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Redundancy Based PMU Placement in State Estimation

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Abstract

An alternative method to obtain a PMU placement with maximum measurement redundancy is proposed in this paper. By considering the number of branches adjacent to each bus, the proposed method can be formulated in a linear and compact way. The effectiveness of the proposed method is shown by comparing its results with those of the existing methods (Two-step Method and Quadratic Programming Method) under the IEEE 14-, 30-, 57- and 118-bus test systems. Different scenarios, such as normal condition, contingencies and consideration of zero-injection buses, are also discussed. Moreover, a post-processing optimization is presented to improve the redundancy distribution. After the optimization, the number of red-1 buses, i.e. buses which are made observable one time only by the measurements, is reduced. In this way the redundancy becomes more evenly distributed and the state estimation is more robust. The short execution time of the post-processing optimization and its effectiveness are verified by using various IEEE test systems.

Index Terms: Observability, PMU placement, redundancy, state estimation.

1 Introduction

The Phasor Measurement Unit (PMU) is becoming widely used via continuous technological improvements ever since it was first introduced in 1980s [1]. By utilizing a uniform clock signal from GPS, PMUs are capable of providing measurements of phasor voltages as well as phasor currents, both magnitudes and angles, with a high degree of accuracy. With this advanced meter, power systems will undergo a great improvement in its operation, monitoring and control [2] [3].

Due to PMU's attractive advantages in measuring phasor angles, State Estimation (SE), the essential functional part in Energy Management System (EMS) will be enhanced both in its algorithm and results [4]. PMU measurements will make the SE algorithm linear and no iteration is needed in getting the solution [5]. In addition, PMUs are highly

accurate devices, which will make the SE results more reliable [5, 6]. Due to PMU's promising accuracy and features, its role in the SE module will be crucial. It is predicted that SE will rely more on PMUs in the future [1]. However due to their high cost, the number of PMUs that can be implemented is limited [8, 9]. Therefore an observability study on PMU placement is necessary prior to the deployment of PMUs.

When investigating the influence of PMUs on SE, the first problem encountered is where to deploy the measurements to make the network observable. And observability analysis is conducted for this purpose. A bus is claimed to be "observable" if the bus's *phasor voltage* is known either by direct measurements or by SE solutions. Research on observability analysis of PMU placement has been performed for many years. Currently the main methods include graph theory with simulated annealing, intelligent algorithm, binary search algorithm and binary integer programming [8-13].

Integer programming on observability analysis was first introduced in [12] and numerical results with a minimum number of PMUs can be obtained. The authors in [8, 9] further improved the observability analysis such that it becomes a linear model using Integer Linear Programming (ILP). Since integer optimization is a well-developed solver in many commercial software packages, ILP is found to be an efficient way to solve the observability based PMU placement problem. It mainly utilizes the connectivity matrix to judge whether the network is observable under a particular PMU placement scheme. The objective function is to minimize the number of PMUs placed in the network. Since the value of the integer variable in the problem is 0 or 1, this integer programming can also be named as Binary Integer Linear Programming (BILP). The drawback of this method is that only one solution can be obtained while an observability based PMU placement is usually a multiple-solution problem [14].

Since measurement redundancy greatly affects SE's accuracy, bad data detection and other important aspects of SE, improvements on the ILP method to solve the PMU placement problem are made to produce a solution not only for considering observability but also for measurement redundancy [13, 15, 16]. References [15, 16] present a binary integer programming technique and reference [13] is based on Genetic Algorithm which is less efficient.

Two important concepts used in this paper are reiterated here:

- Measurement redundancy

This concept reflects the independence degree of “an estimated value” on “the measured value”. Usually, the difference between the number of measurements, N_{meas} , and the number of states to be estimated, N_{state} , is used for calculating this redundancy index, i.e. $N_{meas} - N_{state}$. As N_{state} is a fixed value in a particular network topology, N_{meas} is generally used for calculating the measurement redundancy [13, 15, 16]. Therefore, the placement with the maximum measurement redundancy is the one that can provide the maximum number of measurements.

- Redundancy distribution

This concept is related with the redundancy level of a state, which can be calculated as the number of times a state can be estimated by the measurements. If the concept of “measurement redundancy” is focused on the entire system, the concept of “redundancy distribution” is meant for evaluating every single estimated value. The equation for calculating this index under PMU measurements is given in (1) of Section 2.

In this paper, the number of associated branches of each bus is incorporated as a priority variable in the objective function in the basic ILP model. The proposed formulation is simple and effective in yielding a PMU placement with maximum redundancy and minimum number of PMUs. A comparison is made with the studies reported in [15, 16] to show the validity of the proposed method under various case studies. After this, a post-processing optimization on redundancy distribution is suggested. The optimization process is efficient and improvements can be made to reduce the red-1 buses. The effectiveness is evaluated by comparing the results of BSA in [14].

The paper is organized as follows: Sections 2 will introduce the basic ILP model on PMU placement. Section 3 will discuss the proposed method to achieve a PMU placement with maximum measurement redundancy. Afterwards, a post-processing optimization to enhance the redundancy distribution of the placement strategy will be presented in Section 4. Section 5 will conclude the paper.

2 Basic ILP Based PMU Placement

2.1 Observability Analysis under Normal Conditions

As indicated above, for PMU buses, i.e. buses implemented with a PMU, both voltage magnitude and phase angle are known directly. For other buses, the observability depends on the PMU location and network topology. One concept

used in the observability analysis is “reached”, which means that the phasor voltage of a bus can be obtained by virtue of PMU measurements. Obviously, the PMU bus is directly “reached” by itself and its adjacent buses can also be “reached”. This brings forth a simple way to analyze the network observability, namely, judging the number of times that a bus is reached. If the number is equal to or is larger than one, the bus is observable. If the number is zero, it is unobservable. In this way, the observability analysis model can be built as [10]:

$$\begin{aligned} & \min \sum_{i=1}^{N_{bus}} \mathbf{x}(i) \\ & \text{s.t } \mathbf{RI}(i) = \sum_{j=1}^{N_{bus}} \mathbf{A}(i,j)\mathbf{x}(j) \geq \mathbf{1} \end{aligned} \quad (1)$$

where

N_{bus} Total number of buses.

i, j Bus number, $1 \leq i, j \leq N_{bus}$.

\mathbf{A} Connectivity matrix ($N_{bus} \times N_{bus}$) in which

$$\begin{cases} \mathbf{A}(i, j) = 1 & \text{bus } i \text{ is connected to bus } j. \\ \mathbf{A}(i, j) = 0 & \text{bus } i \text{ is not connected to bus } j, \text{ and} \\ \mathbf{A}(i, i) = 1 \end{cases}$$

$\mathbf{x}(i)$ State variable.

$$\begin{cases} \mathbf{x}(i) = 1 & \text{bus } i \text{ is equipped with a PMU; and} \\ \mathbf{x}(i) = 0 & \text{bus } i \text{ is not equipped with a PMU.} \end{cases}$$

$\mathbf{1}$ Unit vector, $[1, 1, \dots, 1]^T$

$\mathbf{RI}(i)$ Redundancy level of bus i .

2.2 Other Constraints in PMU Placement

An idea to deal with the contingency is to increase the number of times that a bus should be made observable, i.e. to increase the value of the right-hand side of the inequality of (1). In this way the observability under the contingency can be guaranteed.

1) Single-line Outages and Loss of PMUs:

Single-line outages will affect the observability of non-PMU buses whereas the loss of PMUs will influence the observability of both PMU and non-PMU buses. Therefore it can be pointed out that the loss of PMUs is more serious than the single-line outage from the angle of observability analysis for PMU placement problem. When contingencies are considered for the placement problem, the redundancy level in (1) can be changed from $\mathbf{1}$ to $\mathbf{2}$. Besides, the

expression given in (1) is for the same PMU cost. If unequal costs are to consider, the objective function, $\min \sum_{i=1}^{N_{bus}} \mathbf{x}(i)$, will be changed to $\min \sum_{i=1}^{N_{bus}} \mathbf{c}(i)\mathbf{x}(i)$, where $\mathbf{c}(i)$ is the installation cost for placing PMU at bus i . The installation cost considered here is assumed to include the communication infrastructure expenses which is dominant in the total cost to equip PMUs in the system.

2) Zero-injection Buses

Zero-injection can be treated as an additional measurement information and could help reduce the number of PMUs needed to make the system observable. To fully consider the influences of the zero-injection buses when placing PMUs, a bus reordering technique is needed. The permutation matrix \mathbf{P} is needed to merge the set of buses connected to the zero-injection bus. \mathbf{P} will change the connectivity matrix \mathbf{A} into \mathbf{A}_{new} , where $\mathbf{A}_{new} = \mathbf{P}\mathbf{A}$. The above technique and its detailed formulation of the matrices/vectors can be found as Augmented Bus Merging in [10]. After considering this, the constraint will be changed to:

$$\mathbf{R}\mathbf{l}(i) = \sum_{j=1}^{N_{bus}} \mathbf{A}_{new}(i,j)\mathbf{x}(j) \geq \mathbf{b} \quad (2)$$

where $\mathbf{b} = \mathbf{P} \cdot \mathbf{1}$

3 Proposed PMU Placement Strategy with Maximum Measurement Redundancy

Measurement redundancy is an important factor for PMU placement problems as discussed in the Introduction. As an improvement to the basic ILP model, a placement strategy with maximum redundancy will benefit the SE results greatly. In this section, a placement strategy is proposed to achieve the maximum measurement redundancy and the effectiveness will be compared with two typical methods: Two-step Method and Quadratic Programming Method. It can be pointed out that the Two-step Method is complicated and can only produce the results after two rounds of optimization. The Quadratic Programming Method is non-linear and thus less efficient when the problem becomes complex.

In the proposed PMU placement strategy, factors in the objective function include the installation cost of placing PMU at bus i , $\mathbf{c}(i)$ as suggested in Section 2 and the risk cost of placing PMU at bus i , $\mathbf{risk}(i)$ as shown in (3).

$$\min \sum_{i=1}^{N_{bus}} [\mathbf{c}(i) + \mathbf{risk}(i)]\mathbf{x}(i) \quad (3)$$

The ‘‘risk cost’’ used here is to evaluate the reliability of the SE solution by placing a PMU at bus i and it is related to the measurement redundancy. For different placement sites, the measurement redundancy differs since the number of

phasor current measurements related with the sites are different. Robust SE results are attributed by high measurement redundancy, therefore the risk cost of bus i in PMU placement regarding the SE results can be written as:

$$\mathbf{risk}(i) = 1/\mathbf{N}_{branch}(i) \quad (4)$$

where $\mathbf{N}_{branch}(i)$ is the number of branches associated with bus i , $i = 1, 2, \dots, N_{bus}$.

Four IEEE standard test systems, i.e. IEEE 14-, 30-, 57-, and 118-bus systems are used for the case studies. The information about each test system is shown in Table I and the network configuration data can be obtained from [17]. A comparison among the proposed, Two-step and Quadratic Programming methods is shown from Table II to IV under normal conditions, considering zero-injections and contingencies. The results shown in these tables are the same regarding the measurement redundancy and the number of PMUs¹. Hence the validity of the proposed method to obtain a placement with maximum redundancy and minimum number of PMUs can be verified.

Table I IEEE Test Systems Information

IEEE Test System	No. of Branches	No. of Zero injections	Zero injection buses
14	20	1	7
30	41	6	6,9,22,25,27,28
57	78	15	4,7,11,21,22,24,26,34,36,37,39,40,45,46,48
118	179	10	5,9,30,37,38,63,64,68,71,81

4 Post-processing Optimization to Reduce the Number of Red-1 Buses

4.1 Proposed Technique

As discussed in the Introduction, the “optimal redundancy distribution” is focused in this work to improve the redundancy level of each bus. In [14], the method used to balance the redundancy distribution is Binary Search Algorithm (BSA). To better illustrate this point, define the red- p buses as buses reached p times by PMUs, therefore the red-1 buses are referred as those reached only once by the PMU.

¹ An exception is in the IEEE 118-bus system under contingencies

Table II Number of PMUs Needed and Total Redundancy Results of Different Methods under Normal Conditions without Conventional Measurements

IEEE test system	Proposed Method			Two-step Method			Quadratic Method		
	Number of PMUs needed	Measurement Redundancy	Buses Needed to be Placed with PMU	Number of PMUs needed	Measurement Redundancy	Buses Needed to be Placed with PMU	Number of PMUs needed	Measurement Redundancy	Buses Needed to be Placed with PMU
14	4	19	2, 6, 7, 9	4	19	2, 6, 7, 9	4	19	2, 6, 7, 9
30	10	52	2, 4, 6, 9, 10, 12, 15, 18, 25, 27	10	52	2, 4, 6, 9, 10, 12, 15, 18, 25, 27	10	52	2, 4, 6, 9, 10, 12, 15, 19, 25, 27
57	17	72	1, 4, 6, 9, 15, 20, 24, 28, 31, 32, 36, 38, 39, 41, 46, 50, 53	17	72	1, 4, 6, 9, 15, 20, 24, 25, 28, 32, 36, 38, 39, 41, 47, 50, 53	17	72	1, 4, 6, 9, 15, 20, 24, 25, 28, 32, 36, 38, 41, 47, 50, 53, 57
118	32	164	3, 5, 9, 12, 15, 17, 20, 23, 28, 30, 34, 37, 40, 45, 49, 52, 56, 62, 64, 68, 71, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110, 114	32	164	3, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 40, 45, 49, 52, 56, 62, 64, 68, 70, 71, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110, 114	32	164	3, 5, 9, 12, 15, 17, 21, 23, 28, 30, 34, 37, 40, 45, 49, 56, 62, 64, 68, 71, 75, 77, 80, 85, 86, 91, 94, 101, 105, 110, 114

Table III Number of PMUs Needed and Total Redundancy Results of Different Methods with Zero-injections

IEEE test system	Proposed Method			Two-step Method		
	Number of PMUs needed	Measurement Redundancy	Buses Needed to be Placed with PMU	Number of PMUs needed	Measurement Redundancy	Buses Needed to be Placed with PMU
14	3	16	2, 6, 9	3	16	2, 6, 9
30	7	32	2, 4, 10, 12, 18, 23, 27	7	32	2, 4, 10, 12, 18, 24, 27
57	13	68	1, 4, 10, 14, 20, 24, 29, 30, 32, 36, 49, 54, 56	13	68	1, 6, 9, 19, 25, 27, 32, 37, 38, 47, 51, 53, 56
118	28	151	3, 8, 12, 15, 17, 21, 27, 31, 32, 34, 40, 45, 49, 53, 56, 62, 65, 70, 75, 77, 85, 86, 89, 92, 96, 100, 105, 110	28	151	3, 8, 12, 15, 17, 21, 27, 31, 32, 34, 40, 45, 49, 52, 56, 62, 65, 70, 75, 77, 85, 86, 89, 92, 96, 100, 105, 110

Table IV Number of PMU Needed and Total Redundancy Results of Different Methods under Contingencies

IEEE test system	Proposed Method			Two-step Method			Quadratic Method		
	Number of PMUs needed	Measurement Redundancy	Buses Needed to be Placed with PMU	Number of PMUs needed	Measurement Redundancy	Buses Needed to be Placed with PMU	Number of PMUs needed	Measurement Redundancy	Buses Needed to be Placed with PMU
14	8	39	2, 4, 5, 6, 7, 8, 9, 11, 13	8	39	2, 4, 5, 6, 7, 8, 9, 11, 13	8	39	2, 4, 5, 6, 7, 8, 9, 11, 13
30	21	83	2, 3, 4, 6, 7, 9, 10, 11, 12, 13, 15, 17, 19, 20, 23, 24, 25, 26, 27, 28, 30	21	83	2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 18, 19, 23, 24, 25, 26, 27, 30	21	83	1, 2, 3, 5, 6, 9, 10, 11, 12, 13, 15, 16, 18, 19, 22, 24, 25, 26, 27, 28, 29
57	33	130	1, 3, 4, 6, 9, 11, 12, 15, 19, 20, 22, 24, 26, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 41, 45, 46, 47, 50, 51, 53, 54, 56, 57	33	130	1, 3, 4, 6, 9, 11, 12, 15, 19, 20, 22, 24, 25, 27, 28, 29, 30, 32, 33, 34, 36, 37, 38, 39, 41, 44, 46, 47, 50, 51, 53, 54, 56	33	130	1, 3, 4, 6, 9, 11, 12, 15, 19, 20, 22, 24, 25, 26, 28, 29, 30, 32, 33, 35, 36, 37, 38, 41, 45, 46, 47, 50, 51, 53, 54, 56, 57
118	68	309	2, 3, 5, 7, 9, 10, 11, 12, 15, 17, 19, 21, 22, 24, 25, 27, 29, 30, 31, 32, 34, 36, 37, 40, 42, 44, 45, 46, 49, 51, 53, 54, 56, 57, 59, 61, 62, 64, 66, 68, 70, 71, 73, 75, 77, 79, 80, 83, 85, 86, 87, 89, 91, 92, 94, 96, 100, 102, 105, 106, 109, 110, 111, 112, 115, 116, 117, 118	68	309	2, 3, 5, 7, 9, 10, 11, 12, 15, 17, 19, 21, 22, 24, 25, 27, 29, 30, 31, 32, 34, 36, 37, 40, 42, 43, 45, 46, 49, 50, 51, 52, 54, 56, 59, 62, 64, 65, 66, 68, 70, 71, 73, 75, 76, 77, 78, 80, 83, 85, 86, 87, 89, 90, 92, 94, 96, 100, 101, 105, 106, 108, 110, 111, 112, 114, 116, 117	68	308 ²	1, 3, 5, 6, 9, 10, 11, 12, 15, 17, 19, 21, 22, 24, 25, 27, 29, 30, 31, 32, 34, 35, 37, 40, 41, 44, 45, 46, 49, 51, 52, 54, 56, 57, 59, 61, 62, 64, 66, 68, 70, 71, 73, 75, 77, 79, 80, 83, 85, 86, 87, 89, 90, 92, 94, 96, 100, 101, 105, 106, 108, 110, 111, 112, 114, 116, 117, 118

²The solution obtained from Quadratic Programming Method does not have the same maximum measurement redundancy as the other two methods.

For the 7-bus system shown in Fig. 1, if the maximum redundancy is taken as an objective by using the proposed method in Section 3, the minimum number of PMUs needed to satisfy the observability is 3. Buses needed to be placed with PMUs are buses #3, #4 and #6 as shown in Fig. 1 (a). For each bus in the system, the redundancy level can be calculated as: $\mathbf{RI} = [\mathbf{1} \ \mathbf{1} \ \mathbf{3} \ \mathbf{3} \ \mathbf{1} \ \mathbf{3} \ \mathbf{1}]^T$ and the total measurement redundancy is $1+1+3+3+1+3+1 = 13$. The number of red-1 buses is 4. However, if another placement is adopted, i.e. to place PMUs at buses #1, #4 and #6 as shown in Fig. 1 (b), the bus redundancy level becomes: $\mathbf{RI} = [\mathbf{1} \ \mathbf{2} \ \mathbf{3} \ \mathbf{2} \ \mathbf{1} \ \mathbf{2} \ \mathbf{1}]^T$. Though the total measurement redundancy becomes $1+2+3+2+1+2+1 = 12$, the number of red-1 buses now is 3.

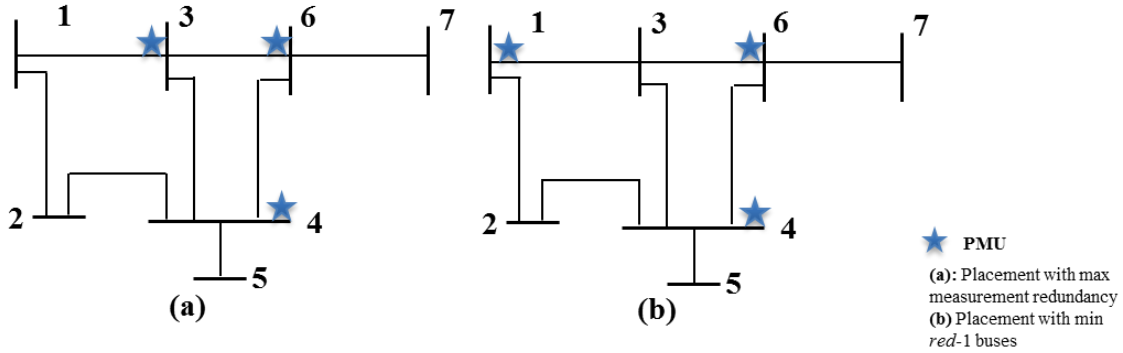


Fig. 1 System configuration and PMU placement for 7-bus case study

The difference between these two placement strategies is: under the placement of Fig. 1 (a), bus #2 is the red-1 bus since it can only be reached by the PMU at bus #4. The corruption/loss of any measurements $\bar{V}_4(|\bar{V}_4|, \theta_{\bar{v}_4})$ or $\bar{I}_{42}(|\bar{I}_{42}|, \theta_{\bar{i}_{42}})$ will lead to bus #2 unobservable or poorly estimated. However for the placement in Fig. 1 (b), any loss of single measurement or data corruption will not affect the estimation of bus #2 since its redundancy level has increased to two. Therefore it can be claimed that the placement in Fig. 1 (b) is more robust regarding the reliability of the execution of SE. The BSA optimization method in [14] involves an unsmooth non-linear objective function which cannot be solved by a normal optimization tool. And the combinatory search method used in BSA is time consuming and cannot be applied to larger or more complex systems. To solve this problem and to better utilize the placement solution with maximum measurement redundancy, a post-processing optimization which reduces the number of red-1 buses is proposed:

- 1) Obtain the solution with maximum measurement redundancy as suggested in Section 3.
- 2) Calculate the redundancy level of each bus by using (1), $\mathbf{RI}(i), 1 \leq i, j \leq N_{bus}$.

- 3) For bus i whose redundancy level is equal to or larger than 2, i.e. $RI(i) \geq 2$, if bus j is bus i 's associated PMU bus, find j 's associated bus/buses and shift the PMU installed at bus j to its associated buses in an attempt to reduce the number of red-1 buses while satisfying the network observability.
- 4) Optimization is terminated when the number of red-1 buses cannot be reduced.

The flow chart of this post-processing optimization process described above can be drawn as in Fig. 2. The post-processing optimization is efficient and the calculation time is also computed for comparison in the following case studies.

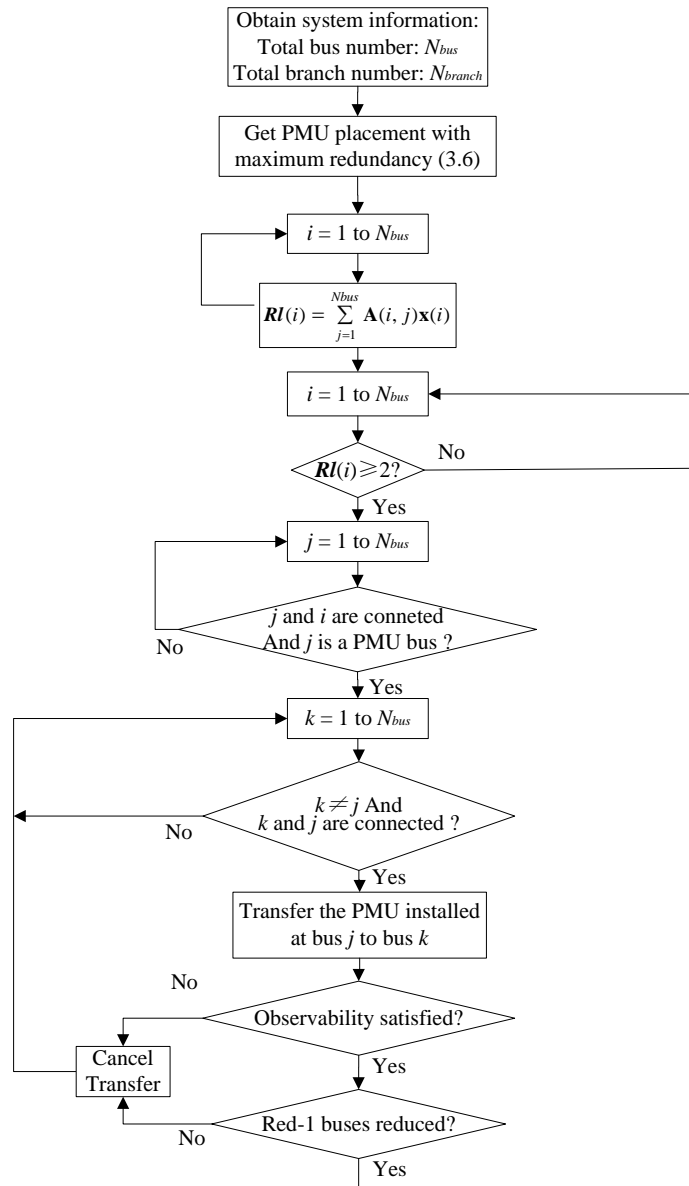


Fig. 2 Flow chart of post-processing optimization

4.2 Case Studies

In this part, various test systems are used to show the reduction of red-1 buses and a comparison with BSA is conducted. Based on the flow chart of the process shown in Fig. 2, the PMU placement with maximum redundancy is optimized and the results of various test systems are presented as follows:

- IEEE 14-bus system

The results are the same as the solution with maximum redundancy.

- IEEE 30-bus system

The PMU placement with maximum measurement redundancy is obtained as:

$$\mathbf{x} = [0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0].$$

Buses need to be placed with PMUs are: #2, #4, #6, #9, #10, #12, #15, #18, #25, #27. The redundancy level vector of each bus is: $\mathbf{RI}(i) = [1 \ 3 \ 1 \ 4 \ 1 \ 5 \ 1 \ 1 \ 3 \ 3 \ 1 \ 3 \ 1 \ 2 \ 3 \ 1 \ 1 \ 2 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 2 \ 1 \ 2 \ 2 \ 1 \ 1]$ which amounts to a total of 18 red-1 buses.

The first successful attempt to reduce the number of red-1 buses occurs on bus #6 whose redundancy level is 5 and bus #4 is found to be an associated PMU bus to bus #6. When shifting the PMU located at bus #4 to its associated bus #3, the placement solution \mathbf{x} now becomes: $\mathbf{x} = [0 \ 1 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0]$. This solution is valid since it can satisfy the observability. Now the number of red-1 buses is reduced to 17. The same procedure can be repeated sequentially and another successful optimization can be achieved at bus #15. The PMU placed at bus #18 which is associated to bus #15, is transferred to bus #19. After checking, the observability can be still satisfied and the number of red-1 buses is reduced to 16. When the optimization process completes, the improved placement solution of \mathbf{x} with 16 red-1 buses can be obtained as:

$$\mathbf{x} = [0 \ 1 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0].$$

The buses which need to be placed with PMUs are: #2, #3, #6, #9, #10, #12, #15, #19, #25, #27.

- Other test systems

A similar process is repeated for the IEEE 57-, 118- and 300-bus systems to examine the improvements on redundancy distribution. The optimization on the measurement redundancy distribution in these systems cannot be executed with BSA due to heavy calculation burden. However, the number of red-1 buses in these systems using the post-processing

optimization is reduced as shown in Table V. Another 72-bus system constructed from the IEEE test system shown in Fig. 3 is carried out and a reduction of 8 red-1 buses is achieved by using the proposed post-processing optimization. The reduction percentage of red-1 buses from the IEEE 30-, 57-, 118-, 300-bus system and the illustrative 72-bus system can be calculated as 11.1%, 2.3%, 1.3%, 1.6% and 17.4% as shown in Table V.

Table V PMU placement with Red-1 Buses Reduced and Their Comparison with BSA

Test Systems	Before Optimization Process	After Optimization Process	Reduction Percentage on Red-1 Buses	BSA
	No. of red-1 buses	No. of red-1 buses		No. of red-1 buses
IEEE-14	10	10	-	10
IEEE-30	18	16	11.1%	16
IEEE-57	43	42	2.3%	Time Out
IEEE-118	80	79	1.3%	Time Out
IEEE-300	193	190	1.6%	Time Out
Illustrative 72	46	38	17.4%	Time Out

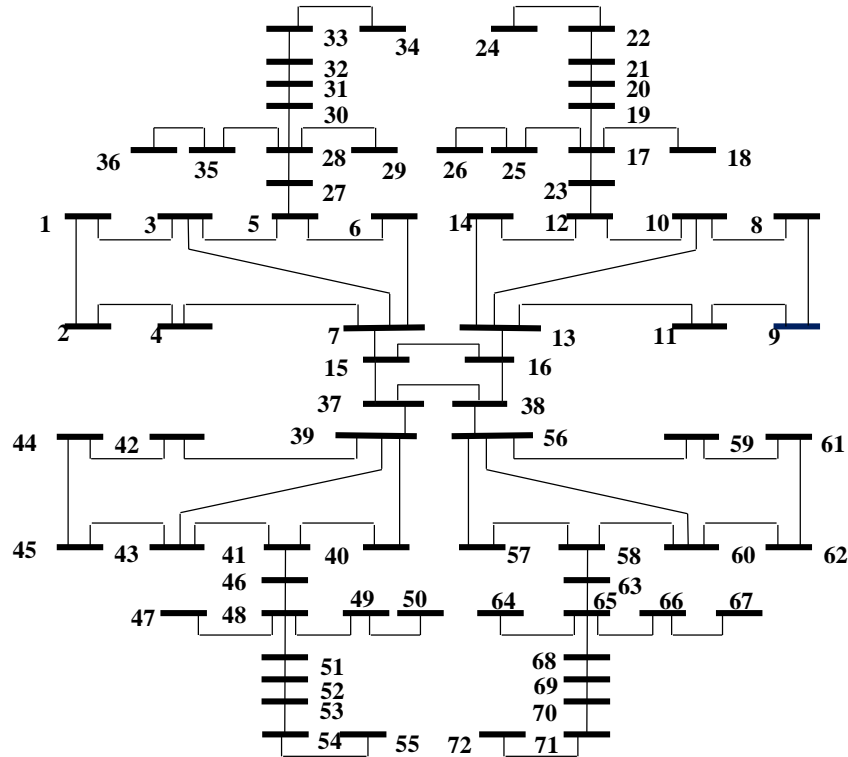


Fig. 3 An illustrative 72-bus system

The computation time of BSA is estimated as the time consumption in processing the combinatory solution. As explained in BSA [14], radial buses³ are excluded from the candidate placement sites and the bus connected to the radial bus should be pre-assigned with a PMU. Therefore, the number of combination in BSA for each case can be calculated in Table VI with the number of radial buses.

³ As defined in [14], a radial bus refers to the bus which has only one connection to other buses.

Take the IEEE 14-bus system as an example. Bus #8 is a radial bus, which has only one connection with other buses as shown in Fig. 4. Therefore bus #8 is excluded from the candidate placement sites. Its associated bus, bus #7 should be assigned with a PMU as explained above. Hence the number of candidate placement sites is $14 - 2 = 12$. Since the minimum number of PMUs needed in the IEEE 14-bus system as computed previously is 4, the combinatory problem is a generation of $^{12}C_3 = 220$ combinations. The computation burden of other systems can be calculated in the same way as shown in Table VI.

Table VI Computation Burden and Time Consumed in BSA

IEEE Test Systems	Number of Radial Buses	Minimum Number of PMUs needed	Number of Combinations	Time Consumed
14	1	4	$^{12}C_3 = 220$	3.2×10^{-4} s
30	3	10	$^{24}C_7 = 346104$	2.79s
57	1	17	$^{55}C_{16}$ out of memory	Time Out
118	7	32	$^{104}C_{25}$ out of memory	Time Out
300	69	87	$^{162}C_{18}$ out of memory	Time Out

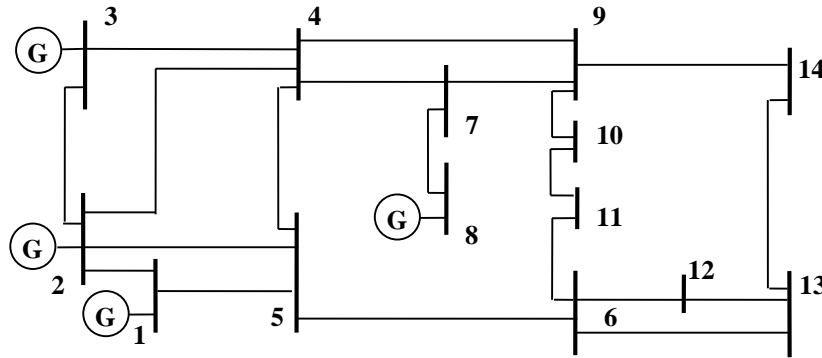


Fig. 4 IEEE-14 bus system configuration

Fig. 5 shows a comparison between the computation time of the post-processing optimization and that of BSA for different case studies. For the IEEE 57-bus system and beyond, dotted lines are used here to indicate that BSA has timed out.

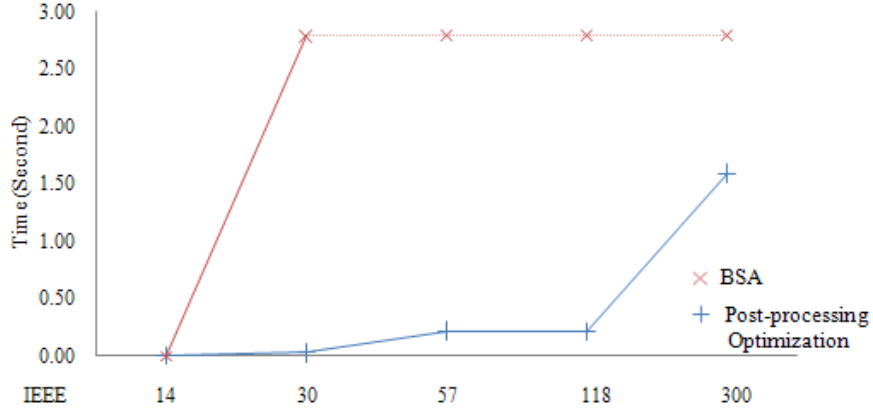


Fig. 5 Comparison on computation time between the post-processing optimization and BSA

5 Discussion and Conclusion

5.1 Discussion on the PMU channel limitation

When there is a channel limitation of PMUs, the PMU placement strategy will be formulated in a different way. The state variables will be augmented as x_{aug} and can be defined as [18]:

$x_{aug}(i, j)$: State variable, $i, j = 1 \dots N_{bus}$. For $i \neq j$, $x_{aug}(i, j) = 1$ indicates that the phasor line current can be measured on line $i - j$, otherwise it cannot be measured. For $i = j$, $x_{aug}(i, j) = 1$ indicates that the phasor voltage can be measured at bus i , namely a PMU is installed at bus i . Otherwise PMU is not installed.

By utilizing the introduced augmented state variables, the PMU placement satisfying the system observability can be obtained via TOMLAB/CPLEX integer programming solver. Afterwards, as suggested from step 1) to 4) in Section 4.1, the redundancy distribution will be improved by reducing the red-1 buses. Take the IEEE 30-Bus system in Section 4 as an example, if the number of channel limitation is set as two, twelve PMUs are needed to be placed at buses #2, #4, #6, #9, #10, #12, #15, #19, #24, #25, #27 and #28. The redundancy level vector of each bus can be calculated as:

$$Rl(i) = [1 \ 1 \ 1 \ 1 \ 1 \ 3 \ 1 \ 1 \ 2 \ 1 \ 1 \ 2 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 2 \ 2 \ 1 \ 1 \ 1 \ 1 \ 1].$$

The number of red-1 buses can be found as 25 and they cannot be reduced further due to the limited measurement channel. If the channel limitation is set as three, ten PMUs are needed to be placed at buses #2, #4, #6, #9, #10, #12, #15, #20, #25, #27. The redundancy level vector of each bus can be calculated accordingly as:

$$RI(i) = [1\ 2\ 1\ 2\ 1\ 3\ 1\ 1\ 2\ 3\ 1\ 2\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 2\ 1\ 1\ 1].$$

The number of red-1 buses is 23. After transferring the PMU at bus #20 to bus #19, the redundancy level of each bus can be obtained as:

$$RI(i) = [1\ 2\ 1\ 2\ 1\ 3\ 1\ 1\ 2\ 2\ 1\ 2\ 1\ 1\ 1\ 1\ 1\ 2\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 2\ 1\ 1\ 1].$$

and the number of red-1 buses is reduced to 22.

After considering the limitation of PMU channels, reduction on the red-1 buses will be less evident than the situation when PMU channels are assumed to be equal to the number of the associated branches. This is because the redundancy level becomes lower when considering the limitation of PMU channels. However, it can be pointed out that the cost of adding additional PMUs or channels will be relatively low once the backbone telecommunication infrastructure is built [19].

5.2 Conclusion

Two improved works on the PMU placement problem are accomplished in this paper:

- (1) A simple and linear alternative formulation which obtains the maximum measurement redundancy with a minimum number of PMUs is proposed in this paper. The installation cost $c(i)$ and risk cost $\mathbf{risk}(i)$ are considered for placing the PMU at bus i in the objective function. Minimization of the combined cost can successfully guarantee a placement with a maximum redundancy and a minimum number of PMUs. The results obtained from the various IEEE test systems under different scenarios show the effectiveness of the proposed method. Besides, from the angle of observability analysis, the loss of PMUs is selected for the case studies since it is more serious than single-line outages.
- (2) A post-processing optimization for bus redundancy distribution is proposed. The process mainly focuses on the reduction of red-1 buses by shifting the PMUs located near the buses with larger redundancy to other buses so as to improve the redundancy level of these buses. In this way, the PMU placement can be made to produce a more robust state estimation. The method is effective and efficient even in larger or complex systems where BSA cannot be applied.

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