CP MC-CDMA

The block diagram of a synchronous CP MC-CDMA system with \( N \) subcarriers and \( N \) users in downlink is shown in Fig. 1. The information data is \( \mathbf{d} = [d_0 \ d_1 \ldots d_{N-1}]^T \) and \( \mathbf{F}_N \) is the \( N \)-point discrete Fourier transform (DFT) matrix. The elements \( F_{n,m} \) of the matrix \( \mathbf{F}_N \) are defined by \( F_{n,m} = e^{-j2\pi nm/N} / \sqrt{N}, \) where \( n, m = 0,1,\ldots, N-1 \). The matrix \( \mathbf{F}_N^{-1} \) for the modulation of the CP part with length \( G \) is the last \( G \) rows of \( \mathbf{F}_N^{-1} \), which is the inverse DFT matrix, i.e. the inverse
of $F_N$. The transmitted signal is defined as:

$$S = [S_0 \, S_1 \ldots \, S_{N+G-1}]^T = \begin{bmatrix} \mathbf{F}_{N}^{-1} \\ \mathbf{F}_{P}^N \end{bmatrix} \mathbf{C} \mathbf{P} \mathbf{d},$$

(1)

where $\mathbf{C} = [c_0 \, c_1 \ldots \, c_{K-1}]$ is the normalized spreading code satisfying $\mathbf{C}^H \mathbf{C} = \mathbf{I}_K$, $\mathbf{I}_K$ is a $K \times K$ identity matrix, $\mathbf{d} = [d_0 \, d_1 \ldots \, d_{K-1}]^T$ and $\mathbf{P} = \text{diag}\{\sqrt{P_0}, \sqrt{P_1}, \ldots, \sqrt{P_{K-1}}\}$ represent modulated data symbols and transmission power for $K$ users respectively. For simplicity, we assume that the number of users is equal to $N$ (i.e., $K = N$). For spreading factor (SF) $L$, $\mathbf{C}$ will be a block diagonal matrix and each block $\mathbf{C}_k$ on the diagonal consists of $L$ orthogonal spreading codes, i.e., $\mathbf{C}_{L}^H \times \mathbf{C}_{L} = \mathbf{I}$.

The quasi-static multipath channel with $Q$ paths is applied. The equivalent channel impulse response is modeled as (2).

$$h(\tau) = \sum_{q=0}^{Q-1} h_q \delta[\tau - \tau_q] \quad \text{and} \quad \sum_{q=0}^{Q-1} E[|h_q|^2] = 1.$$  

(2)

In the discrete time domain, we can assume that the delays $\tau_q = q$ is an integer from 0 to $Q-1$, which represents a multipath of the sample period. Hence, the discrete channel is $\mathbf{h}_Q = [h_0 \, h_1 \ldots \, h_{Q-1}]^T$. The frequency response of the channel $H$ can be obtained by taking an $N$-point DFT on $\mathbf{h}_Q$ after $N$-Q zeros are padded to $\mathbf{h}_Q$.

The received one OFDM symbol $y_M = [y_0 \, y_1 \ldots \, y_{M-1}]^T$, where $M=N+G+Q$, can be represented by the convolution of equivalent channel impulse response and the transmitted signals $S$ as follows,

$$y_M = \mathbf{A}_M(\varepsilon) \mathbf{h}_M \begin{bmatrix} \mathbf{F}^{-1}_{P} \\ \mathbf{F}^N \end{bmatrix} \mathbf{C} \mathbf{P} \mathbf{d},$$

(3)

where $\mathbf{h}_M$ is an $M \times M$ circular matrix with the first column $\mathbf{h} = [h_0 \, h_{Q-1} \ldots \, 0]^T$. $\mathbf{0}_Q$ is a $Q \times N$ zero matrix. $\varepsilon$ is the residual carrier frequency offset (CFO) between the transmitter and receiver after frequency offset compensation. The diagonal matrix $\mathbf{A}_M(\varepsilon) = \text{diag}\{1, e^{j2\pi q/N}, e^{j2\pi 2q/N}, \ldots, e^{j2\pi (M-1)/N}\}$ represents the effects of the residual CFO. Thus, at the receiver side, the transmitted OFDM symbol overlaps in the guard interval after passing through a multipath channel. The received signals can be represented as

$$r_{k,n} = \begin{cases} y_{k,n} + y_{k-1,n+G} + v_n, & 0 \leq n < G \\ y_{k,n} + v_n, & G \leq n < N + G \end{cases}$$

(4)

where $y_{k,n}$ denotes the $k$-th OFDM symbol before overlap and $r_{k,n}$ denotes the received $k$-th OFDM symbol after a multipath and additive white Gaussian noise (AWGN) channel. $\mathbf{v} = [v_0 \, v_1 \ldots \, v_{N+G-1}]^T$ denotes zero mean Gaussian noise with a variance of $\sigma^2$. For the conventional MMSE receiver in [5]-[6], the CP part is removed to eliminate the ISI. After an $N$-point DFT, in the frequency domain, the $N \times 1$ information data $\mathbf{R}_N = [R_0 \, R_1 \ldots \, R_{N-1}]^T$ can be expressed by

$$\mathbf{R}_N = \mathbf{H}(\varepsilon) \mathbf{F}_N \mathbf{A}_N(\varepsilon) \mathbf{F}_N^{-1} \mathbf{C} \mathbf{P} \mathbf{d} + \mathbf{V}_N,$$

(5)

where $\mathbf{H}(\varepsilon)$ is a diagonal matrix with $H = \mathbf{F}_N \mathbf{A}_N(\varepsilon) \mathbf{h} = [H_0(\varepsilon) \ldots H_{N-1}(\varepsilon)]^T$ on the main diagonal, and $\mathbf{V}_N = \mathbf{F}_N \mathbf{V}_N = \mathbf{F}_N [v_0 \, v_1 \ldots \, v_{N-1}]^T$ denotes zero mean Gaussian noise with variance $\sigma^2$ in the frequency domain. $H_n(\varepsilon)$ is the channel in frequency domain with offset $\varepsilon$. Since the CFO cannot be completely compensated and the residual CFO $\varepsilon$ is unknown to the receiver, it can be assumed that $\varepsilon=0$ for the subsequent analyses. The residual CFO has been introduced in the simulation for the BER performance comparison. Thus, the conventional MMSE equalizer can be expressed by

$$\hat{\mathbf{d}} = (\mathbf{HCP})^H (\mathbf{HCP} \mathbf{\Psi} \mathbf{P}^H \mathbf{C}^H \mathbf{H}^H + \mathbf{I} \sigma^2)^{-1} \mathbf{R}_N,$$

(6)

where $\mathbf{\Psi} = E[dd^H]$ and $(.)^H$ denotes the conjugate transpose operation. For the quadrature phase shift key (QPSK) modulation, $\mathbf{\Psi} = \mathbf{I}$. The ZF equalizer can be achieved by setting $\sigma^2=0$ in (6).

### III. PROPOSED FDO BASED MMSE RECEIVER

Since the CP is removed for the ISI elimination in the conventional MC-CDMA receiver and the energy of the CP is wasted, if the received symbol with CP is oversampled in the frequency domain, the total useful signal energy will be increased and spreads over all fractional spaced subcarriers, which will be beneficial to the data recovery from a multipath channel. However, the CP part cannot be directly used due to the ISI from the preceding symbol. So we have an idea to reconstruct the CP part for the ISI elimination. Hence, for the proposed FDO receiver, we can firstly reconstruct the received symbol including the CP to remove the ISI by using the result of conventional equalization. Then, it can be followed by oversampling the reconstructed symbol in the frequency domain. Finally, the MMSE in the oversampled frequency domain will be adopted to estimate the transmitted data.

#### A. The Proposed Receiver

The detailed procedures of proposed receiver are shown as follows.
1) Perform conventional MMSE equalization and estimate the transmitted symbol \( \{ \hat{y}_k \}_{n=0}^{M-1} \). In order to achieve a better performance, the equalized data \( \hat{d} \) will be reversely mapped first to get the estimated binary data, and then map the estimated binary data again to the modulated data \( d \) to reduce the noise power. Substituting \( d \) for \( d \) in (1) and combining with (3), the estimated data of \( k \)-th symbol \( \{ \hat{y}_k \}_{n=0}^{M-1} \) can be obtained in the time domain.

2) Reconstruct the \( k \)-th OFDM symbol \( \{ \hat{y}_k \}_{n=0}^{M-1} \) to reduce the ISI. We have

\[
\hat{y}_{k,n} = \begin{cases} 
  r_{k,n} - \hat{y}_{k-1,n} + N - G, & 0 \leq n < G \\
  r_{k,n}, & G \leq n < N + G \\
  r_{k,n} - \hat{y}_{k+1,n} - N - G, & N + G \leq n < M 
\end{cases}
\]  

(7)

3) Oversample the new reconstructed \( k \)-th symbol \( \{ \hat{y}_k \}_{n=0}^{M-1} \) in the frequency domain (i.e., the DFT size is \( M \)) and then retrieve the transmitted data \( d \) of the \( k \)-th symbol by using the proposed FDO MMSE equalizer in section III. B. The retrieved transmitted data \( d \) shall be more accurate than the result in step 1). For the CP MC-CDMA system, it is unnecessary to repeat the step 2) and 3) for more iterations because it can quickly converge at the first iteration.

B. FDO-MMSE Equalizer

If we oversample the reconstructed symbol in the frequency domain by using an \( M \)-point DFT of \( \{ \hat{y}_k \}_{n=0}^{M-1} \) in (7) with the assumption of \( \epsilon = 0 \), since \( \epsilon \) is unknown, the symbol in the frequency domain can be represented as

\[
Y = F_M h^*_M \begin{bmatrix} F^{-1}_N & \text{CP}d + F_M \nu + F_M w \\ 0_Q \end{bmatrix} 
= H_M ACPd + F_M \nu + F_M w, 
\]

where \( F_M \) denotes an \( M \times M \) DFT matrix, \( \nu \) denotes zero mean Gaussian noise with variance \( \sigma^2 \), \( H_M \) is a diagonal matrix with oversampled channel \( H = F_M h \) on the main diagonal. We define \( A = F_M \begin{bmatrix} F^{-1}_N & \text{CP}d + F_M \nu + F_M w \\ 0_Q \end{bmatrix} \) and \( w = \begin{bmatrix} r_{k,Q} - \hat{y}_{k-1,Q} \\ 0_Q \end{bmatrix} \), where \( A \) is a pre-defined matrix, \( w \) denotes the ISI from the adjacent symbols where \( y_{k-1,Q} \) and \( y_{k+1,Q} \) are the estimation of \( y_{k-1,Q} \) and \( y_{k+1,Q} \), respectively. \( y_{k-1,Q} \) and \( y_{k+1,Q} \) denotes the last \( Q \) points of the \( k-1 \)-th symbol and \( y_{k+1,Q} \) denotes the first \( Q \) points of the \( k+1 \)-th symbol. Since the number of nonzero elements of \( w \) is only \( 2Q \), for the simplicity of the FDO based equalizers, it can be assumed that the ISI is zero, i.e., \( w = 0 \). For a linear equalizer, the transmitted data \( d \) of the \( k \)-th symbol can be estimated from the \( Y \) by multiplying an optimal weight matrix \( W_H \), i.e.

\[
d = W_H Y, 
\]

(9)

where \( Y \) is estimated from (7) by taking an \( M \)-point DFT and can be represented as (8) mathematically. The optimal linear MMSE equalizer can be derived according to the principle of orthogonality, i.e., the estimation error \( d - \hat{d} \) shall be orthogonal to the known random variable \( Y \). i.e. \( E((d - \hat{d})Y^H) = 0 \), where \( E() \) denotes expectation. Defining \( \Phi = H_M ACP \), the FDO MMSE equalizer \( W_H \) can be expressed as

\[
W^H = \Phi^H(\Phi \Phi^H + I \sigma^2)^{-1}, 
\]

(10)

where \( ( )^+ \) represents the Moore-Penrose pseudo-inverse that is used to replace the inverse of the matrix. Since the oversampling operation with the oversampling rate of \( M/N \) leads to an over-determined system due to \( M > N \), the matrix \( \Phi \Phi^H + I \sigma^2 \) is rank-deficient and is not invertible. \( H_M \) in \( \Phi \) can be replaced with the estimated channel. For the simplicity of the analysis, it is assumed that the transmission power will not be changed and \( P \) equals to \( I \). To reduce the complexity, the equalizer in (10) can be rewritten as (11) to reduce the size of the matrix to be inverted from \( M \times M \) to \( N \times N \).

\[
W^H = (\Phi^H \Phi + I \sigma^2)^{-1} \Phi^H, 
\]

(11)

which can be verified by using the singular value decomposition of \( \Phi \) in both (10) and (11).

The FDO technique enables the MMSE equalizer to more efficiently exploit the received signal and channel state information in the high dimensional space and greatly suppress the interference, hence to improve the system BER performance for the frequency selective channels. However, the complexity of implementation could be high due to the computation of matrix inverse with a large DFT size. The FDO-MMSE equalizer is valuable for the applications where the real-time processing is not highly required.

IV. SIMULATION RESULTS AND DISCUSSION

In order to prove our design, we have conducted simulations, where it is assumed that the synchronous MC-CDMA in downlink has \( N = 64 \), \( K = 64 \), \( G = 8 \), QPSK modulation, and Walsh-Hadamard spreading codes. And the quasi-static multipath channel has \( Q = 8 \) linearly decaying multipath components with normalized delays 0, 1, 2, 3, 4, 5, 6, 7 and powers 8/36, 7/36, 6/36, 5/36, 4/36, 3/36, 2/36, 1/36. The total power of the multipath components is 1. Each component follows a complex normal distribution. And the channel coefficients without frequency offset information can be perfectly estimated. The received signals are reversely mapped with a hard decision.

Fig. 2 shows the BER performance of the conventional ZF, the MMSE and the proposed FDO-MMSE receivers. It can be found that the FDO based MMSE receiver for CP MC-CDMA has much better BER performance than those of the conventional ZF and MMSE equalizers at all the SNR levels, and is even slightly better than the FDO-MMSE for ZP MC-CDMA (derived from the algorithms in [11] and [12]). If the guard interval increases from \( N/8 \) to \( N/4 \), the BER of the proposed FDO based MMSE equalizer can be further reduced. The reason is that the information of the guard interval can be fully exploited by the proposed equalizer. Longer the guard interval can offer the better BER performance.

Fig. 3 shows the BER performance over a multipath channel with a residual carrier frequency offset \( \varepsilon = 0.02 \), which is 2% of the subcarrier spacing reasonable for a conventional frequency offset compensation algorithm. The proposed FDO based MMSE equalizer can still offer a better BER performance than
the conventional ZF and the MMSE equalizers at all the SNR levels.

Fig. 4 shows the BER performance of the equalizers with different SFs. The conventional MMSE equalizer can benefit from a large SF. It has the best BER performance when the SF is equal to 64 and has the similar BER performance to the ZF equalizer when the SF is 1. But the proposed FDO based MMSE equalizer can obtain better BER performance than the conventional MMSE equalizer for all SFs. The BER improvement is becoming larger when the SF reduces from \( N \) to 1.

V. CONCLUSIONS

In this paper, we have proposed a novel FDO based MMSE receiver for the CP MC-CDMA systems over a multipath fading channel with a frequency offset. Compared with the conventional ZF and MMSE equalizers, the FDO based MMSE equalizer can offer a better BER performance at all the SNR levels. It can also be found that the BER performance of it benefits from the longer CP part. The proposed FDO based MMSE equalizer can improve the BER performance of the conventional MMSE equalizers with all SFs, and it has the largest BER improvement when SF is equal to 1.

REFERENCES