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A Robust Optimization Approach for Energy Generation Scheduling in Microgrids

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Abstract

1 In this paper, a cost minimization problem is formulated to intelligently sched-
2 ule energy generations for microgrids equipped with unstable renewable sources
3 and combined heat and power generators. In such systems, the fluctuant net de-
4 mands (i.e., the electricity demands not balanced by renewable energies) and heat
5 demands impose unprecedented challenges. To cope with the uncertainty nature
6 of net demand and heat demand, a new flexible uncertainty model is developed.
7 Specifically, we introduce reference distributions according to predictions and
8 field measurements and then define uncertainty sets to confine net and heat de-
9 mands. The model allows the net demand and heat demand distributions to fluc-
10 tuate around their reference distributions. Another difficulty existing in this prob-
11 lem is the indeterminate electricity market prices. We develop chance constraint
12 approximations and robust optimization approaches to firstly transform and then
13 solve the prime problem. Numerical results based on real-world data evaluate the
14 impacts of different parameters. It is shown that our energy generation scheduling
15 strategy performs well and the integration of combined heat and power generators

16 effectively reduces the system expenditure. Our research also helps shed some
17 illuminations on the investment policy making for microgrids.

Keywords: microgrid, energy generation scheduling, demand uncertainties,
robust optimization, uncertainty set, reference distribution.

1. Introduction

18 The electricity grid is being restructured to allow high penetration of dis-
19 tributed generators to become more environment friendly and cost effective [1].
20 The growth and evolution of the power grids is expected to come with the plug-
21 and-play of the basic structure called microgrid. Microgrids can operate in grid-
22 connected mode, in which they are allowed to import power from the electricity
23 grid, or in islanded mode, where they are isolated from the upstream power grid
24 and use their local generators as the source of power supply when needed. There
25 are world-wide deployments of pilot microgrids, especially in Europe, e.g., those
26 reported in [2] and [3]. Reference [2] investigates the key drivers enabling the
27 market feasibility of microgrids in different contexts. While in [3], the technical,
28 social, economic, and environmental benefits provided by microgrids are studied.

29 Energy generation scheduling to achieve robust and economic power supply
30 is an essential component in microgrids. Two features of microgrids are the
31 integration of large-scale renewable sources and the use of combined heat and
32 power (CHP) generators. Such features, however, impose significant difficulties
33 on the design of intelligent control strategies for microgrids. Traditional gener-
34 ation scheduling schemes are typically based on perfect prediction of future de-

35 mands [4], which is hardly the case in the microgrids since small-scale demands
36 are hard to predict and renewable energies are highly volatile. Furthermore, al-
37 though the integration of CHP generators can bring great economic benefits to mi-
38 crogrids by simultaneous production of useful heat and electricity outputs, thereby
39 increasing the overall efficiency and bringing environmental benefits, it brings
40 new uncertainties to the scheduling problem: the heat demand exhibits a new
41 stochastic pattern and makes it more difficult to predict the overall energy de-
42 mands. On top of these, the real-time pricing in electricity market yields another
43 uncertainty dimension to the scheduling problem. The microgrid has to make a
44 proper strategic decision on the amount of power to be imported so as to cope
45 with the financial risks brought by price uncertainty. Because of these unique
46 challenges, it remains an open issue to design robust and cost-effective energy
47 generation scheduling strategies for microgrids.

48 *1.1. Related Work*

49 Energy generation scheduling is the process of effectively scheduling different
50 energy sources (local generators, central grid, renewable energy generations, etc.)
51 to meet the energy requests at the minimum cost subject to various physical con-
52 straints of the power systems. It is a classic problem in electricity system which is
53 composed of two aspects, namely unit commitment (UC) [5] and economic dis-
54 patch (ED) [6], respectively. The UC problem involves determining the start-up
55 and shut-down schedules for generator units to be used to meet forecast demand
56 over a short time in future. It is a complex optimization problem with both integer

57 and continuous variables and has been shown to be NP-complete in general. The
58 basic UC methods reported in literature include priority listing method [7], where
59 the generator units are committed according to a priority order based on unit av-
60 erage full load cost; dynamic programming method [8], where the complicated
61 scheduling problem is broken down into a sequence of decision steps over time
62 in a recursive manner; Lagrangian relaxation method [9], where the Lagrangian
63 dual of the UC is maximized with standard sub-gradient techniques and a reserve-
64 feasible dual solution is computed; and integer programming method [10] [11],
65 where binary variables are adopted to model the startup, shutdown and on/off
66 states for every generator unit and every time period, etc. Once the UC problem
67 has determined the start-up and shut-down schedules, the ED problem seeks to
68 find the optimal allocation of electric power outputs from various available gen-
69 erators without alternating their on/off status. In [5], a genetic algorithm (GA)
70 solution to the UC problem is presented. Authors of [6] propose a particle swarm
71 optimization (PSO) method for solving the ED problem in power systems. Read-
72 ers can refer to comprehensive surveys on UC [12] and ED [13] for more details,
73 in which different methods used in the UC and ED problem-solving techniques
74 are summarized and analyzed.

75 Conventional energy generation scheduling is typically conducted 24 hours
76 in advance (day ahead) and based on the fact that the system load can be fore-
77 cast with reasonably good accuracy one day in advance. In microgrids, however,
78 this is no longer the case due to the fact that accurate predictions of small-scale
79 electricity and heat demands, renewable energy supplies and electricity market

80 prices are very difficult, as we stated earlier. Some recent literature has investi-
81 gated energy generation scheduling of microgrids [14, 15, 16, 17, 18]. In [14],
82 a multi-objective optimization of economic load dispatch for a microgrid is in-
83 vestigated using evolutionary computation. The paper aims at minimizing the
84 emission of the thermal generators and minimizing the total operating cost. In
85 [15], a generalized formulation for intelligent energy management of microgrid is
86 proposed using artificial intelligence techniques jointly with linear-programming-
87 based multi-objective optimization. Similarly, in [16], an intelligent energy man-
88 agement system is proposed for optimal operation of a CHP-based microgrid over
89 a 24-hour time interval. Authors of [17] and [18] also propose different energy
90 management strategies based on different assumptions. The limitation of these
91 results, however, is that they all assume that the energy demands and supplies are
92 known ahead of time, which is rarely the case in practice.

93 There also exist some studies considering demand and supply uncertainties
94 when scheduling the energy generation. Such work can be categorized into two
95 groups: the stochastic optimization based approaches [19, 20, 21, 22, 23, 24, 25]
96 and robust optimization based approaches [26, 27, 28, 29, 30]. In [19], a stochas-
97 tic programming approach is adopted in the development of the proposed bid-
98 ding strategies for microgrid producers and loads. In [20], the authors develop
99 a solution method for scheduling units of a power-generating system to produce
100 electricity by taking into consideration the stochastic nature of the hourly load
101 and its correlation structure. In [21], a stochastic model for the long-term solu-
102 tion of security-constrained unit commitment is proposed. A more complicated

103 scenario can be found in [22], in which an efficient stochastic framework is de-
104 veloped to investigate the effect of uncertainty on the operation management of
105 microgrids. The proposed stochastic framework considers the uncertainties of
106 load forecast error, wind turbine generation, photovoltaic generation and market
107 price concurrently. Authors of [23] examine the impact of the stochastic nature
108 of wind on planning and dispatch of a system. Similarly, authors of [24] compare
109 stochastic and reserve methods and evaluate the benefits of a combined approach
110 for the efficient management of uncertainty in the unit commitment problem. In
111 [25], a two-stage stochastic objective function aiming at minimizing the expected
112 operational cost is implemented. Note that the stochastic optimization approach
113 explicitly incorporate a probability distribution function of the uncertainty, and
114 they often rely on enumerating discrete scenarios of the uncertainty realizations.
115 Such approaches mainly have two practical limitations. First, it may be difficult
116 and costly to obtain an accurate probability distribution of uncertainty. Second,
117 the solution only provides probabilistic guarantees to the system reliability. To ob-
118 tain a highly reliable guarantee requires a huge number of samples, which poses
119 substantial computational challenges.

120 In recent literature, robust optimization has received growing attentions as
121 a modeling framework for optimization under uncertainty. In [26], a two-stage
122 adaptive robust unit commitment model is proposed for the security constrained
123 unit commitment problem in the presence of nodal net injection uncertainty. In
124 [27], a robust optimization approach is proposed to accommodate wind output
125 uncertainty, with the objective of providing a robust unit commitment schedule

126 for the thermal generators in the day-ahead market. In [28], a power schedul-
127 ing approach is proposed based on robust optimization to address the intrinsically
128 stochastic availability of renewable energy sources. References [29] and [30] also
129 present robust optimization based approaches for optimal microgrid management
130 considering wind power or energy consumption uncertainties. Instead of postu-
131 lating explicit probability distribution, robust optimization confines the random
132 variable in a pre-defined uncertainty set containing the worst-case scenario. For
133 instance, in [26, 27, 28, 31, 32, 29, 30], uncertainties in price prediction or renew-
134 able energy generation are presented as interval values with deterministic lower
135 and upper bounds, and the framework developed in [33] and [34] is incorporated
136 to solve the problem. Without requiring an explicit probability distribution, the
137 uncertainty can be characterized more flexibly. In addition, the conservativeness
138 of the solution can easily be controlled and the problem is always computationally
139 tractable both practically and theoretically even for large scale problems.

140 In our study, the robust optimization concept is also applied to tackle the un-
141 certainties in energy generation scheduling problem of microgrids. Different from
142 the previous robust optimization works [26, 27, 28, 31, 32, 29, 30] which confine
143 the uncertainty within a lower and upper bounds, in our work, we propose a new
144 uncertainty model to characterize the renewable energy and user demand uncer-
145 tainties, which can provide more statistical details in describing the underlying
146 uncertainty. Moreover, the proposed uncertainty model is also flexible enough
147 that we can incorporate more information into the uncertainty model when such
148 information is available.

149 *1.2. Main Contributions*

150 In this paper, we consider a robust optimization based energy generation schedul-
151 ing problem in a CHP-microgrid scenario considering the net demand (the elec-
152 tricity demand not balanced by renewable energy) uncertainty, heat demand un-
153 certainty and electricity price uncertainty. The main contributions of this paper
154 can be briefly summarized as follows:

- 155 • We propose a new flexible uncertainty model to capture the fluctuant na-
156 ture of the net demand and heat demand. Specifically we extract reference
157 distributions as useful references and allow the actual distributions of net
158 demand and heat demand to vary around their references. To the best of
159 our knowledge, this is the first time that distribution uncertainty model is
160 adopted to depict the indeterminacy nature of net demand and heat demand.
- 161 • We develop chance constraint approximation and robust optimization ap-
162 proaches based on our uncertainty model to transform the constraints with
163 random variables into typical linear constraints. Then an iterative algorithm
164 is designed to solve the problem.
- 165 • Price uncertainty is addressed by adopting robust optimization techniques,
166 which allows the degree of conservatism to be controlled easily. We fi-
167 nally transform the prime problem into a mixed integer linear programming
168 (MILP) problem, which can be solved efficiently by commercial solvers.
- 169 • Numerical results based on real-world data evaluate the impacts of different
170 parameters and help provide some insights on designing investment policies

171 for microgrid. It is also shown that the proposed energy generation schedul-
172 ing strategy achieves considerable cost savings and the integration of CHP
173 generators can effectively reduce the system expenditure.

174 The remainder of this paper is organized as follows. Section 2 introduces the
175 particulars of the system operation. In Section 3, we introduce the mathematical
176 depiction of the energy generation scheduling problem and the uncertainty models
177 of net and heat demands. Section 4 presents the chance constraint approximation
178 and robust optimization approach for handling the demand balancing and price
179 uncertainty. The simulation results and discussions are shown in Section 5. The
180 parameters and calibration data are drawn from real-world statistics. Finally, we
181 conclude our paper in Section 6.

182 **2. System Model**

183 We consider a microgrid comprising a number of homogeneous CHP gen-
184 erators, a renewable energy generation system and a local heating system. The
185 microgrid is operated on the grid-connected mode, such that it can purchase elec-
186 tricity from the external utility grid when needed. The illustration of the microgrid
187 system is shown in Fig. 1. The main symbols utilized in the paper and their mean-
188 ings are listed in Table 1. The particulars of the system operation are explained in
189 the following subsections.

Table 1: Notations used in this paper

Symbol	Defination
\mathcal{A}	set of CHP generators
a	index of CHP generator, $a \in \mathcal{A}$
c_a^s	start up cost of turning on the generator a
c_a^b	sunk cost of maintaining the generator a
c_a^m	marginal cost for the generator a
\mathcal{H}	the set of time slots
\mathbf{x}_a	energy generation scheduling vector of CHP a
\mathbf{y}_a	state vector of CHP a (binary)
E_a^{min}	the minimum stable output capacity of CHP a
E_a^{max}	the maximum electricity output capacity of CHP a
η_a	heat-electricity ratio for the generator a
p_g	price of heating system for providing one unit of heat
U^h	amount of heat generated from heating system at time h
p_s^h	electricity market price at time h
\hat{p}_s^h	lower bound of the predicted electricity market price at time h
d^h	uncertainty range of electricity market price at time h
V^h	electricity obtained from outside power grid at time h
L^h	net demand at time h (random variable)
S^h	heat demand of the microgrid at time h (random variable)
$f_0(L^h)$	electricity demand distribution at time h
$g_h(L^h)$	reference distribution of $f_0(L^h)$
D_h	distance limit of $f_0(L^h)$'s uncertainty set
$\mathbf{U}_r(\cdot)$	uncertainty set based on KL divergence
ϵ	fault tolerance limit of the power grid

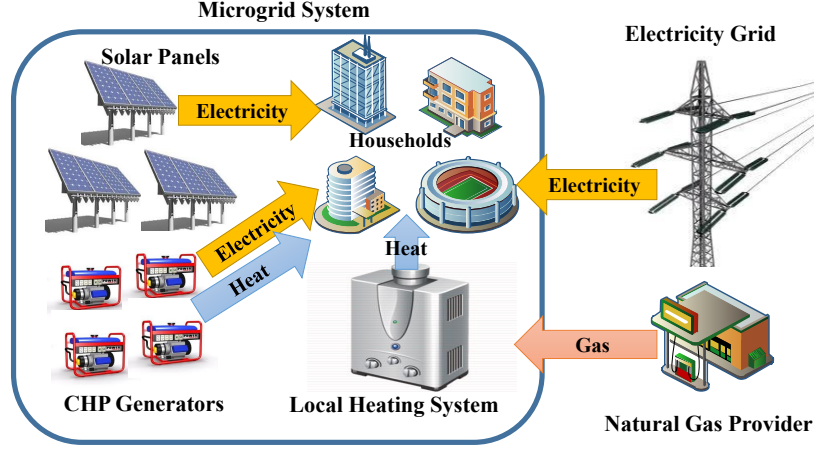


Figure 1: An illustration of a typical microgrid system.

190 *2.1. CHP Generators*

191 We divide time into discrete time slots with equal length. Let \mathcal{A} denote the set
 192 of CHP generators. Further denote the start up cost of turning on a generator a as
 193 c_a^s , the sunk cost of maintaining the generator a in active mode for one time unit as
 194 c_a^b , and the marginal cost for the generator a to produce one unit of electricity as
 195 c_a^m . Adopting a general generator model, we define energy generation scheduling
 196 vector \mathbf{x}_a and state vector \mathbf{y}_a as follows:

$$\mathbf{x}_a = [x_a^1, x_a^2, \dots, x_a^H] \text{ and } \mathbf{y}_a = [y_a^1, y_a^2, \dots, y_a^H], \quad (1)$$

197 where $H \geq 1$ is the scheduling horizon which indicates the number of time slots
 198 ahead that are taken into account for decision making in the energy generation
 199 scheduling. For each coming time slot $h \in \mathcal{H} = [1, 2, \dots, H]$, let a binary variable
 200 $y_a^h = 0/1$ denote the state of generator a (on/off) and a variable x_a^h denote the
 201 dispatched load to generator a . For each generator a with the maximum electricity

202 output capacity E_a^{max} and the minimum stable generation E_a^{min} , we have

$$y_a^h \cdot E_a^{min} \leq x_a^h \leq y_a^h \cdot E_a^{max}. \quad (2)$$

203 The CHP generators can efficiently generate electricity and useful heat energy
204 simultaneously. Let η_a denote the heat-electricity ratio for generator a , which
205 means that the CHP generator a can supply η_a units of heat for free when gener-
206 ating one unit of electricity. Alternatively, heat can be supplied by local heating
207 system at a price of p_g per unit. We use the variable U^h to denote the amount
208 of heat generated from local gas heaters at time slot h . Note that in this paper,
209 we omit the ramping-up and ramping-down constraints of CHP generators since
210 we consider fast response CHP generators such as gas turbines or microturbines,
211 which have fast ramping rates and are able to start from cold to full capacity in
212 1-10 mins [35].

213 2.2. Electricity from External Utility Grid

214 The microgrid can import electricity from outside electricity grid for the un-
215 balanced power demand in an on-demand manner. We assume that the electricity
216 market price at time h is p_s^h , which is a bounded random variable that takes value
217 in $[\hat{p}_s^h, \hat{p}_s^h + d^h]$. \hat{p}_s^h denotes the lower bound of the predicted price. $d^h > 0$ denotes
218 that there exists price uncertainty (financial risks) at time h while $d^h = 0$ indicates
219 the price at time h is known in advance. The amount of electricity obtained from
220 electricity grid at time h is denoted as V^h .

221 *2.3. Fluctuant Electricity and Heat Demand*

222 Renewable energy generation can be regarded as a non-positive demand [4].
223 Denote the net demand at time h as L^h , which is a random variable of which
224 the probability distribution may not be known. Similarly, the heat demand of the
225 microgrid S^h is also random. Accurate prediction of small-scale demands and
226 renewable energy generation is difficult to obtain due to limited management re-
227 sources and their unpredictable nature. We need a proper uncertainty model to
228 capture the indeterminacy properties of net and heat demands. A central require-
229 ment to the microgrid is to set the generation source power such that the electricity
230 and heat supplies could meet the demands. This statement can be described as

$$V^h + \sum_{a \in \mathcal{A}} x_a^h \geq L^h \quad (3)$$

$$U^h + \sum_{a \in \mathcal{A}} \eta_a \cdot x_a^h \geq S^h. \quad (4)$$

231 **3. Problem Formulation**

232 In this section, a cost minimization problem formulation which incorporates
233 CHP generation constraints, uncertain net demand, uncertain heat demand and
234 time varying electricity prices is first given. The uncertainty model for describing
235 the randomness of net demand and heat demand is then demonstrated.

236 *3.1. Cost Minimization Formulation*

237 The microgrid aims to minimize the operation cost of the whole system over
 238 the entire time horizon. The cost minimization formulation is defined as follows:

$$\begin{aligned}
 & \min_{\mathbf{X}, \mathbf{Y}, \mathbf{V}, \mathbf{U}} \sum_{h=1}^H \left\{ p_g \cdot U^h + p_s^h \cdot V^h + \right. & (5) \\
 & \left. \sum_{a \in \mathcal{A}} \left[c_a^m \cdot x_a^h + c_a^b \cdot y_a^h + c_a^s \cdot (y_a^h - y_a^{h-1})^+ \right] \right\} \\
 & \text{s.t.} \quad (2) \ (3) \ (4), \ y_a^h \in \{0, 1\} \\
 & \quad \quad x_a^h, V^h, U^h \in \mathbb{R}_0^+, h \in \mathcal{H}, a \in \mathcal{A},
 \end{aligned}$$

239 where $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_a, \dots]^T$ and $\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_a, \dots]^T$ are matrices of de-
 240 cision vectors \mathbf{x}_a and \mathbf{y}_a for $a \in \mathcal{A}$, respectively; $\mathbf{V} = [V^1, V^2, \dots, V^h, \dots]$ and
 241 $\mathbf{U} = [U^1, U^2, \dots, U^h, \dots]$ are vectors of decision variables V^h and U^h for $h \in \mathcal{H}$,
 242 respectively; $(\cdot)^+$ is a function where $(x)^+ = \max(0, x)$. The cost function com-
 243 prises the cost of electricity from outside power grid, the cost of generating heat
 244 from local heat generators, and the operation and start-up cost of CHP generators
 245 for the entire time horizon H .

246 A difficulty in solving this problem lies in the correlation term $(y_a^h - y_a^{h-1})^+$.
 247 By introducing an auxiliary variable z_a^h into the problem formulation, an equiva-

248 lent expression can be obtained as:

$$\begin{aligned}
& \min_{\mathbf{x}, \mathbf{Y}, \mathbf{Z}, \mathbf{V}, \mathbf{U}} && \sum_{h=1}^H \left\{ p_g \cdot U^h + p_s^h \cdot V^h + \right. && (6) \\
& && \left. \sum_{a \in \mathcal{A}} \left[c_a^m \cdot x_a^h + c_a^b \cdot y_a^h + c_a^s \cdot z_a^h \right] \right\} \\
& \text{s.t.} && z_a^h \geq 0, z_a^h \geq y_a^h - y_a^{h-1} \\
& && (2) (3) (4), y_a^h, z_a^h \in \{0, 1\} \\
& && x_a^h, V^h, U^h \in \mathbb{R}_0^+, h \in \mathcal{H}, a \in \mathcal{A},
\end{aligned}$$

249 where $\mathbf{Z}_{|\mathcal{A}| \times H}$ is the matrix of auxiliary variable z_a^h for $a \in \mathcal{A}$, $h \in \mathcal{H}$. The
250 objective for introducing an auxiliary variable z_a^h into problem formulation (5) is
251 to have an equivalent, solvable problem without the correlation term $(y_a^h - y_a^{h-1})^+$.
252 Another difficulty in solving problem (5) is the indeterminacy of net demand L^h
253 and heat demand S^h existing in constraints (3) and (4). Note that to optimize over
254 the space defined by (3) and (4) amounts to solving an optimization problem with
255 potentially large or even infinite number of constraints. Obviously, this realization
256 of uncertainties is intractable. Next, we develop a practical and flexible model to
257 capture the uncertainties of L^h and S^h .

258 3.2. Probability Distribution Measure of Uncertainties

259 It is generally difficult to characterize the net demand and heat demand. In our
260 optimization model, operations on the random variables L^h and S^h are cumber-
261 some and computationally intractable. Moreover, in practice, we may not know
262 the precise distributions of L^h and S^h . Solutions based on assumed distribu-
263 tions hence may not be justified. We usually measure the variability of a random

264 variable using its variance or second moments which, however, may not provide
 265 sufficient details in describing the random variables. In this paper, we extract
 266 a reference distribution, rather than moment statistics, from historical data and
 267 predictable information, to capture the distribution properties. Since net demand
 268 and heat demand distributions may fluctuate over time and hard to be described
 269 in closed-form expressions, we adopt empirical distributions as useful references
 270 and allow the actual distributions to fluctuate around them. For example, we may
 271 assume that the net demand distribution $f_0(L^h)$ is shifting around a known dis-
 272 tribution $g_h(L^h)$, which can be obtained based on predictions and long-term field
 273 measurements. In the following part of this paper, we only show the way to deal
 274 with random variable L^h . The method to tackle with random variable S^h is exactly
 275 the same.

276 The discrepancy between $f_0(L^h)$ and its reference $g_h(L^h)$ can be described
 277 by a probabilistic distance measure: the Kullback-Leibler (KL) divergence [36],
 278 which is a non-symmetric measure of the difference between two probability dis-
 279 tributions. Name these two distributions as $f(L^h)$ and $g(L^h)$, respectively. Gen-
 280 erally, one of the distributions, say, $f(L^h)$, represents the real distribution through
 281 precise modeling, while the reference $g(L^h)$ is a closed-form approximation based
 282 on the theoretic assumptions and simplifications. The definition of the KL diver-
 283 gence between two continuous distributions is given as follows:

$$\begin{aligned}
 D_{KL}(f(L^h), g(L^h)) = & \hspace{15em} (7) \\
 & \int_{L^h \in S} [\ln f(L^h) - \ln g(L^h)] f(L^h) dL^h,
 \end{aligned}$$

284 where S is the integral domain. When distributions $f(L^h)$ and $g(L^h)$ are close to
 285 each other, the distance measure is close to zero. Adopting the KL divergence, we
 286 define the distribution uncertainty set as follows:

$$\begin{aligned} \mathbf{U}_r(g(L^h), D_0) = & \hspace{15em} (8) \\ \{f(L^h) \mid \mathbb{E}_f[\ln f(L^h) - \ln g(L^h)] \leq D_0\}, & \end{aligned}$$

287 where $D_0 > 0$ represents a distance limit which may be obtained from empirical
 288 data or real-time measurement. It indicates net demand's variation level. If the net
 289 demand is highly volatile, we have less confidence on the reference distribution
 290 and thus may set a larger distance limit.

291 Considering the electricity demand distribution $f_0(L^h)$ with reference distribu-
 292 tion $g_h(L^h)$ and distance limit D_h , we have the following constraints for electricity
 293 demand distribution $f_0(L^h)$:

$$\mathbb{E}_{f_0}[\ln f_0(L^h) - \ln g_h(L^h)] \leq D_h \quad (9)$$

$$\mathbb{E}_{f_0}[1] = 1. \quad (10)$$

294 Equation (10) represents the fact that the integral of a probability density function
 295 over the entire space is equal to 1. With (9) and (10), we are now ready to trans-
 296 form the constraint (3) (similarly for (4)) to allow efficient solution of the problem
 297 (6).

298 Note that in the proposed approach, renewable energy is treated as a non-
 299 positive demand. We integrate user demand and renewable energy generation
 300 together and denote it as the net demand. The combined uncertainties from both

301 user and supply sides are described by an uncertainty set as defined in (9) and
302 (10).

303 The proposed model also allows some convenient extensions to include and
304 handle more components in the microgrid systems. For example, to incorporate
305 the reserve constraint into the proposed model, we only need to add the reserve
306 constraints, which are linear functions, into the formulation (5) and then add a
307 quadratic reserve cost into the objective function [37]. The new problem could
308 still be transformed into a mixed integer programming (MIP) problem and the
309 algorithm to be introduced in the next section can still be applied with virtually no
310 change.

311 **Remark:** Proper estimations of reference distribution and distance limit may
312 be obtained by various methods, for instance, the Kernel Density Estimation (KDE),
313 which is a non-parametric way to estimate the probability density function of a
314 random variable [38, 39]. KDE handles the fundamental data smoothing problem
315 where inferences about the population are made based on finite data sampling.
316 Adopting such a method typically involves analyzing a large amount of historical
317 data. Detailed discussions on such approaches, however, are beyond the scope of
318 this paper.

319 4. Optimization Algorithms

320 In this section, we present the optimization algorithms for solving problem (6).
321 We first develop a robust approach for handling constraints (3) and (4), and then
322 decompose (6) into a subproblem and a main problem to allow easier solution.

323 Finally, a robust approach for tackling the financial risk inducted by time varying
 324 electricity market clearing prices is demonstrated.

325 4.1. Robust Approach for Constraints (3) and (4)

326 As shown in (3), the net demand balance can be expressed as $V^h + \sum_{a \in \mathcal{A}} x_a^h \geq$
 327 L^h . In practice, a decision criterion is to properly set decision $V^h + \sum_{a \in \mathcal{A}} x_a^h$ to
 328 allow good confidence that (3) is satisfied. To achieve that, we may introduce a
 329 small value ϵ to control the degree of conservatism and change the above expres-
 330 sion into a chance constraint:

$$\mathbf{P}(L^h \geq V^h + \sum_{a \in \mathcal{A}} x_a^h) \leq \epsilon \quad (11)$$

331 where ϵ is the fault tolerance limit of the power grid, representing the acceptable
 332 probability that the desirable power supply is not attained. Then we can have this
 333 expression that

$$\max_{f_0(L^h) \in \mathcal{U}_r(g_h, D_h)} \mathbf{P}(L^h \geq V^h + \sum_{a \in \mathcal{A}} x_a^h) \leq \epsilon, \quad (12)$$

334 which is equivalent to:

$$\max_{f_0(L^h) \in \mathcal{U}_r(g_h, D_h)} \int_{V^h + \sum_{a \in \mathcal{A}} x_a^h}^{+\infty} f_0(L^h) dL^h \leq \epsilon. \quad (13)$$

335 Defining $\mathcal{L}^h = V^h + \sum_{a \in \mathcal{A}} x_a^h$ as the robust electricity supply (ES) decision,
 336 which equals the amount of electricity generated and imported at time slot h , we
 337 introduce an auxiliary function as follows:

$$h(L^h, \mathcal{L}^h) = \begin{cases} 0, & L^h \leq \mathcal{L}^h; \\ 1, & L^h > \mathcal{L}^h. \end{cases} \quad (14)$$

338 The left part of inequality (13) then can be formulated into an optimization prob-
 339 lem:

$$\max_{f_0(L^h)} \int_0^{+\infty} h(L^h, \mathcal{L}^h) \cdot f_0(L^h) dL^h \quad (15)$$

$$\text{s.t.} \quad \mathbb{E}_{f_0}[\ln f_0(L^h) - \ln g_h(L^h)] \leq D_h \quad (16)$$

$$\mathbb{E}_{f_0}[1] = 1 \quad (17)$$

340 Define $K_f^h(\mathcal{L}^h) = \max_{f_0(L^h) \in U_r(g_h, D_h)} \int_0^{+\infty} h(L^h, \mathcal{L}^h) \cdot f_0(L^h) dL^h$ as the worst-
 341 case fault probability. We can then get a worst-case mapping \mathcal{M}_{wc}^h which maps
 342 the robust ES decision \mathcal{L}^h to $K_f^h(\mathcal{L}^h)$:

$$\mathcal{M}_{wc}^h : \quad \mathcal{L}^h \longrightarrow K_f^h(\mathcal{L}^h). \quad (18)$$

343 Note that the degree of conservatism depends on the values of fault tolerance
 344 limit ϵ and the distance limit of uncertainty set D_h . When a less conservative
 345 control sequence is desired, we shall set a higher fault tolerance limit and a more
 346 lenient distance limit. A tradeoff exists between the degree of conservation and
 347 the reliability of the decision making.

348 4.2. Sub-Problem: Determine the Robust ES Decision Threshold

349 Since there exists a random variable L^h in the constraint, we cannot solve
 350 energy generation scheduling problem (6) directly. As aforementioned, we de-
 351 compose the problem into a subproblem and a main problem. The goal of the
 352 sub-problem is to determine the robust ES decision threshold \mathcal{L}^{h*} so that the con-
 353 straint (3) can be transformed into a solvable form.

354 **Theorem 1:** Problem (15)-(17) is a convex optimization problem.

355 The proof of this theorem is shown in Appendix-A. Through Theorem 1 and

356 Slater's condition, we can see that strong duality holds for problem (15)-(17).

357 Adopting the Lagrangian method, we can obtain the worst-case fault probability

358 $K_f^h(\mathcal{L}^h)$ as follows:

$$K_f^h(\mathcal{L}^h) = \min_{\tau, \eta} \max_{f_0(L^h)} \mathbb{E}_{f_0} \left[h(L^h, \mathcal{L}^h) - \eta - \tau \ln \frac{f_0(L^h)}{g_h(L^h)} \right] + \tau D_h + \eta,$$

359 where $\tau \geq 0$ and η are Lagrangian multipliers associated with constraints (16)

360 and (17), respectively. Let

$$\mathcal{P}(L^h, f_0, \tau, \eta) = \mathbb{E}_{f_0} \left[h(L^h, \mathcal{L}^h) - \eta - \tau \ln \frac{f_0(L^h)}{g_h(L^h)} \right],$$

361 the derivative of $\mathcal{P}(L^h, f_0, \tau, \eta)$ with respect to f_0 can be derived as

$$\begin{aligned} \frac{\partial \mathcal{P}}{\partial f_0} &= \lim_{t \rightarrow 0} \frac{1}{t} \left[\mathcal{P}(f_0(L^h) + t \cdot g_0(L^h)) - \mathcal{P}(f_0(L^h)) \right] \\ &= \int_0^{+\infty} \left(h(L^h, \mathcal{L}^h) - \tau \ln \frac{f_0(L^h)}{g_h(L^h)} - \eta - \tau \right) g_0(L^h) dL^h. \end{aligned}$$

362 Adopting the Karush-Kuhn-Tucker (KKT) optimality condition, we have

$$h(L^h, \mathcal{L}^h) - \tau \ln \frac{f_0(L^h)}{g_h(L^h)} - \eta - \tau = 0 \quad (19)$$

$$\int_0^{+\infty} f_0(L^h) dL^h = 1 \quad (20)$$

$$\mathbb{E} \left[\ln \frac{f_0(L^h)}{g_h(L^h)} \right] - D_h \leq 0 \quad (21)$$

$$\tau \cdot \left(D_h - \mathbb{E} \left[\ln \frac{f_0(L^h)}{g_h(L^h)} \right] \right) = 0 \quad (22)$$

363 From (19), the optimal distribution function can be expressed as follows:

$$f_0^*(L^h) = g_h(L^h) \exp \left(\frac{h(L^h, \mathcal{L}^h) - \eta}{\tau} - 1 \right). \quad (23)$$

364 The dual variables (τ, η) in (23) should be chosen properly such that conditions
 365 (20)-(22) are satisfied. Specifically, we have the following results.

366 **Theorem 2:** The choice of (τ, η) is a solution of the following nonlinear equa-
 367 tions.

$$H_1(\tau, \eta) = R(\mathcal{L}^h)e^{-\eta/\tau} + S(\mathcal{L}^h)e^{(1-\eta)/\tau} - 1 = 0 \quad (24)$$

$$H_2(\tau, \eta) = S(\mathcal{L}^h)e^{(1-\eta)/\tau} - \eta - \tau(1 + D_h) = 0, \quad (25)$$

368 where $S(\mathcal{L}^h) = (1 - G_h(\mathcal{L}^h)) \exp(-1)$, $R(\mathcal{L}^h) = G_h(\mathcal{L}^h) \exp(-1)$, and $G_h(\mathcal{L}^h) =$
 369 $\int_{L^h \leq \mathcal{L}^h} g_h(L^h) dL^h$ denotes the cumulative distribution function of reference dis-
 370 tribution $g_h(L^h)$.

371 The proof for Theorem 2 is straightforward by substituting (23) to (20)-(22).
 372 However, it is still rather difficult to obtain an explicit solution from (24) and (25).
 373 Hence we propose the Newton iteration method as detailed in Algorithm 1.

374 Once we determine the solutions for (24) and (25) in Theorem 2, we can obtain
 375 the worst-case fault probability from (19) and (22) as follows:

$$K_f^h(\mathcal{L}^h) = \mathbb{E}_{f_0^*}[h(L^h, \mathcal{L}^h)] = (1 + D_h)\tau + \eta. \quad (26)$$

376 Our next step is to find the robust ES decision threshold \mathcal{L}^{h*} such that $K_f^h(\mathcal{L}^{h*}) =$
 377 ϵ , which involves the calculation of inverse function of $K_f^h(\mathcal{L}^h)$ that is not directly
 378 possible from (26). The following property of the function $K_f^h(\mathcal{L}^h)$, however, may
 379 help us design such a search method.

380 **Theorem 3:** The worst-case fault probability $K_f^h(\mathcal{L}^h)$ is non-decreasing with
 381 respect to the robust ES decision \mathcal{L}^h .

382 It is straightforward to derive Theorem 3 since $dK_f^h(\mathcal{L}^h)/d\mathcal{L}^h =$
383 $d\mathbb{E}_{f_0^*}[h(L^h, \mathcal{L}^h)]/d\mathcal{L}^h = f_0^*(\mathcal{L}^h) \geq 0$. Though direct solution is not available, the
384 monotonicity of $K_f^h(\mathcal{L})$ enlightens us a bisection method to search for the solution
385 for $K_f^h(\mathcal{L}^h) = \epsilon$. The main idea is to perform the search within an interval of $[0, \rho]$,
386 where ρ is an empirical constant such that $K_f^h(\rho) > \epsilon$.

387 Details of the algorithm for searching the robust ES decision threshold are
388 presented in Algorithm 1. Note that, from the 3rd to the 11th lines of the algo-
389 rithm, we use Newton iteration to solve the equation in Theorem 2 and obtain the
390 worst-case probability with fixed robust ES decision. Then we compare the worst-
391 case probability at \mathcal{L}^{h-} and \mathcal{L}^{h+} with the fault tolerance limit ϵ , respectively. The
392 comparison results help shrink the search region as shown in lines 12-14.

393 Once the robust ES decision threshold \mathcal{L}^{h*} for the constraint (3) is obtained
394 (and similarly, robust heat supply (HS) decision threshold \mathcal{S}^{h*} for constraint (4) is
395 obtained), we can approximate (3) and (4) with the following two constraints:

$$V^h + \sum_{a \in \mathcal{A}} x_a^h \geq \mathcal{L}^{h*} \quad (27)$$

$$U^h + \sum_{a \in \mathcal{A}} \eta_a \cdot x_a^h \geq \mathcal{S}^{h*}. \quad (28)$$

396 4.3. Main Problem: Robust Approach for the Uncertain Electricity Prices

397 There exist financial risks associated with real time electricity price uncer-
398 tainty where p_s^h are unknown quantities. We adopt certain intervals at the α -confidence
399 level for prices $p_s^h \in [\hat{p}_s^h, \hat{p}_s^h + d^h]$, $h \in \mathcal{H}$ and formulate the well defined robust
400 model [33] [34]. Specifically, we tackle the following optimization problem rather

401 than the original formulation (6):

$$\begin{aligned}
\min \quad & \sum_{h=1}^H \left\{ p_g \cdot U^h + \hat{p}_s^h \cdot V^h + \sum_{a \in \mathcal{A}} \left[c_a^m \cdot x_a^h \right. \right. \\
& \left. \left. + c_a^b \cdot y_a^h + c_a^s \cdot z_a^h \right] \right\} + \phi \cdot \Gamma + \sum_{h \in J_0} e^h \tag{29} \\
\text{s.t.} \quad & \phi + e^h \geq d^h \cdot k^h, \quad \forall h \in J_0 \\
& -k^h \leq V^h \leq k^h \\
& e^h \geq 0, \quad k^h \geq 0, \quad \phi \geq 0, \quad z_a^h \geq 0, \quad \forall h \in J_0 \\
& z_a^h \geq y_a^h - y_a^{h-1} \\
& \text{(2) (27) (28), } y_a^h, z_a^h \in \{0, 1\} \\
& x_a^h, V^h, U^h, k^h, e^h, \Gamma \in \mathbb{R}_0^+, h \in \mathcal{H}, a \in \mathcal{A}.
\end{aligned}$$

402 Robust problem (29) is obtained using duality properties and exact linear
403 equivalences. It represents the worse case while considering that electricity prices
404 can be uncertain in at most Γ slots. $J_0 = \{h \mid d^h > 0\}$ is the set of electricity
405 price p_s^h , $h \in \mathcal{H}$ that are subject to parameter uncertainty. Variable e^h is the dual
406 variable of the initial problem (6) used to consider the known bounds of electric-
407 ity prices, while ϕ and k^h are auxiliary variables used to obtain equivalent linear
408 expression. Readers can refer to Appendix-B for detailed description of how to
409 obtain this robust problem from problem (6). Γ is a parameter that controls the
410 level of robustness in the objective function. This parameter is assumed to be inte-
411 ger and takes value in the set $\{0, 1, 2, \dots, |J_0|\}$, i.e., between zero and the number
412 of unknown electricity prices. In this case, when $\Gamma = 0$, the influence of price
413 uncertainty in the objective function is ignored; when $\Gamma = |J_0|$, all possible price

414 deviations are taken into account, which is the most conservative case. In general,
415 a higher value of Γ increases the level of robustness at the expense of a higher
416 cost. Note that constraints (3) and (4) with random variables in the initial formu-
417 lation (6) are approximated and replaced by (27) (28) with no random variable.
418 This problem is a mixed integer linear programming (MILP) problem, which can
419 be effectively tackled by cutting plane method, branch and bounded method, etc.

420 **5. Simulation Results and Discussions**

421 In this section, we present simulation results based on real world traces to
422 assess the performance of the proposed energy generation scheduling scheme and
423 evaluate the effects of different parameters.

424 *5.1. Parameters and Settings*

425 *5.1.1. Net Demand and Heat Demand Trace*

426 We obtain the electricity and gas demand statistics from [40]. We focus on a
427 college at Forecasting Climate Zone (FCZ) 09. The electricity within this zone is
428 supplied by the Southern California Edison company. This trace contains hourly
429 electricity demand and heat demand of the college in year 2002. We assume there
430 are solar panels in the microgrid system. The area of solar panel in this microgrid
431 system is set to be $3.75 \times 10^4 m^2$. The energy conversion efficiency is 0.8. The
432 solar radiation intensity data is adopted from [41]. We employ electricity demand,
433 heat demand and solar power data of a typical month in winter (January) and esti-
434 mate the distributions of net demand (electricity demand minus solar energy) and

435 heat demand in each hour based on the samples using Kernel Density Estimation
436 [42]. We find that in all the time slots (hours), the distribution functions of net
437 demand and heat demand are close to be normal distribution. Thus, the reference
438 distribution of net demand and heat demand is set to be normal distribution.

439 5.1.2. CHP Generator Characteristics

440 The parameters of CHP generators are set based on the statistics in [43]. The
441 maximum output of a CHP generator is $E_a^{max} = 3.5$ MWh and the minimum
442 stable output is $E_a^{min} = 1.5$ MWh. The marginal cost for producing one unit
443 of electricity is $c_a^m = 0.051$ \$/KWh, which is obtained using the fuel price and
444 the energy conversion efficiency. The sunk cost for CHP generator keeping in
445 active mode is $c_a^b = 110$ \$/h, which includes the capital cost, operation cost and
446 maintenance cost. We set the start up cost to be $c_a^s = 560$ \$ and the heat-electricity
447 ratio to be $\eta = 2.065$ [43]. Finally, it is assumed there are 8 CHP generators in
448 this microgrid system unless otherwise stated.

449 5.1.3. Electricity and Gas Prices

450 The electricity price trace is obtained from [44] and the gas price data is ob-
451 tained from [45]. In our paper, we adopt the electricity market prices of central
452 New York Control Area (NYCA) on a typical day in January. We set \hat{p}_s^h and d^h be
453 equal to the lower bound and variation range of electricity market price at hour h ,
454 respectively. In addition, the natural gas price is set to be $p_g = 6.075$ \$/mmBTU.

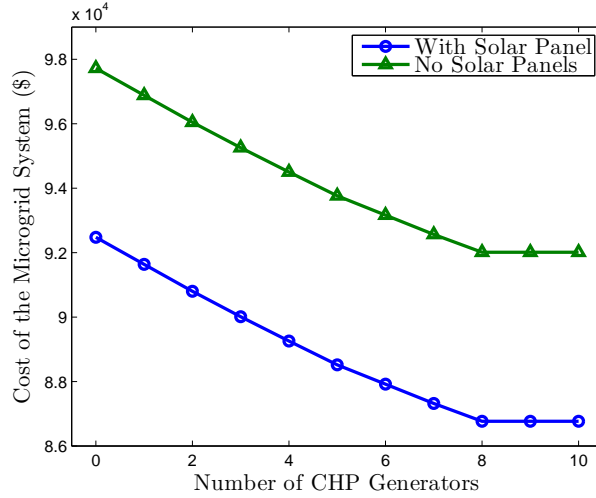


Figure 2: Cost reduction for different number of CHPs

455 *5.2. Results and Discussions*

456 *5.2.1. Robust ES Threshold and Robust HS Threshold*

457 We first solve the sub-problem and obtain the robust ES threshold \mathcal{L}^{h^*} and
 458 robust HS threshold \mathcal{S}^{h^*} for solving the main problem. The reference distributions
 459 of net demand and heat demand are normal and are estimated from sample data.
 460 The distance limit of net demand and heat demand uncertainty sets is 10^{-1} . The
 461 fault tolerance limit of net demand supply is 10^{-2} while the fault tolerance limit
 462 of heat demand supply is 10^{-1} . Given reference distributions, distance limits, and
 463 fault tolerance limits, we obtain \mathcal{L}^{h^*} and \mathcal{S}^{h^*} based on **Algorithm 1**. The results
 464 are shown in Table 2.

Table 2: Parameters of distribution uncertainty sets and corresponding ES and HS thresholds (unit: MWh for electricity and mmBTU for heat. \bar{m}_E^h and σ_E^h are mean and standard deviation of net demand reference distribution, respectively. \bar{m}_H^h and σ_H^h are mean and standard deviation of heat demand reference distribution, respectively)

Time Slot	\bar{m}_E^h	σ_E^h	\mathcal{L}^{h*}	\bar{m}_H^h	σ_H^h	\mathcal{S}^{h*}
1	18.44	0.1059	18.98	63.88	8.3372	81.65
2	18.08	0.0965	18.57	51.96	5.0481	62.72
3	18.06	0.1005	18.58	43.63	1.7780	47.42
4	18.43	0.1246	19.07	46.62	1.8902	50.64
5	20.60	0.1456	21.34	50.39	1.7311	54.08
6	24.67	0.3807	26.61	80.35	7.5946	96.53
7	32.18	1.6355	40.52	124.93	1.4380	127.99
8	44.08	1.9485	53.50	283.69	8.0012	300.74
9	64.06	3.7971	78.77	285.91	6.4596	299.67
10	58.64	2.2394	60.54	254.82	7.5097	270.82
11	59.28	2.3199	58.88	219.39	10.7104	242.21
12	58.73	2.2730	56.27	195.55	10.1975	217.28
13	58.68	2.2121	54.18	183.64	11.0907	207.27
14	58.77	2.3731	56.66	177.02	11.6296	201.79
15	58.70	2.4761	61.38	171.43	12.0786	197.17
16	57.91	2.5475	64.65	167.69	12.1597	193.59
17	57.32	2.2805	67.77	166.47	12.6110	193.34
18	55.41	2.0156	65.69	169.83	14.0442	199.75
19	53.16	2.2647	64.72	176.10	14.0746	206.09
20	47.58	2.5553	60.62	184.35	14.3077	214.83
21	41.59	3.3157	58.51	190.49	15.3283	223.14
22	35.99	3.4268	53.47	198.32	15.0698	230.43
23	27.40	2.9277	42.34	111.43	10.2832	133.33
24	20.05	0.2638	21.40	78.80	7.7375	95.29

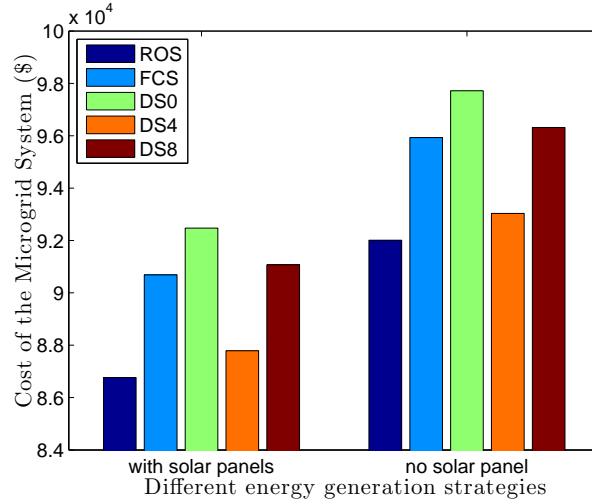


Figure 3: Costs of different generation scheduling strategies

465 *5.2.2. Potential Benefits of CHP Generators and Solar Panels*

466 Once we obtain robust ES threshold \mathcal{L}^{h*} and robust HS threshold \mathcal{S}^{h*} , we are
 467 ready to adopt the robust optimization approach to study the energy generation
 468 scheduling problem (29) with respect to real time electricity market prices. Prob-
 469 lem (29) is solved using the data provided in the previous subsection 5.2.1. The
 470 problem is solved using MOSEK optimization toolbox 7.0 on an Intel workstation
 471 with 6 processors clocking at 3.2 GHZ and 16 GB of RAM.

472 We first try to investigate the potential savings with CHP generators and solar
 473 panels. In particular, we conduct two sets of experiments. Both sets of experi-
 474 ments have nearly the same default settings, except that solar panels in the micro-
 475 grid are enabled in the first set, but not in the second one. We vary the number of
 476 CHP generators installed in the microgrid from 0 to 10 and compute the total cost

477 of the system in a day. The results are shown in Fig. 2. It is observed that having
478 8 CHP generators with full capacity 28 MW is sufficient to obtain nearly all the
479 cost saving benefits. Thus, we may suggest that installed CHP generator capacity
480 should be about half of the peak demand (The peak demand of a day in January
481 is around 60 MW.). The intuitive reason is that most of the time, demands are
482 much lower than the peaks. This result can shed some light on making investment
483 decisions in microgrids. Note that the leftmost points in the two curves denote
484 the case where microgrid only uses external electricity and local heat generators
485 (without CHP generators). System cost in this case can be interpreted as a cost
486 benchmark. The results show that CHP can bring a saving of 6.2% (around \$5700
487 per day) to the system. Finally, by comparing the two curves in Fig. 2, we find
488 that the one day cost reduction achieved by solar panels is about 6.05% (around
489 \$5200 per day).

490 5.2.3. Comparisons of Different Generation Scheduling Strategies

491 We compare 3 energy generation scheduling strategies: (1) the proposed ro-
492 bust optimal strategy (ROS); (2) fixed choice strategy (FCS): making one fixed
493 choice of the generation level for entire duration for each generator. The sys-
494 tem cost induced by this strategy has been used as a benchmark in literature [46];
495 (3) deterministic strategy (DS): A fixed number of CHP generators are switched
496 on for the entire time horizon. The microgrid has to properly schedule the out-
497 put level of active CHP generators, imported energy and local heat generators
498 to meet electricity and heat demand. Specifically, we consider 3 schemes with

499 0, 4 and 8 CHP generator(s) in active mode and termed as DS0, DS4 and DS8,
500 respectively. We emphasize that the microgrid always tries to find the optimal
501 control sequences under any of these three generation scheduling strategies and
502 the scheduling choices of the last two methods for comparison (i.e., FCS and DS)
503 are made in hindsight. In addition, all the three scheduling strategies adopt the
504 same parameter settings. The cost comparison results are depicted in Fig. 3.

505 As we observe in Fig. 3, ROS can achieve a cost saving of 4.5% (about \$3900
506 per day), 6.5% (about \$5700 per day), 1.2% (about \$1000 per day) and 5.0%
507 (about \$4300 per day) compared with FCS, DS0, DS4 and DS8, respectively
508 (equipped with solar panels). Moreover, we note that only using external elec-
509 tricity (DS0) or switching on all the local generators (DS8) are not economical.
510 Another interesting observation is that the cost of DS8 is lower than that of DS0.
511 This shows that when all the CHP generators are switched on, although a signif-
512 icant amount of electricity may be wasted in the off-peak time slots, the strategy
513 nevertheless still achieves better performance than the case where all electricity is
514 imported from outside power grid. This justifies the economic potential of using
515 local CHP generators. Obviously, DS4 achieves more cost savings than DS0 and
516 DS8. This is because that when half of the CHP generators are turned on, a con-
517 siderable proportion of the electricity demand can be supplied by CHP generators
518 and the energy loss in off-peak hours is relatively low than that in DS8.

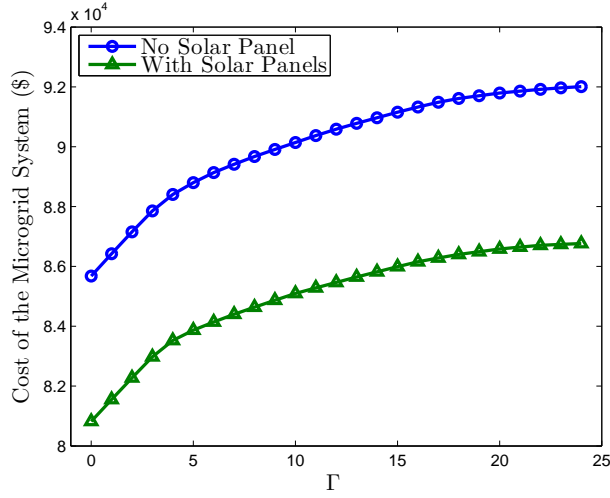


Figure 4: System cost with respect to robustness level Γ .

519 *5.2.4. The Impact of Robustness Level Γ*

520 The sensitivities of the electricity cost with respect to robustness level Γ are
521 depicted in Fig. 4. We set $|J_0| = 24$, i.e., price uncertainty may exist in all time
522 slots of the day. We are interested in finding an optimal solution which optimizes
523 against all scenarios under which a number Γ of the electricity prices can vary in
524 such a way as to maximally influence the objective. We vary the value of Γ from
525 0 to 24 in formulation (29) and obtain the optimal system cost. Remember that
526 the value of Γ indicates the number of worst-case prices during the 24 time slots.
527 $\Gamma = 0$ corresponds to the lowest robustness level while $\Gamma = 24$ corresponds to the
528 highest robustness level. Apparently, the system cost is an increasing function of
529 Γ . The incremental cost when the robustness level grows is the price for tackling
530 the financial risks. We observe that to fully overcome the financial risks (i.e. the

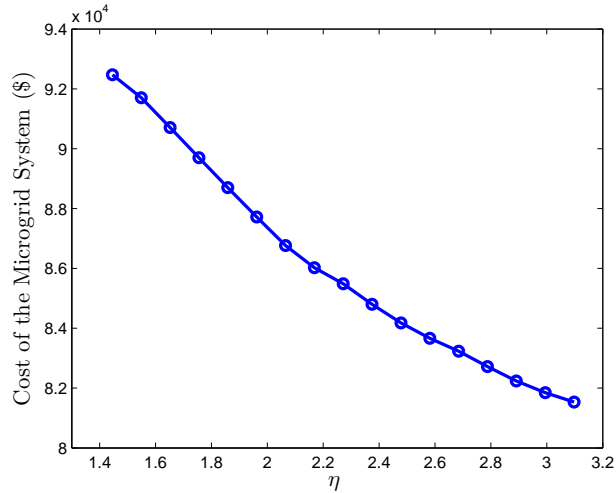


Figure 5: Cost profile with respect to different η

531 most conservatism condition), the microgrid has to pay additional 7.35% (about
 532 \$5900 per day) expenditures. However the rise rate of the cost curve slows down
 533 when Γ increases. The reason is that when Γ increases, the protection level for
 534 the robust solution increases, then the probability that the robust solution is not
 535 favorable declines. Hence, it becomes less costly to protect the microgrid against
 536 the financial risk. We also compare the costs of two scenarios where solar panels
 537 are available and not available, respectively. The difference between these costs
 538 is called cost gap. It is interesting to note that cost gap only rises marginally when
 539 Γ increases. This shows that the uncertainty of solar energy has little impacts on
 540 the financial risks of the system since the indeterminacy of it has been alleviated
 541 by the proposed robust approach in the sub-problem.

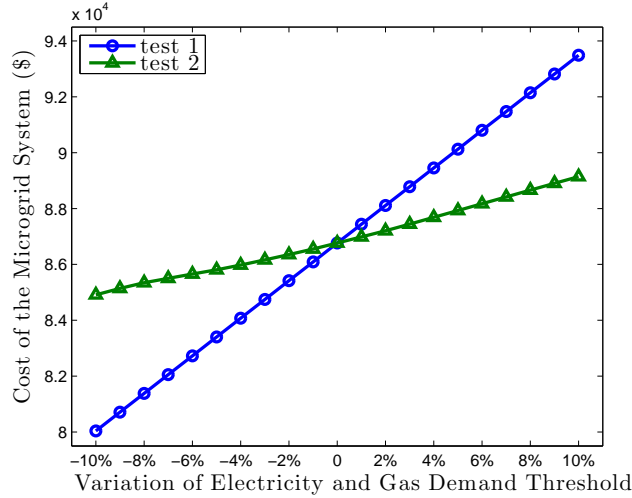


Figure 6: Cost sensitivity with the variation of \mathcal{L}^* and \mathcal{S}^*

542 *5.2.5. The Impacts of Heat-Electricity Ratio η*

543 Figure 5 depicts the reduction in cost versus heat-electricity ratio η . It appears
544 that system cost decreases when η grows. The reason is that a larger η means
545 CHP generators can provide more heat for free. In this case, the microgrid can
546 reduce the reliance on local heat generators, which can be seen from Eq. (28).
547 Meanwhile, we observe that the decrease rate slows down when η increases. This
548 observation is intuitive since when η is large, nearly all the heat demands can be
549 supplied by CHP generators for free. Therefore, additional free heat cannot bring
550 significant benefits as the heat may be wasted.

551 *5.2.6. System Cost Sensitivity to the Robust ES and HS Thresholds*

552 In Fig. 6, we illustrate the relationship between the system cost and variation
553 of \mathcal{L}^{h*} and \mathcal{S}^{h*} . Specifically, we conduct two tests. In the first test, \mathcal{S}^{h*} remains

554 unchanged and we vary the value of \mathcal{L}^{h^*} ; while in the second one, \mathcal{L}^{h^*} remains
 555 constant and \mathcal{S}^{h^*} varies. It is observed that the system cost has a nearly linear
 556 relationship with \mathcal{L}^{h^*} and \mathcal{S}^{h^*} , which is consistent with the theoretical formulation
 557 (29). From (29) we see that the objective function have linear relationships with
 558 variables V^h , U^h and $\sum_{a \in \mathcal{A}} x_a^h$, $h \in \mathcal{H}$. However, due to the tradeoff between
 559 using local CHP generators and outside electricity when we vary \mathcal{L}^{h^*} and \mathcal{S}^{h^*} , the
 560 relation between system cost and \mathcal{L}^{h^*} (\mathcal{S}^{h^*} as well) is only approximately linear.
 561 Also note that system cost is more sensitive to the variation of \mathcal{L}^{h^*} . Since a large
 562 proportion of heat demands are satisfied by CHP generators for free, the system
 563 expenditure on heating is much lower than that on generating or buying electricity.
 564 Hence, the variation of heat demand has lower impacts on the system cost.

565 6. Conclusions

566 In this paper, we studied the energy generation scheduling problem in a mi-
 567 crogrid scenario to minimize the cost and maintain system stability. To tackle
 568 the randomness of net demand and heat demand, we introduced reference distri-
 569 butions and then defined distribution uncertainty sets to confine the fluctuations.
 570 Such a model allows convenient handling of volatile demands as long as the de-
 571 mand profiles are not too intensely different from the predictions or empirical
 572 knowledge. The uncertainty in electricity price was addressed by bounded random
 573 variables. We developed chance constraint approximations and robust optimiza-
 574 tion algorithms to firstly transform and then solve the problem. Numerical results
 575 based on real-world data indicate the satisfactory efficiency of the proposed en-

576 ergy scheduling strategy and the cost benefits of CHP generators. Moreover, the
 577 impacts of different parameters have been carefully evaluated. Such evaluations,
 578 as we believe, shall provide useful insights helping microgrid operators develop
 579 rational investment strategies.

580 In our future work, we will consider a microgrid where there are energy stor-
 581 ages (batteries and heat accumulators) in the system. In such a system, the energy
 582 storages will impose their own cost; meanwhile they may to a certain extent al-
 583 leviate the uncertainty problem caused by the fluctuations of the net demand and
 584 heat demand, especially when the storages are of a large enough capacity. The
 585 optimization problem for this scenario therefore becomes significantly different
 586 from the one we considered in this paper and worth further studies.

587 **Appendix A. Proof of Theorem 1**

588 *Proof.* Rewrite (15)-(17) as follows:

$$\begin{aligned}
 \max_{f_0(L^h)} \quad & \int_0^{+\infty} h(L^h, \mathcal{L}^h) \cdot f_0(L^h) dL^h & (A.1) \\
 \text{s.t.} \quad & \int_0^{+\infty} [\ln f_0(L^h) - \ln g_h(L^h)] f_0(L^h) dL^h \leq D_h \\
 & \int_0^{+\infty} f_0(L^h) dL^h = 1.
 \end{aligned}$$

589 We can see that the objective function and equality constraint function are affined
 590 with respect to $f_0(L^h)$. Next we show that the inequality constraint function is
 591 convex.

592 *Lemma:* If $f : \mathbf{R}^n \rightarrow \mathbf{R}$ is convex, then the perspective of f , which is

593 denoted as a function $g : \mathbf{R}^{n+1} \longrightarrow \mathbf{R}$ that

$$g(x, t) = tf(x/t), \quad (\text{A.2})$$

594 with domain

$$\mathbf{dom} g = \{(x, t) | x/t \in \mathbf{dom} f, t > 0\} \quad (\text{A.3})$$

595 preserves convexity.

596 That is to say, if f is a convex function, so is its perspective function g . Sim-
597 ilarly, if f is concave, so is g . This can be proved in several ways, e.g., by direct
598 verification of the defining inequality or using epigraphs and the perspective map-
599 ping on \mathbf{R}^{n+1} . Readers can refer to [47] for more detailed discussions.

600 We consider the convex function $f(x) = -\ln x$ on \mathbf{R}_{++} . Its perspective is

$$g(x, t) = -t \ln(x/t) = t \ln(t/x) = t(\ln t - \ln x) \quad (\text{A.4})$$

601 and it is convex on \mathbf{R}_{++}^2 . The function g is called the relative entropy of t and
602 x . Then we have that the KL divergence $\int_{x \in S} [\ln f(x) - \ln g(x)] f(x) dx$ between
603 distribution $f(x)$ and $g(x)$ is convex in $f(x)$ (and $g(x)$ as well). In this case, we
604 claim that the inequality constraint is convex with respect to distribution $f_0(L^h)$.

605 □

606 **Appendix B. Reformulation of Problem (6)**

607 Specifically, the robust counterpart of Problem (6) is as follows:

$$\begin{aligned}
\min_{\mathbf{x}, \mathbf{Y}, \mathbf{Z}, \mathbf{V}, \mathbf{U}} \quad & \sum_{h=1}^H \left\{ p_g \cdot U^h + \hat{p}_s^h \cdot V^h + \right. \\
& \left. \sum_{a \in \mathcal{A}} \left[c_a^m \cdot x_a^h + c_a^b \cdot y_a^h + c_a^s \cdot z_a^h \right] \right\} \\
\text{s.t.} \quad & + \max_{\{W_0 | W_0 \subseteq J_0, |W_0| \leq \Gamma\}} \left\{ \sum_{h \in W_0} d^h \cdot V^h \right\} \\
& z_a^h \geq 0, \quad z_a^h \geq y_a^h - y_a^{h-1} \\
& (2) (3) (4), \quad y_a^h, z_a^h \in \{0, 1\} \\
& x_a^h, V^h, U^h \in \mathbb{R}_0^+, \quad h \in \mathcal{H}, a \in \mathcal{A},
\end{aligned} \tag{B.1}$$

608 **Theorem 4:** Problem (B.1) has an equivalent MIP formulation as (29).

609 *Proof.* Given a vector \mathbf{V}^* , we can convert the last part of Problem (B.1)'s objec-
610 tive function to a linear one as follows:

$$\begin{aligned}
\beta_0(\mathbf{V}^*) &= \max \left\{ \sum_{h \in W_0} d^h \cdot V^{h*} : W_0 \subseteq J_0, |W_0| \leq \Gamma \right\} \\
&= \max \left\{ \sum_{h \in J_0} d^h \cdot V^{h*} \cdot \phi_h : \sum_{h \in J_0} \phi_h \leq \Gamma, \right. \\
&\quad \left. 0 \leq \phi_h \leq 1, \forall h \in J_0 \right\}.
\end{aligned} \tag{B.2}$$

611 Next, the dual of Problem (B.2) is:

$$\begin{aligned}
\min \quad & \sum_{h \in J_0} e^h + \Gamma \cdot \phi \\
\text{s.t.} \quad & \phi + e^h \geq d^h \cdot V^{h*} \\
& \phi \geq 0, e^h \geq 0, \forall h \in J_0.
\end{aligned} \tag{B.3}$$

612 By strong duality, we have:

$$\beta_0(\mathbf{V}^*) = \min \left\{ \sum_{h \in J_0} e^h + \Gamma \cdot \phi : \right. \quad (\text{B.4})$$
$$\left. \phi + e^h \geq d^h \cdot V^{h*}, \phi \geq 0, e^h \geq 0, \forall h \in J_0 \right\}.$$

613 Substituting (B.4) to Problem (B.1), we obtain that Problem (B.1) is equivalent to
614 Problem (29). □

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Algorithm 1 Search for robust ES decision threshold \mathcal{L}^{h*}

Input: Reference distribution $g_h(L^h)$;

Distance limit D_h ; Search radius ρ ;

Load balance fault tolerance limit ϵ ; Tolerance ε .

Output: Robust ES decision threshold such that $K_f^h(\mathcal{L}^{h*}) = \epsilon$;

- 1: **Begin**
 - 2: **initialize** $\mathcal{L}^{h-} = 0$, $\mathcal{L}^{h+} = \rho$, and set $\mathbf{H}(\tau, \eta) = [H_1(\tau, \eta), H_2(\tau, \eta)]^T$
 - 3: **while** $|\mathcal{L}^{h-} - \mathcal{L}^{h+}| > \varepsilon$
 - 4: **set** $\bar{\mathcal{L}}^h = \frac{\mathcal{L}^{h-} + \mathcal{L}^{h+}}{2}$, initiate the time iteration $k = 1$
 - 5: **while** $\mathbf{H}(\tau, \eta) > \varepsilon$
 - 6: **evaluate** $\mathbf{H}(\tau, \eta)$ and Jacobian matrix $\mathbf{J}(\tau, \eta)$
 - 7: **solve** $\mathbf{J}(\tau, \eta)\Delta\mathbf{x}_k = -\mathbf{H}(\tau, \eta)$
 - 8: **update** $\tau_{k+1} = [\tau_k + \Delta\tau_k]^+$, $\eta_{k+1} = \eta_k + \Delta\eta_k$
 - 9: **set** $k = k + 1$
 - 10: **end while**
 - 11: **update** $K_f^h(\bar{\mathcal{L}}^h) = (1 + D_h)\tau_{k+1} + \eta_{k+1}$
 - 12: **if** $(K_f^h(\bar{\mathcal{L}}^h) - \epsilon)(K_f^h(\mathcal{L}^{h-}) - \epsilon) < 0$
 - 13: **then set** $\mathcal{L}^{h-} = \bar{\mathcal{L}}^h$ **else set** $\mathcal{L}^{h+} = \bar{\mathcal{L}}^h$ **end if**
 - 14: **if** $|K_f^h(\bar{\mathcal{L}}^h) - \epsilon| < \varepsilon$ **break end if**
 - 15: **end while**
 - 16: **set** $\mathcal{L}^{h*} = \bar{\mathcal{L}}^h$
 - 17: **End**
-