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Transport properties of a two-dimensional electron gas dressed by light

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(Received 9 December 2014; revised manuscript received 10 March 2015; published 29 April 2015)

We show theoretically that the strong interaction of a two-dimensional electron gas (2DEG) with a dressing electromagnetic field drastically changes its transport properties. Particularly, the dressing field leads to a giant increase of conductivity (which can reach thousands of percents), resulting in nontrivial oscillating dependence of conductivity on the field intensity, and suppressing the weak localization of 2DEG. As a consequence, the developed theory opens an unexplored way to control transport properties of 2DEG by a strong high-frequency electromagnetic field. From an experimental viewpoint, this theory is applicable directly to quantum wells exposed to a laser-generated electromagnetic wave.

DOI: 10.1103/PhysRevB.91.155312 PACS number(s): 73.63.Hs, 72.20.Ht, 72.10.—d

I. INTRODUCTION

Transport properties of a two-dimensional electron gas (2DEG) in nanostructures exposed to a high-frequency electromagnetic field have been studied in the deep past and taken a deserved place in textbooks (see, e.g., Refs. [1–3]). However, the most attention in previous studies on the subject was paid to the regime of weak light-matter interaction. Following the conventional terminology of quantum optics, in this regime an electron energy spectrum is assumed to be unperturbed by photons. Correspondingly, the weak electron-photon interaction results only in electron transitions between unperturbed electron states, which are accompanied by absorption and emission of photons. As a consequence, the regime of weak electron-photon interaction in solids leads to photovoltaic effects, high-frequency conductivity, and other well-known electronic transport phenomena which are accompanied by absorption of field energy by conduction electrons. However, the interaction between electrons and a strong electromagnetic field (the regime of strong light-matter interaction) cannot be described as a weak perturbation. In this case, the system “electron + electromagnetic field” should be considered as a whole. Such a bound electron-photon system, which was called “electron dressed by photons” (dressed electron), became a commonly used model in modern physics [4,5]. For a long time the main objects for studying the physical properties of dressed electrons were atoms and molecules. The field-induced modification of the energy spectrum and wave functions of dressed electrons—the so-called dynamic Stark effect—was discovered in atoms many years ago [6] and has been studied in detail in various atomic and molecular systems. These studies of strong electron-photon processes formed such an exciting field of modern physics as quantum optics [4,5]. In nanostructures the research activity in the area of quantum optics was focused on exciton-polaritonic effects in microcavities with quantum wells [7–13] and quantum dots [14–16], physical properties of dressed electrons in graphene [17–19] and quantum wires [20], a variety of technological applications [21], including novel types of the lasers [22,23], optical switches and logic gates [24,25], all-optical integrated circuits [26], and others. As to transport properties of dressed 2DEG, they are still waiting for detailed research. The present article is aimed to partially fill this interdisciplinary gap which takes place at the border between physics of nanostructures and quantum optics.

II. MODEL

For definiteness we will restrict our consideration to the case of 2DEG with a parabolic electron energy spectrum

\[ \varepsilon_k = \frac{\hbar^2 k^2}{2m}, \]  

where \( k \) is the electron wave vector, and \( m \) is the electron effective mass. Let the 2DEG be subjected to a plane monochromatic electromagnetic wave propagating perpendicularly to the 2DEG plane (see Fig. 1). In what follows, we will assume that the wave frequency \( \omega \) meets two conditions. First, the wave frequency is far from resonant electron frequencies corresponding to interband electron transitions and, therefore, the interband absorption of the wave by the 2DEG is absent. Second, the wave frequency is high enough in order to satisfy the inequality

\[ \omega \tau_0 \gg 1, \]

where \( \tau_0 \) is the electron relaxation time in an unirradiated 2DEG. It is well known that the intraband (collisional) absorption of wave energy by conduction electrons is negligibly small under condition (2) (see, e.g., Refs. [27–29]). Thus, the considered electromagnetic wave can be treated as a purely dressing (nonabsorbable) field. It follows from the basic principles of quantum optics that the strong coupling of electrons to such a dressing field leads to the renormalization of all physical quantities describing the electrons [4,5]. Particularly, it is well known that a high-frequency electromagnetic field can strongly affect the scattering of conduction electrons and change electronic transport characteristics [30,31]. Recently this approach was extended for the case of a two-dimensional electron gas subjected to a purely dressing field which cannot
be absorbed and emitted by conduction electrons [29]. Namely, the scattering probability of a dressed electron between electron states with wave vectors \( \mathbf{k} \) and \( \mathbf{k}' \) per unit time has the form [29]

\[
w_{\mathbf{k}\mathbf{k}'} = \frac{2\pi}{\hbar} J_0^2 \left( f_{\mathbf{k}\mathbf{k}'} \right) |U_{\mathbf{k}\mathbf{k}'}|^2 \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}}),
\]

where \( J_0(z) \) is the zeroth order Bessel function of the first kind, and \( U_{\mathbf{k}\mathbf{k}'} \) is the matrix element of the scattering potential \( U(\mathbf{r}) \), which arises from a macroscopically large number of scatterers in the conductor. In the case of a linearly polarized dressing field [29], the argument of the Bessel function is given by

\[
f_{\mathbf{k}\mathbf{k}'} = \frac{e E_0 (\mathbf{k} - \mathbf{k}')}{m \omega^2}, \quad \text{(4)}
\]

where \( E_0 \) is the amplitude of the electric field of the wave, \( E_0(t) = E_0 \sin \omega t \). In the case of a circularly polarized dressing field (see the Appendix), this argument is

\[
f_{\mathbf{k}\mathbf{k}'} = \frac{2 E_0 e \mathbf{k}}{m \omega^2} \sin \left( \frac{\theta_{\mathbf{k}\mathbf{k}'}}{2} \right), \quad \text{(5)}
\]

where \( \theta_{\mathbf{k}\mathbf{k}'} = \left( \mathbf{k}, \mathbf{k}' \right) \) is the angle between electron wave vectors \( \mathbf{k} \) and \( \mathbf{k}' \). The formal difference between the scattering probability of a dressed electron (3) and the conventional expression for the scattering probability of a bare electron [32] consists of the Bessel-function factor \( J_0^2 \left( f_{\mathbf{k}\mathbf{k}'} \right) \), where \( f_{\mathbf{k}\mathbf{k}'} \) depends on the dressing field amplitude \( E_0 \) and the dressing field frequency \( \omega \) [see Eqs. (4) and (5)]. Just this factor results in nontrivial dependence of electronic transport properties on \( \omega \) in nontrivial dependence of electronic transport properties on the dressing field. In what follows, we will focus our attention on the conductivity and the weak localization of dressed 2DEG.

### III. Conductivity of Dressed 2DEG

Assuming the temperature to be zero, let us apply a stationary (dc) electric field \( \mathbf{E} \) to the 2DEG. It follows from the conventional Boltzmann equation for conduction electrons (see, e.g., Ref. [33]) that electric current density \( \mathbf{J} \) is given by the well-known expression

\[
\mathbf{J} = \frac{e^2}{2\pi^2} \int_{\mathbf{k}} \left[ \mathbf{E} \cdot \mathbf{v}(\mathbf{k}) \right] \tau(\mathbf{k}) \mathbf{v}(\mathbf{k}) \delta(\varepsilon_{\mathbf{k}} - \varepsilon_F) d^2\mathbf{k},
\]

where \( \mathbf{v}(\mathbf{k}) = (1/\hbar) \nabla_{\mathbf{k}} \varepsilon_{\mathbf{k}} \) is the electron velocity, \( \varepsilon_F = \hbar^2 k_F^2 / 2m \) is the electron Fermi energy, and \( \tau(\mathbf{k}) \) is the relaxation time. In the most general case of anisotropic electron scattering, this relaxation time is given by the equation [34]

\[
\frac{1}{\tau(\mathbf{k})} = \sum_{\mathbf{k}} \left[ 1 - \frac{\tau(\mathbf{k}) |\mathbf{E} \cdot \mathbf{v}(\mathbf{k})|}{\tau(\mathbf{k}) |\mathbf{E} \cdot \mathbf{v}(\mathbf{k})|} \right] w_{\mathbf{k}\mathbf{k}}.
\]

Since absorption of a high-frequency field satisfying inequality (2) is negligibly small, the field does not change the equilibrium distribution function of electrons [29]. Therefore, the field influences on the stationary (dc) transport of dressed electrons only through the renormalization of the scattering probability \( w_{\mathbf{k}\mathbf{k}} \) given by Eq. (3). It should be stressed that condition (2) is crucial in order to consider a 2DEG subjected to the field as an equilibrium system. Otherwise, the photon-assisted scattering of electrons is accompanied by absorption of field energy and leads to heating 2DEG. In this case, there are no stationary transport characteristics of the 2DEG and the problem should be reformulated in terms of the nonequilibrium electron transport (see, e.g., Refs. 30, 31).

Substituting the scattering probability of dressed electron (3) into Eq. (7), we can obtain from Eqs. (6) and (7) the conductivity of dressed 2DEG, \( \sigma_{ij} = J_i / E_j \).

To simplify calculations, let us consider the electron scattering within the \( s \)-wave approximation [32], where the matrix elements \( U_{\mathbf{k}\mathbf{k}'} \) do not depend on the angle \( \theta_{\mathbf{k}\mathbf{k}'} = \left( \mathbf{k}, \mathbf{k}' \right) \). Substituting Eq. (5) into Eqs. (3), (6), and (7), we arrive at the isotropic conductivity of 2DEG dressed by circularly polarized field \( \sigma_{ij} \), which is given by the expression

\[
\frac{\sigma_{ij}}{\sigma_0} = 2\pi \left[ \int_0^{2\pi} \left[ 1 - \cos \theta \right] J_0^2 \left( \frac{2e E_0 k_F}{m \omega^2} \sin \frac{\theta}{2} \right) d\theta \right]^{-1},
\]

where \( \sigma_0 \) is the conductivity of an unirradiated 2DEG. In the case of a linearly polarized dressing field, the conductivity tensor \( \sigma_{ij} \) has a simple diagonal form \( \sigma_{ii} = (\sigma_{\parallel}, \sigma_{\perp}) \) in the basis of two axes which are parallel (\( \parallel \)) and perpendicular (\( \perp \)) to the wave field \( \mathbf{E} \), respectively. Substituting Eq. (4) into Eqs. (3), (6), and (7), we arrive at

\[
\frac{\sigma_{\parallel}}{\sigma_0} = \frac{1}{\pi} \int_0^{2\pi} \frac{\tau_\parallel(\theta)}{\tau_0} \cos^2 \theta d\theta,
\]

and

\[
\frac{\sigma_{\perp}}{\sigma_0} = \frac{1}{\pi} \int_0^{2\pi} \frac{\tau_\perp(\theta)}{\tau_0} \sin^2 \theta d\theta,
\]

where \( \tau_\parallel(\theta) \) and \( \tau_\perp(\theta) \) are the relaxation time (7) at the Fermi energy for the dc electric field \( \mathbf{E} = (E_0, 0) \) and \( \mathbf{E} = (0, E_\perp) \), respectively [see Fig. 1(a)], and \( \theta = \left( \mathbf{k}, \mathbf{E} \right) \).

The conductivities of irradiated 2DEG [Eqs. (8)-(10)], which can be easily calculated numerically, are plotted in Fig. 2. It is seen that the irradiation of 2DEG by a dressing light results in a giant increase of conductivity, which can reach thousands of percents. Physically this increase follows from the fact that the dressing field significantly decreases the scattering probability of dressed 2DEG (3). From the mathematical viewpoint, this is a consequence of the rapid decrease of the Bessel function in Eq. (3) versus the dressing field amplitude \( E_0 \). As to the oscillations of the plotted conductivity, this is a formal consequence of the oscillating behavior of the Bessel function in the probability (3). In order to give a qualitative physical explanation of this behavior of conductivity, we have to stress that the Born scattering
The frequency quantum well irradiated by a dressing electromagnetic field with the well-known weak localization (WL) of conduction electrons manifesting itself as the self-interference of electron waves propagating along a closed trajectory in mutually opposite—clockwise and counterclockwise—directions. As a result of the interference, the well-known weak localization (WL) of conduction electrons appears [35–37]. To find the WL correction to the conductivity of dressed 2DEG, \( \Delta \sigma \), one needs to consider all possible closed electron trajectories in the 2DEG irradiated by a dressing electromagnetic field. Going this way, the sought correction can be calculated using the conventional expression [35]

\[
\frac{\Delta \sigma}{\sigma_0} = \frac{\hbar}{m} \int_{t_F}^{t_F + \tau_F} C(t) dt,
\]

where \( C(t) \) is the probability of finding an electron in the initial point of its trajectory at time \( t \) averaged over the 2DEG plane, \( \tau_F \) is the mean free time of conduction electron at the Fermi energy, which is given by the expression

\[
\frac{1}{\tau_F} = \sum_{k_p} w_{k_p},
\]

and \( \tau_p \) is the effective breaking time of the electron phase. Generally, the electron phase can be broken by both inelastic scattering and a magnetic field which destroy WL [38–43]. The breaking time of the electron phase caused by a magnetic field \( B \) is \( \tau_{pB} = \hbar/4eDB \), where \( D \) is the diffusion constant [43]. Correspondingly, the effective breaking time of the electron phase \( \tau_p \) can be written as \( 1/\tau_p = 1/\tau_{pe} + 1/\tau_{p\perp} \), where \( \tau_{pe} \) is the characteristic breaking time arisen from inelastic processes. As to the mean free time of conduction electron \( \tau_F \), it can be found by substituting the scattering probability of dressed electron (3) into Eq. (12). It has been mentioned above that this scattering probability is anisotropic for 2DEG dressed by a linearly polarized field. Therefore, the time (12) depends on the initial electron wave vector \( \mathbf{k}_F \). To describe this anisotropy, it is suitable to introduce the two mean free times \( \tau_{F\parallel} \) and \( \tau_{F\perp} \) for the cases of \( \mathbf{k}_F \parallel \mathbf{E} \) and \( \mathbf{k}_F \perp \mathbf{E} \), respectively [see Fig. 1(a)]. Certainly, in the case of a circularly polarized dressing field, the anisotropy vanishes and the corresponding WL correction \( \Delta \sigma_c \) does not depend on direction in the 2DEG plane.

Within the conventional theory of WL, the probability \( C(t) \) in Eq. (11) can be described by the Feynman path integral [36, 44]

\[
C(t) = \int DxDy \exp \left[ - \int_{-\epsilon_t/2}^{\epsilon_t/2} \frac{\dot{x}^2}{4D_x} + \frac{\dot{y}^2}{4D_y} - \frac{i}{\hbar} [\mathbf{E}(t') - \mathbf{E}(-t')] dt' \right],
\]

where the integration should be performed over closed electron trajectories per unit plane of 2DEG, and \( \mathbf{r} = (x, y) \) is the electron radius vector in the 2DEG plane. In the integrand of the path integral (13), the first two terms in the exponent describe the kinetic energy of the diffusion propagation of a dressed electron along the path, where \( D_{x,y} \) are the diffusion constants along the \( x,y \) axes. Generally these constants can be written as \( D_x = v_F^x/2 \tau_{F\parallel} \) and \( D_y = v_F^y/2 \tau_{F\perp} \) (see, e.g., Ref. [43]), where \( v_F \) is the Fermi velocity, and the \( x \) axis is assumed to be directed along the electric field \( \mathbf{E} \) of a linearly polarized dressing field. In the case of a circularly polarized dressing field, we have \( \tau_{F\parallel} = \tau_{F\perp} \) and, therefore, the \( x, y \) axes
polarized field is within 1% and cannot be easily detected experimentally. Therefore, we have \( \Delta \sigma_{||} \approx \Delta \sigma_{\perp} \) for the plots in Fig. 3.

V. CONCLUSION

We demonstrated theoretically that the strong coupling of 2DEG to a dressing electromagnetic field results in a set of such unexpected transport phenomena as a giant increase of conductivity, oscillating behavior of conductivity versus intensity of dressing field, and suppression of weak localization effects. These phenomena open a way to control the transport properties of 2DEG by a strong high-frequency electromagnetic field and, therefore, form a basis for optoelectronic nanodevices.

ACKNOWLEDGMENTS

The work was partially supported by FP7 IRSES projects POLATER and QOCaN, FP7 ITN project NOTEDEV, Tier 1 project “Polaritons for novel device applications,” Rannis project BOFEHYSS, RFBR project 14-02-00033, and the Russian Ministry of Education and Science.

APPENDIX: SCATTERING OF 2DEG DRESSED BY A CIRCULARLY POLARIZED ELECTROMAGNETIC FIELD

For definiteness we will restrict our consideration to the case of a two-dimensional electron system subjected to a circularly polarized electromagnetic wave propagating perpendicularly to the system. Let the system lie in the plane \((x,y)\) at \(z = 0\) and the wave propagate along the \(z\) axis. Then the electron properties of the system in the absence of scatterers are described by the Schrödinger equation

\[
i \hbar \frac{\partial \psi}{\partial t} = \hat{H}_0 \psi, \tag{A1}\]

with the Hamiltonian \(\hat{H}_0 = [\hat{p} - e A(t)]^2/2m\), where \(\hat{p} = (\hat{p}_x, \hat{p}_y)\) is the operator of electron momentum, \(m\) is the effective electron mass, \(e\) is the electron charge,

\[
A(t) = e_x (E_{\omega}/\omega) \sin \omega t + e_y (E_{\omega}/\omega) \cos \omega t
\]

is the vector potential of the wave, \(E_{\omega}\) is the electric field amplitude of the wave, \(\omega\) is the frequency of the wave, and \(e_{x,y}\) are the unit vectors along the \(x, y\) axis. The Schrödinger equation (A1) can be solved accurately and the exact wave function of the electron is

\[
\psi_{\mathbf{r},t} = \exp \left[ -i \left( \frac{\varepsilon_{\mathbf{k}}}{\hbar} + \frac{E^2_{\omega} e^2 t}{2m \omega^2 \hbar^2} - \frac{E_{\omega} e k_x}{m \omega^2} \sin \omega t \right) \frac{E_{\omega} e k_y}{m \omega^2} \cos \omega t \right] \varphi_{\mathbf{k}}(\mathbf{r}), \tag{A2}\]

where \(\varphi_{\mathbf{k}}(\mathbf{r}) = V^{-1/2} \exp (i \mathbf{k} \cdot \mathbf{r})\) is the plane electron wave, \(\mathbf{k} = (k_x, k_y)\) is the electron wave vector, \(\mathbf{r} = (x, y)\) is the electron radius vector, \(V\) is the normalization volume, and \(\varepsilon_{\mathbf{k}} = \hbar^2 k^2/2m\) is the energy spectrum of a free electron. Evidently the wave function (A2) can be easily verified by direct substitution into the Schrödinger equation (A1). Introducing the polar system \(\{k, \varphi\}\), we can write the electron
wave vectors as $k_x = k \cos \varphi$ and $k_y = k \sin \varphi$. Then the wave function (A2) takes the simplest form

$$
\psi_{k}(r, t) = \exp \left[ -i \left( \frac{E_k t}{\hbar} + \frac{E_{k}^2 t^2}{2m\hbar} + \frac{E_{k} e^{i \varphi}}{\hbar} \cos(\omega t + \varphi) \right) \right] \varphi_{k}(r).
$$

Let an electron move in a total scattering potential $U(r)$ in the presence of the same field. Then the wave function of the electron $\Psi(r, t)$ satisfies the Schrödinger equation

$$
i\hbar \frac{\partial \Psi(r, t)}{\partial t} = \left[ \hat{H}_0 + U(r) \right] \Psi(r, t).
$$

Assuming the scattering potential energy $U(r)$ to be a small perturbation, we can apply the conventional perturbation theory to describe the electron scattering. Since the functions (A3) with different wave vectors $k$ form the complete function system for any time $t$, we can seek solutions of the Schrödinger equation (A4) as an expansion

$$
\Psi(r, t) = \sum \alpha_k(t) \varphi_k(r, t).
$$

Let an electron be in the state (A3) with the wave vector $k$ at the time $t = 0$. Correspondingly, $\alpha_k(0) = \delta_{k, k}$, where $\delta_{k, k}$ is the Kronecker symbol. In what follows, we will assume that the wave frequency $\omega$ is large enough to satisfy the inequality

$$\omega \tau_0 \gg 1,$$

where $\tau_0$ is the characteristic relaxation time of the conduction electron in the absence of the wave. Under the condition (A6), we can neglect the absorption (emission) of field energy by a scattered electron (see, e.g., Refs. [27, 28]). Within this approximation, the scattering potential $U(r)$ mixes only electron states $k$ and $k'$ with the same energy $E_k = E_{k'}$ [29]. Therefore, it is enough to take into account only terms with $k' = k$ in the expansion (A5). Substituting the expansion (A5) into the Schrödinger equation (A4) and restricting the accuracy by the first order of perturbation theory, we arrive at the expression

$$\alpha_{k}(t) = -i \frac{U_{k k}}{\hbar} \int_0^t e^{i(\omega - \omega')t/\hbar} f_{k k} e^{i(\varphi + \varphi')/2} dt'.
$$

where

$$
U_{k k} = \langle \varphi_{k}(r) | U(r) | \varphi_{k}(r) \rangle
$$

is the matrix element of the scattering potential,

$$
f_{k k} = \frac{2E_{k} e^{i \varphi}}{m\omega^2} \sin \left( \frac{\theta}{2} \right),
$$

and $\theta = \varphi - \varphi' = (k, k')$ is the scattering angle. Let us apply the Jacobi-Anger expansion,

$$e^{i\varphi} = \sum_{n=\infty} J_n(\varphi) e^{in\gamma},$$

in order to rewrite Eq. (A7) as

$$|\alpha_{k}(t)|^2 = \frac{|U_{k k}|^2}{\hbar^2} \sum_{n=\infty} J_n(f_{k k}) e^{i(\omega - \omega + n\hbar\omega)/2\hbar} \times e^{i(n\omega + \omega')/2} \left\| \int_{t/2}^{t/2} e^{i(\omega - \omega + n\hbar\omega)t/\hbar} dt' \right\|^2,
$$

where $J_n(z)$ is the $n$th order Bessel function of the first kind. Since the integrals in Eq. (A10) for long time $t$ turn into the $\delta$ function

$$\delta(\varepsilon) = \frac{1}{2\pi\hbar} \lim_{t \to \infty} \int_{-t/2}^{t/2} e^{i\varepsilon t/\hbar} dt',
$$

the expression (A10) takes the form

$$|\alpha_{k}(t)|^2 = 4\pi^2 |U_{k k}|^2 \sum_{n=\infty} J_n^2(f_{k k}) \delta(\varepsilon_k - \varepsilon_k + n\hbar\omega).
$$

To transform the square $\delta$ functions in Eq. (A11), we can apply the conventional procedure,

$$\delta^2(\varepsilon) = \delta(\varepsilon) \delta(0) = \frac{\delta(\varepsilon)}{2\pi\hbar} \lim_{t \to \infty} \int_{-t/2}^{t/2} e^{i\omega t} dt' = \frac{\delta(\varepsilon)}{2\pi\hbar}.
$$

Keeping in mind that $\varepsilon_k = \varepsilon_k$, the probability of the electron scattering between the states (A2) with the wave vectors $k$ and $k'$ per unit time, $w_{k k'} = d|\alpha_{k}(t)|^2/\hbar dt$, is given by

$$w_{k k'} = \frac{2\pi}{\hbar} J_0^2(f_{k k}) |U_{k k}|^2 \delta(\varepsilon_k - \varepsilon_k).
$$

Since Eq. (A9) depends only on the electron energy $\varepsilon_k = \hbar^2 k^2/2m$ and the scattering angle $\theta$, the probability (A12) describes the isotropic scattering which can be tuned by the wave amplitude $E_{\omega}$ and the wave frequency $\omega$.


