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On the effect of demand randomness on inventory, pricing and profit[1]

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Abstract

We consider a stocking-factor-elasticity approach for pricing newsvendor facing multiplicative demand uncertainty with lost sales. For a class of iso-elastic demand curves, we prove that optimal order quantity decreases in demand uncertainty for zero salvage value. This contrasts with fixed-price newsvendor results which depend on the critical ratio. Numerical tests show that optimal order quantity increases in demand uncertainty for high salvage value, low marginal cost, and low price-elasticity. We also report results on optimal price, service level, and profit.

Keywords: demand randomness, pricing newsvendor

1. Introduction

Consider a pricing newsvendor facing multiplicative demand uncertainty with lost sales. We study the effect of demand randomness on the optimal price and order quantity, as well as on the optimal service level (i.e. normalized stocking factor). The impact of demand uncertainty on the firm's optimal decisions has been well studied, as summarized in Table 1. Gerchak and Mossman

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[1] first studied the fixed-price newsvendor with lost sales and found that while the optimal service level is independent of the demand uncertainty, optimal order quantity increases for high critical ratios and decreases for low critical ratios. For the pricing newsvendor, results depend on whether unsatisfied demand is lost or backlogged and demand uncertainty is multiplicative or additive. For lost sales under certain conditions, Li and Atkins [2] and Xu et al. [3] found that both optimal price and service level increase in demand variability for multiplicative demand uncertainty, whereas they both decrease in demand variability for additive demand uncertainty. Agrawal and Seshadri [4] considered backlogged demand satisfied by a more expensive emergency supplier. They found that under multiplicative demand uncertainty, optimal price is higher with uncertainty than without uncertainty while optimal order quantity are lower with uncertainty than without uncertainty. Under additive demand uncertainty, they found that optimal price and order quantity are independent of demand uncertainty. For both additive and multiplicative demand uncertainty, they also found that with demand uncertainty, optimal service level is lower for high critical ratios and higher for low critical ratios.

Table 1: Summary of the literature on the effect of demand uncertainty on optimal decisions

Demand models			Price	Service level*	Order quantity
Fixed-price newsvendor [1]			N.A.	no change	↑ for high critical ratio, ↓ otherwise
Pricing newsvendor	Lost sales [2][3]	Multiplicative	↑	↑	N.A.
		Additive	↓	↓	N.A.
	Backlogged demand [4]	Multiplicative	↑	↓ for high critical ratio, ↑ otherwise**	↓**
		Additive	no change		no change**

* Service level is defined as the normalized stocking factor.
** Results not explicitly claimed but inferred from the paper's results

In recent years, elasticity-based approaches are gaining popularity in the study of the pricing newsvendor problem because demand elasticities are fundamental to the microeconomic analysis of pricing problems. Moreover, different elasticity approaches can be used to address different problems. For instance, Kocabıyıkoğlu and Popescu [5] show that the price-elasticity of lost-sales rate provides a general framework for establishing uniqueness of pricing newsvendor solutions. They also characterize how elasticity affects price and inventory, and vice versa. Another example is Salinger and Ampudia [6] who use price-elasticity of expected sales to generalize the Lerner relationship to price-setting newsvendors. This result provides a unified framework to understand the different effects of additive and multiplicative demand uncertainty. In this paper, we use both price-elasticity of demand and the stocking-factor-elasticity of expected sales used earlier in Petruzzi et al. [7].

Because of our focus on multiplicative demand with lost sales, our elasticity approach allows us to discover new relationships as well as closed forms for optimal decisions and profit of special cases. As summarized in Table 2, our contributions are as follows. For general demand curves, we discover a relationship between the price-elasticity of demand and the stocking-factor-elasticity of expected sales. We provide a simpler elasticity-based proof for the result that optimal price is increasing in demand uncertainty, and generalize Li and Atkins' [2] result for linear demand curve that the optimal service level is increasing in demand uncertainty. For a class of iso-elastic demand curves, we obtain the first explicit result for optimal order quantity of a pricing newsvendor with lost sales. We find that when salvage value is zero, optimal order quantity decreases in demand uncertainty. This result complements Agrawal and Seshadri's [4] result for backlogged demand. Moreover, this result holds even when the critical ratio is high, hence it contrasts with Gerchak and Mossman's [1] result for fixed-price newsvendor. Finally, numerical tests show that optimal order quantity increases in demand uncertainty when salvage value is high, marginal cost is low, and price-elasticity is low. These findings persist beyond iso-elastic demand curves, e.g. demand curve with linear form.

Table 2: Summary of our contributions on the effect of demand uncertainty on optimal decisions

Demand model	Price	Service level	Order quantity
Multiplicative demand with lost sales	New proof	Generalized	New results

2. Model and Results

Facing a random price-dependent demand, a firm's decision is to choose order quantity q and selling price p. We focus on the case where a change in price affects the scale of the demand distribution. In particular, uncertainty is incorporated into demand according to a multiplicative fashion as follows.

$$D(p, \xi) = y(p)\xi$$

where $y'(p) \leq 0$. An economic interpretation for this model is that ξ represents the uncertainty of the market size and $y(p)$ is the demand curve. See Petruzzi and Dada [8] and Li and Atkins [2] for more explanation on the validity of the model. We consider a general $y(p)$ by only assuming that it satisfies the property of increasing price-elasticity. Specifically, the *price-elasticity of demand* $\eta(p) = -py'(p)/y(p)$ is increasing in p. (Throughout this paper, we use increasing and decreasing in their weak sense.) This property is satisfied by various demand curves in the literature, including both the power (i.e. $y(p) = ap^{-b}$) and exponential (i.e. $y(p) = ae^{-p}$) forms in [8] and the linear (i.e. $y(p) = a - bp$) form in [2].

To study the effect of demand randomness, we consider a family of random variables

$$\xi_\beta = \beta\xi + (1 - \beta)\mu$$

such that the mean and variance of ξ are μ and σ^2, respectively, and $0 \leq \beta \leq 1$. As β increases, the mean of ξ_β remains unchanged while the variance increases. For this reason, it is called the *mean-preserving transformation*, which is extensively used in microeconomics and is drawing increasing attention from the operations management community (e.g. [1] and [2]). Note that for any $\beta_1 \geq \beta_2$, ξ_{β_1} is *more variable* than ξ_{β_2} (see [1] for details), that is $\xi_{\beta_1} \geq_v \xi_{\beta_2}$. We let $f(x)$ (resp, $f_\beta(x)$), $F(x)$ (resp, $F_\beta(x)$) and $\bar{F}(x)$ (resp, $\bar{F}_\beta(x)$) be the probability density function, the cumulative distribution function and the complementary cumulative distribution function, respectively, for ξ (resp, ξ_β). For ease of exposition, we define the failure rate of ξ as $h(x) = f(x)/\bar{F}(x)$

and assume that ξ has increasing failure rate (IFR). This assumption is not restrictive as it is satisfied by a large range of probability distributions, including but not limited to the uniform, Weibull, normal, and exponential distributions, and their truncated versions. We further define the generalized failure rate of ξ_β as $g_\beta(x) = xf_\beta(x)/\bar{F}_\beta(x)$.

At the beginning of the selling season, the firm stocks q units of inventory at marginal cost c. At the end of the selling season, the leftover is salvaged at a unit value $s < c$. Given selling price p and market uncertainty ξ_β, the expected sales is $E\min\{q, y(p)\xi_\beta\}$ and the expected leftover is $q - E\min\{q, y(p)\xi_\beta\}$. Thus, the firm's expected profit is

$$\begin{aligned}\pi_\beta(p,q) &= pE\min\{q, y(p)\xi_\beta\} + s[q - E\min\{q, y(p)\xi_\beta\}] - cq \\ &= (p-s)E\min\{q, y(p)\xi_\beta\} - (c-s)q\end{aligned}$$

For ease of analysis, we transform the decision variables from (p, q) to (p, z) where $z = \frac{q}{y(p)}$ is called the *stocking factor*. It follows that letting $S_\beta(z) = E\min\{z, \xi_\beta\}$,

$$\hat{\pi}_\beta(p,z) = (p-s)y(p)S_\beta(z) - (c-s)zy(p). \tag{1}$$

We denote the *stock-factor-elasticity of expected sales* as $\epsilon_\beta(z) = z\bar{F}_\beta(z)/S_\beta(z)$. Also, let the optimal decisions be p^*_β, q^*_β and z^*_β. The optimal profit will be $\pi^*_\beta = \hat{\pi}^*_\beta$. We now present our first result.

Lemma 1. *If ξ is IFR, then for any β,*

(a) $\epsilon'_\beta(z) < 0$,

(b) *There exists a unique solution (p^*_β, z^*_β) (equivalently, (p^*_β, q^*_β)) that satisfies*

$$\left[\frac{y(p)}{y'(p)} + (p-s)\right]S_\beta(z) = (c-s)z \tag{2}$$

$$\bar{F}_\beta(z) = \frac{c-s}{p-s} \tag{3}$$

Moreover, price-elasticity of demand $\eta(p)$ and stocking-factor-elasticity of expected sales $\epsilon_\beta(z)$ are related as follows.

$$\frac{p}{p-s}\cdot\frac{1}{\eta(p)} + \epsilon_\beta\left(\bar{F}^{-1}_\beta\left(\frac{c-s}{p-s}\right)\right) = 1. \tag{4}$$

Proof: For (a), by definition, $F_\beta(x) = P[\beta\xi + (1-\beta)\mu \le x] = P[\xi \le [x-(1-\beta)\mu]/\beta] = F([x-(1-\beta)\mu]/\beta)$. Thus, $g_\beta(z) = \frac{zf_\beta(z)}{\bar{F}_\beta(z)} = [t+(1-\beta)\mu/\beta]\frac{f(t)}{\bar{F}(t)}$, where $t = [z-(1-\beta)\mu]/\beta$. Thus, if ξ is IFR, then ξ_β is IGFR for any β. From Petruzzi et al. [7], the result follows. For (b), (2) and (3) are obtained by differentiating $\hat{\pi}_\beta$ with respect to p and z, respectively. Combining (2) and (3) and by definitions of $\eta(p)$ and $\epsilon_\beta(z)$, we arrive at (4). For uniqueness, due to part (a) and the fact that $\eta(p)$ is increasing in p and $\bar{F}_\beta(x)$ is a decreasing function, the left-hand side of (4) is strictly decreasing in p. Hence, the optimal solution is unique. □

We note that Kocabıyıkoğlu and Popescu [5] deals with a more general model and the price-elasticity of lost-sales rate can be written as $\eta(p)\cdot g_\beta(x/y(p))$ in our setting. This implies that our uniqueness result can be also adapted from their approach. However, our analysis is simpler and our result also sheds light on the choice of optimal price. In particular, Equation (4) characterizes the tradeoff between the price-elasticity of demand and stocking-factor-elasticity of expected sales. Also, when there is no demand uncertainty, the second term in the left-hand side of (4) becomes $\frac{c-s}{p-s}$. Thus, (4) becomes the classical result that optimal price occurs at price-elasticity of demand equals to $\frac{p}{p-c}$.

Next, we will examine the effect of randomness on the optimal decisions and profit. Because demand uncertainty ξ_β also contains a deterministic portion $(1-\beta)\mu$, it is useful to further transform the decision variables from (p,z) to (p,A) where $A = \frac{z-(1-\beta)\mu}{\beta}$ is the *normalized stocking factor*. Substituting $z = \beta A + (1-\beta)\mu$ into (1), the firm's expected profit becomes

$$\begin{aligned}\bar{\pi}_\beta(p,A) &= (p-s)y(p)[\beta S(A) + (1-\beta)\mu] - (c-s)y(p)[\beta A + (1-\beta)\mu] \\ &= (1-\beta)(p-c)y(p)\mu + \beta y(p)[(p-s)S(A) - (c-s)A].\end{aligned}$$

Observe that profit can be seen as a weighted sum of profit from deterministic demand and expected profit from stochastic demand. Then, the normalized stocking factor A can be interpreted as a service level for the stochastic part as in Li and Atkins [2]. From here onwards, we shall refer to A as the service level. As β varies, we are interested to know how the firm should adjust the optimal service level A^*_β and the optimal price p^*_β, and also how the optimal profit $\pi^*_\beta = \bar{\pi}^*_\beta$ changes. It turns out that while the optimal stocking factor z^*_β is not necessarily monotonic in β, the optimal normalized stocking factor A^*_β is increasing in β. The following proposition summarizes the results and the proof is in the appendix.

Proposition 1. (a) *The optimal service level A^*_β is increasing in β.*
(b) *The optimal price p^*_β is increasing in β.*
(c) *The expected profit π^*_β is decreasing in β.*

Proof: (a) As $A = \frac{z-(1-\beta)\mu}{\beta}$, it is easy to see that $\bar{F}_\beta(z) = \bar{F}(A)$ and $S_\beta(z) = \beta S(A) + (1-\beta)\mu$, where $S(A) = S_1(A) = E\min\{A, \xi\}$. Substituting into (2) and (3) in Lemma 1, we get these first-order conditions.

$$\left[\frac{y(p)}{y'(p)} + (p-s)\right][(1-\beta)\mu + \beta S(A)] = \hat{c}[A\beta + (1-\beta)\mu] \tag{5}$$

$$(p-s)\bar{F}(A) = \hat{c} \tag{6}$$

where $\hat{c} = c - s$. From (6), $p = s + \hat{c}/\bar{F}(A)$. Substituting into (5), so A^*_β satisfies

$$\left[\left(1+\frac{\bar{F}(A^*_\beta)s}{\hat{c}}\right)\frac{-1}{\eta(s+\hat{c}/\bar{F}(A^*_\beta))}+1\right][(1-\beta)\mu+\beta S(A^*_\beta)] = \bar{F}(A^*_\beta)[A^*_\beta\beta+(1-\beta)\mu].$$

Taking derivative with respect to β on both sides,

$$\left[\left(1+\frac{\bar{F}(A^*_\beta)s}{\hat{c}}\right)\frac{\eta'(s+\hat{c}/\bar{F}(A^*_\beta))}{\eta^2(s+\hat{c}/\bar{F}(A^*_\beta))}\frac{\hat{c}f(A^*_\beta)}{\bar{F}^2(A^*_\beta)}\frac{dA^*_\beta}{d\beta} + \frac{f(A^*_\beta)s}{\hat{c}}\frac{1}{\eta(s+\hat{c}/\bar{F}(A^*_\beta))}\frac{dA^*_\beta}{d\beta}\right]\times$$
$$[(1-\beta)\mu+\beta S(A^*_\beta)] + \left[-\frac{1+\bar{F}(A^*_\beta)s/\hat{c}}{\eta(s+\hat{c}/\bar{F}(A^*_\beta))}+1\right]\left[\beta\bar{F}(A^*_\beta)\frac{dA^*_\beta}{d\beta} + S(A^*_\beta) - \mu\right]$$
$$= -f(A^*_\beta)\frac{dA^*_\beta}{d\beta}\left[A^*_\beta\beta + (1-\beta)\mu\right] + \bar{F}(A^*_\beta)\left[\frac{dA^*_\beta}{d\beta}\beta + A^*_\beta - \mu\right].$$

Note that $-\frac{1+\bar{F}(A^*_\beta)s/\hat{c}}{\eta(s+\hat{c}/\bar{F}(A^*_\beta))} + 1 = \frac{\bar{F}(A^*_\beta)[A^*_\beta\beta+(1-\beta)\mu]}{(1-\beta)\mu+\beta S(A^*_\beta)} = \epsilon_\beta(z^*_\beta) > 0$. After some algebraic manipulation,

$$\left\{\left[\left(1+\frac{\bar{F}(A^*_\beta)s}{\hat{c}}\right)\frac{\eta'(s+\hat{c}/\bar{F}(A^*_\beta))}{\eta^2(s+\hat{c}/\bar{F}(A^*_\beta))}\frac{\hat{c}f(A^*_\beta)}{\bar{F}^2(A^*_\beta)} + \frac{f(A^*_\beta)s}{\hat{c}}\frac{1}{\eta(s+\hat{c}/\bar{F}(A^*_\beta))}\right]\times\right.$$
$$\left.[(1-\beta)\mu+\beta S(A^*_\beta)] + \epsilon_\beta(z^*_\beta)\beta\bar{F}(A^*_\beta) + f(A^*_\beta)[A^*_\beta\beta+(1-\beta)\mu] - \beta\bar{F}(A^*_\beta)\right\}\frac{dA^*_\beta}{d\beta}$$
$$= \bar{F}(A^*_\beta)[A^*_\beta - \mu] - \epsilon_\beta(z^*_\beta)[S(A^*_\beta) - \mu]. \tag{7}$$

From Lemma 1(a), $\epsilon'_\beta(z) = \epsilon_\beta(z)[1-\epsilon_\beta(z)-g_\beta(z)]/z < 0$, hence $\epsilon_\beta(z)+g_\beta(z) > 1$. Thus, $\epsilon_\beta(z^*_\beta)\beta\bar{F}(A^*_\beta) + f(A^*_\beta)[A^*_\beta\beta + (1-\beta)\mu] - \beta\bar{F}(A^*_\beta) = \beta\bar{F}(A^*_\beta)[\epsilon_\beta(z^*_\beta) + g_1(A^*_\beta) - 1] + (1-\beta)\mu f(A^*_\beta) = \beta\bar{F}(A^*_\beta)[\epsilon_\beta(z^*_\beta) + g_\beta(z^*_\beta) - 1] > 0$, where the second equation is because $g_\beta(z^*_\beta) = g_1(A^*_\beta) + \frac{(1-\beta)\mu f(A^*_\beta)}{\beta\bar{F}(A^*_\beta)}$ and the inequality is because of $\epsilon_\beta(z^*_\beta) + g_\beta(z^*_\beta) > 1$. Hence, the coefficient of $\frac{dA^*_\beta}{d\beta}$ on the left-hand side of (7) (i.e. the entire expression inside the {...}) is positive. Moreover, the right-hand side of (7) is equal to $\bar{F}(A^*_\beta)[A^*_\beta - \mu] - \frac{\bar{F}(A^*_\beta)[A^*_\beta\beta+(1-\beta)\mu]}{(1-\beta)\mu+\beta S(A^*_\beta)}[S(A^*_\beta)-\mu] = \frac{\bar{F}(A^*_\beta)}{(1-\beta)\mu+\beta S(A^*_\beta)}[A^*_\beta - S(A^*_\beta)]\mu \geq 0$. Thus, $\frac{dA^*_\beta}{d\beta} \geq 0$.

(b) The result for the optimal price is because $p^*_\beta = s + \hat{c}/\bar{F}(A^*_\beta)$.

(c) Let $h(x) = (p-s)E\min\{q, y(p)x\} - (c-s)q$, it is easily verified that $h(x)$ is concave in x. Since for any $\beta_1 \geq \beta_2$, $\xi_{\beta_1} \geq_v \xi_{\beta_2}$, from Corollary 8.5.2 in Ross ([9], p.271), we have $Eh(\xi_{\beta_1}) \leq Eh(\xi_{\beta_2})$. Then, $\bar{\pi}_{\beta_1}(p^*_{\beta_1}, A^*_{\beta_1}) \leq \bar{\pi}_{\beta_2}(p^*_{\beta_1}, A^*_{\beta_1})$. Hence, $\bar{\pi}_{\beta_1}(p^*_{\beta_1}, A^*_{\beta_1}) \leq \bar{\pi}_{\beta_2}(p^*_{\beta_2}, A^*_{\beta_2})$; namely, the expected profit is decreasing in demand variability. □

The proposition implies that as demand variability increases, the firm's optimal decision is to increase both the service level and the price. Moreover, the firm will receive less profit. For part (a), Li and Atkins [2] proved it for the special case of linear demand curve (i.e. $y(p) = a - bp$), while we generalize it to the class of demand curves with increasing price-elasticity. In addition, our proof method is different from theirs as we do not employ any second-order derivatives. Our method works because the stocking-factor-elasticity of expected sales is a decreasing function as shown in Lemma 1(a). This result further allows us to prove part (b) through a simple newsvendor formula, namely, $\bar{F}(A^*_\beta) = \frac{c-s}{p^*_\beta - s}$. We must note however that both Salinger and Ampudia [6] and Xu et al. [3] obtain the same result on price. Our result complements the literature by considering the service level which provides an operational reason for the change of price under uncertainty. Part (c) is also an existing result (see [3]), but we include it for completeness and for use in Proposition 2 later.

For practitioners as well as researchers, a more interesting problem is how the order quantity changes with demand variability. To answer this question, we first focus on a class of iso-elastic demand curves.

$$y(p) = p^{-b}, b > 1$$

Note that the iso-elastic demand curve is widely used in the operations management literature (e.g. Petruzzi and Dada [8], Monahan et al. [10], Wang et al. [11]). For this class of demand curves, one can find an explicit solution for the pricing newsvendor problem. More importantly, it allows us to characterize the effect of demand uncertainty on optimal order quantity.

Proposition 2. *Consider any iso-elastic demand curve $y(p) = p^{-b}$, $b > 1$.*

(a) *For any ξ_β, the optimal solution is $p^*_\beta = s + \frac{c-s}{\bar{F}_\beta(z^*_\beta)}$ and $q^*_\beta = z^*_\beta[s + \frac{c-s}{\bar{F}_\beta(z^*_\beta)}]^{-b}$, and the associated expected profit is $\pi^*_\beta = (c-s) \cdot q^*_\beta[\frac{1}{\epsilon_\beta(z^*_\beta)} - 1]$, where $\epsilon_\beta(z^*_\beta) = 1 - \frac{1}{b} - \frac{\bar{F}_\beta(z^*_\beta)s}{b(c-s)}$.*

(b) *When $s = 0$ and for any ξ_β, the optimal solution is $p^*_\beta = c/\bar{F}_\beta(z^*_\beta)$ and $q^*_\beta = z^*_\beta[\bar{F}_\beta(z^*_\beta)/c]^b$, and the associated expected profit is $\pi^*_\beta = \frac{c}{b-1}q^*_\beta$, where $z^*_\beta = \epsilon^{-1}_\beta(1 - \frac{1}{b})$.*

(c) *When $s = 0$, the optimal order inventory q^*_β is decreasing in β.*

Proof: (a) The optimal price follows from (3) in Lemma 1 while optimal order quantity follows from the definition of stocking factor, the optimal price, and the iso-elastic nature of the demand curve. It is easy to see that $\frac{y(p)}{y'(p)} = \frac{-p}{b}$. Substituting into (2), we get $\left[\frac{-p}{b} + (p-s)\right] S_\beta(z) = (c-s)z$. With some algebraic manipulation, we obtain $\epsilon_\beta(z) = \frac{\bar{F}_\beta(z)}{c-s}\left[(p-s)(1-\frac{1}{b}) - \frac{s}{b}\right]$. Substituting the optimal price will yield the elasticity at the optimal stocking level. Finally, optimal profit follows by substituting the first-order conditions and $z = \frac{q}{p^{-b}}$ into (1) and simplifying the expressions.

(b) The result follows by substituting $s = 0$ into (a).

(c) As $\pi^*_\beta = \frac{c}{b-1}q^*_\beta$, the result is straightforward from Proposition 1(c). □

To our best knowledge, Proposition 2 is the first explicit result for the optimal order quantity of the pricing newsvendor problem with lost sales. From Proposition 1(c), we know that π^*_β is decreasing in β. Proposition 2(a) implies that this decrease will be due to q^*_β decreasing in β if $\frac{1}{\epsilon_\beta(z^*_\beta)} - 1$ is increasing β. This condition is not easy to satisfy because in general, neither z^*_β nor $\epsilon_\beta(\cdot)$ has monotonicity properties. However, when $s = 0$, Proposition 2(b) shows that this condition holds, hence q^*_β is decreasing in β. This result complements the literature on pricing newsvendor with backlogged demand, where Agrawal and Seshadri [4] show that the optimal order quantity with uncertainty is lower than without uncertainty.

It is interesting to compare our pricing newsvendor result with the fixed-price newsvendor. When the price is exogenous, Gerchak and Mossman [1] show that the order quantity is increasing in demand variability if and only if the critical ratio $\gamma = c_u/(c_o + c_u) > F(\mu)$, where c_u and c_o are the unit underage and overage costs, respectively. Does this rule hold for the price-setting newsvendor? Specifically, does $\gamma > F(\mu)$ indicate that order quantity is increasing in demand variability?

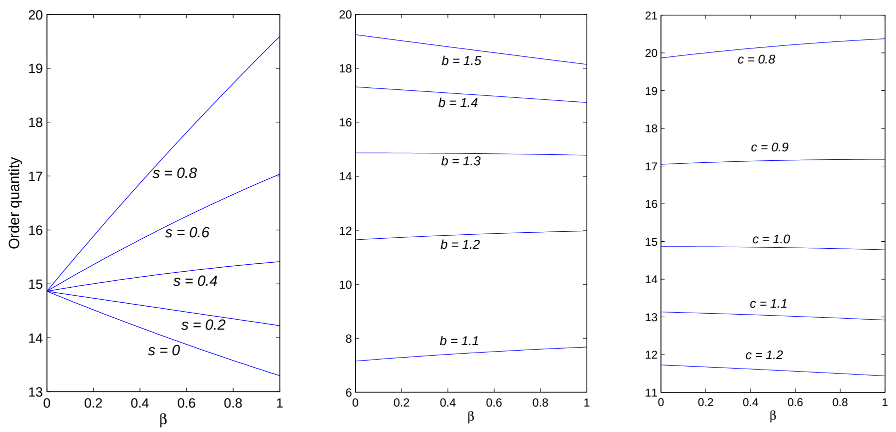
To answer this problem , consider the case when the demand is deterministic (i.e. $\beta = 0$) and there is no salvage value (i.e. $s = 0$). Then, the order quantity $q = y(p)\mu$ and the profit $\pi = (p - c)p^{-b}\mu$. It is easy to see that the optimal solution is $p^*_0 = \frac{b}{b-1}c$. Hence, the corresponding critical ratio $\gamma_0 = (p^*_0 - c)/p^*_0 = 1/b$. From Proposition 1(b), we know p^*_β is increasing in β. Hence, if $\gamma_0 > F(\mu)$, then $(p^*_\beta - c)/p^*_\beta \geq (p^*_0 - c)/p^*_0 > F(\mu) = F_\beta(\mu)$ for any $0 \leq \beta \leq 1$. We summarize the result as follows.

Proposition 3. *Consider any iso-elastic demand curve $y(p) = p^{-b}$, $b > 1$, and $s = 0$. If $bF(\mu) < 1$, then the critical ratio $\gamma_\beta > F_\beta(\mu)$ for any $\beta \in [0, 1]$.*

Proposition 3 shows that if $bF(\mu) < 1$, then the critical ratio $\gamma_\beta > F_\beta(\mu)$. Note that Gerchak and Mossman [1] shows that for fixed-price newsvendor the order quantity increases in demand variability if $\gamma_\beta > F_\beta(\mu)$ and decreases otherwise, but Proposition 2(c) indicates that the order quantity here is still decreasing. Hence, the simple comparison between the critical ratio and $F_\beta(\mu)$ is not enough to explain the effect of demand randomness on order quantity for the pricing newsvendor. Next, we will further explore the underlying driving forces behind the change in optimal order quantity.

3. Numerical Analysis

While Proposition 2 tells us that order quantity is decreasing in demand variability for iso-elastic demand curves and zero salvage value, what about when salvage value is positive? Moreover, how do the marginal cost and demand curve influence the change in order quantity? To that end, we let the demand curve $y(p) = p^{-b}$ and $\xi \sim N(100, 30^2)$. We then numerically show the change in order quantity for different values of salvage value, price-elasticity and marginal cost. The results are shown in Figure 1.



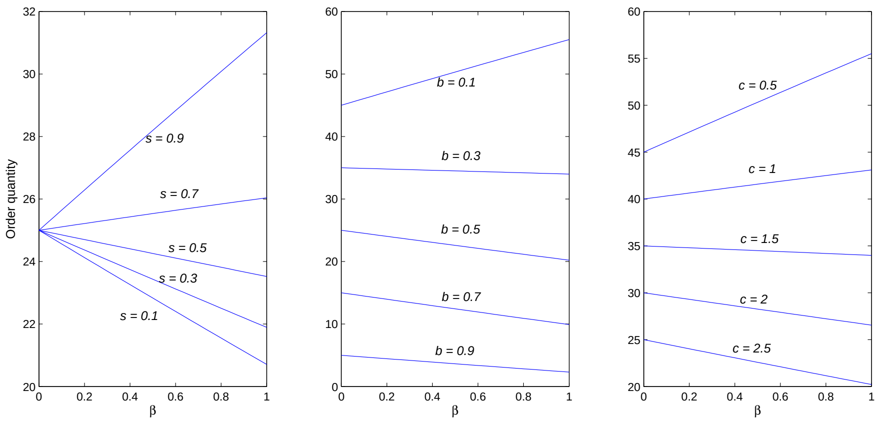
(a) $b = 1.3$ and $c = 1$ **(b)** $c = 1$ and $s = 0.3$ **(c)** $b = 1.3$ and $s = 0.3$

Figure 1: Order quantity w.r.t β for $y(p) = p^{-b}$

In Figure 1(a), when the salvage value increases, the slope for the change in order quantity (i.e., $dq^*_\beta/d\beta$) increases. In particular, if the salvage value is zero, as Proposition 2(c) demonstrates, the order quantity decreases in demand variability. However, as the salvage value becomes greater (e.g., $s = 0.4$), the order quantity changes direction and becomes increasing in demand variability. To understand this, when the price is fixed at p, Gerchak and Mossman ([1]) show that the change in order quantity $dQ^*_\beta/d\beta = F^{-1}(\frac{p-c}{p-s}) - \mu$ is increasing

s. Figure 1(a) suggests that the pricing newsvendor inherits this behavior from the fixed-price newsvendor. For Figure 1(b), the slope for the change in order quantity (i.e., $dq^*_\beta/d\beta$) is decreasing in the price-elasticity b. The reason is that as b increases, pricing becomes a more effective tool so that the newsvendor can rely more on pricing rather than on quantity. Hence, the marginal effect on order quantity decreases, i.e. $dq^*_\beta/d\beta$ is decreasing in b. Figure 1(c) shows that the slope for the change in order quantity is decreasing in marginal cost, and the underlying reason is similar to the effect of salvage value.

Finally, to test the robustness of these results, we consider the case when the demand curve is linear. Without loss of generality, let $y(p) = 1 - bp$. Analogously, we show the change in order quantity for different values of salvage value, price-elasticity and marginal cost in Figure 2. On the effect of salvage value and marginal cost, it is clear that Figure 2 and Figure 1 are qualitatively the same. The seeming difference for the effect of price-elasticity is due to the fact that the influence of demand variability on order quantity itself (not change in order quantity) for the two demand curves are in opposite directions.



(a) $b = 0.5$ and $c = 1$ **(b)** $c = 1$ and $s = 0$ **(c)** $b = 0.2$ and $s = 0$

Figure 2: Order quantity w.r.t β for $y(p) = 1 - bp$

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References

[1] Y. Gerchak, D. Mossman, On the effect of demand randomness on inventories and costs, Operations Research 40 (1992) 804–807.

[2] Q. Li, D. Atkins, On the effect of demand randomness on a price/quantity setting firm, IIE Transactions 37 (2005) 1143–1153.

[3] M. Xu, Y. Chen, X. Xu, The effect of demand uncertainty in a price-setting newsvendor model, European Journal of Operational Research 207 (2010) 946–957.

[4] V. Agrawal, S. Seshadri, Impact of uncertainty and risk aversion on price and order quantity in the newsvendor problem, Manufacturing & Service Operations Management 2 (2000) 410–423.

[5] A. Kocabıyıkoğlu, I. Popescu, An elasticity approach to the newsvendor with price-sensitive demand, Operations Research 59 (2011) 301–312.

[6] M. Salinger, M. Ampudia, Simple economics of the price-setting newsvendor problem, Management Science 57 (2011) 1996–1998.

[7] N. Petruzzi, K. Wee, M. Dada, The newsvendor model with consumer search costs, Production and Operations Management 18 (2009) 693–704.

[8] N. Petruzzi, M. Dada, Pricing and the newsvendor problem: a review with extensions, Operations Research 47 (1999) 183–194.

[9] S. Ross, Stochastic Processes, Wiley, 1983.

[10] G. Monahan, N. Petruzzi, W. Zhao, The dynamic pricing problem from a newsvendor's perspective, Manufacturing & Service Operations Management 6 (2004) 73–97.

[11] Y. Wang, L. Liang, Z. Shen, Channel performance under consignment contract with revenue sharing, Management Science 50 (2004) 34–47.