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Keywords: Time reversal; ultra-wideband; broadband; SINR; correlation; channel estimation error

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Abstract: In this letter, we analyze the performance of multi-user multiple-input single-output time reversal (MU MISO TR)-based systems in a generalized framework. The considered propagation is correlated in both the space and the frequency domains, and channel estimation errors (CEEs) at the transmitter side are also taken into account. We derive a novel average signal-to-interference-plus-noise (SINR) formula in closed-form for TR-based systems. This formula is based on the exact expressions of the expectations of the powers of the desired signal, the inter-symbol interference (ISI) and the inter-user interference (IUI) components. These expressions allow us to reveal, for the first time, properties of time reversal which remained unknown up to now. More precisely, CEE has no influence on the ISI in terms of average power, whereas it degrades the power of the desired message-bearing signal. Moreover, irrespective of the central tap, the other taps of the IUI are independent from the effects of both inter-user correlation and CEE. Our analytical results are validated in both ultra-wideband and conventional broadband channels. Finally, these results are numerically compared to previous works.

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Vu Tran-Ha		2014/12/11
Hieu Nguyen		2014/12/11
Een-Kee Hong		2014/12/11

Response to Review of Paper:

Generalized Analysis of MU-MISO Time Reversal-based Systems over Correlated Multipath Channels with Estimation Error

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We would like to thank all the reviewers and the editor for their thorough reviews and constructive comments. We have carefully revised the paper to clarify and address the reviewers' comments. In the document that follows, we describe changes made to the preivous version and address the comments of the reviewers.

I. Response to reviewer 1

The authors would like to thank the reviewer for his/her valuable comments and suggestions. Below please find the detailed modifications we have made to the original manuscript to address all the comments, and the rationale used for each modification as well.

Reviewer's comments	Authors' responses
The quality of this paper has improved significantly after the second revision. However, I found new problems in equations (9) and (especially) (18):	
In the signal model given by (9) the dimensions of y_n and s_n need to be defined carefully.	We would like to clarify the dimensions as follows $\mathbf{y}_n \in \mathbb{C}^{(2L-1)\times 1}$, $s_n \in \mathbb{C}$ and $s_{n'} \in \mathbb{C}$
After reviewing the cited references, equation (18) only applies to real jointly Gaussian random variables. This equation is used subsequently in the derivations. Does (18) apply to complex random variables as well? (the extension does not seem trivial). Also, are all the variables in the derivations jointly gaussian? If so, please explain.	In [23], the authors also proved for the validation of (18) in the case of complex jointly Gaussian random variables. Please visit the page 869 and see *Lemma 1* in [23] for more details.
Minor comments are attached in a .pdf document. These problems need to be addressed before publication. Thus, I recommend that the paper is accepted after a minor revision.	We have addressed the reviewer's comments. Especially, for the following comment, "The variables inside the expectation are scalars, and should not be in bold font." We have corrected all related equations in the paper.

II. Response to reviewer 2

The authors would like to thank the reviewer for his/her valuable comments and suggestions. Below please find the detailed modifications we have made to the original manuscript to address all the comments, and the rationale used for each modification as well.

Reviewer's comments	Authors' responses
Thank you for taking into account my previous comments. The paper has been greatly improved. The work is visible. It still needs some minor corrections. Please take into account my last comments, given in the attached 'annotated' manuscript (AEUE-D-14-00856R2_comments.pdf file).	We appreciate all comments by the reviewer. In the revised manuscript, we have incorporated the suggestions.

*Manuscript

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***Abstract*—In this letter, we analyze the performance of multi-user multiple-input single-output time reversal (MU MISO TR)-based systems in a generalized framework. The considered propagation is correlated in both the space and the frequency domains, and channel estimation errors (CEEs) at the transmitter side are also taken into account. We derive a novel average signal-to-interference-plus-noise (SINR) formula in closed-form for TR-based systems. This formula is based on the exact expressions of the expectations of the powers of the desired signal, the inter-symbol interference (ISI) and the inter-user interference (IUI) components. These expressions allow us to reveal, for the first time, properties of time reversal which remained unknown up to now. More precisely, CEE has no influence on the ISI in terms of average power, whereas it degrades the power of the desired message-bearing signal. Moreover, irrespective of the central tap, the other taps of the IUI are independent from the effects of both inter-user correlation and CEE. Our analytical results are validated in both ultra-wideband and conventional broadband channels. Finally, these results are numerically compared to previous works.**

***Index Terms*—Time reversal, ultra-wideband, broadband, SINR, correlation, channel estimation error.**

I. Introduction

Due to the fast development of wireless communications, trends for the design of energy efficient and green networks are emerging [1], [2]. The time reversal (TR) technique is one of the most popular forms of linear precoding, especially for ultra-wideband (UWB) systems over multipath channels. It is considered as a beamforming technique [2]-[14]. TR tightly focuses the energy of all the taps of the propagation channel, in the time and space domains at the intended terminals by utilizing the time-reversed of channel impulse response (CIR) to prefilter transmit signals. Since the TR technique exploits the channel propagation diversity at the transmitter side, to perform focusing, complex equalizers as well as large numbers of antennas can be avoided at the receiver side.

Due to the focalization property of TR, much research efforts have been put recently on the analysis of TR-based transmission [5]-[11]. Popovski *et al.* [6] give approximate derivations of ISI and IUI for TR-UWB system. The performance analysis of a MISO TR UWB system with a decision feedback equalizer is also found in [7]. The analysis conducted in [2], [5] and [11] consider specific wideband channels where the average power of each tap decays exponentially. However, the applications of TR techniques can be completely extended to conventional broadband systems as shown in [11]-[13]. In fact, a channel with a narrower bandwidth has a smaller multipath delay spread [14]. Such channel has a lower diversity and a lower multiplexing gain [15], [16]. In broadband systems, the transmission bandwidth is reduced as well as the amount of scattering in the propagation channel, in comparison with the bandwidth and the channel of UWB systems. For TR-based broadband systems in particular as well as for generalized TR-based systems, the correlation should therefore be taken into account. Furthermore, in practice, the accuracy of the estimated channel plays an importance role in determining the performance of TR systems. Although the effects of CEE on TR have been studied in [8]-[10], there have been no works conducting the analysis for multi-user systems assuming imperfect CIR condition.

In this letter, we take into account a MU MISO TR-based system model that is *generalized* by considering (*i*) the frequency selective channel that has arbitrary power in each tap, (*ii*) the correlation at both transmitter and user sides, and (*iii*) the channel estimation errors. In fact, most previous works do not analyze this general system. This directly motivates us to determine in this letter the exact generalized closed-form expressions for the desired signal, ISI and IUI terms, respectively. Based on such closed-form derivations, we first show that CEE and inter-user correlation do not impact the average power of ISI whereas CEE explicitly decreases the average signal power. For the IUI component, we find that the effect of correlation on the central tap is distinct from the effects of the other taps. More precisely, the average power of the taps of the IUI excluding the power of the main tap is immune to the effects of inter-user correlation and CEE. Finally, the validity of our analysis is verified by means of Monte-Carlo simulation and the comparison with a previous work [5].

II. System Description

We consider a TR-based system consisting of a BS equipped with M transmit antennas and N single-antenna users. In the multipath channel, we assume that the maximum length of each CIR is L. Thus, the CIR between the m-th transmit antenna and the n-th user is

$$h_{mn}(t)=\sum_{l=1}^{L}\alpha_{mn}^{(l)}\delta(t-\tau_{mn}^{(l)}),\qquad \left(1\le l\le L\right) \tag{1}$$

where $\alpha_{mn}^{(l)}$ and $\tau_{mn}^{(l)}$ are the amplitude and the delay of the l-th tap, respectively. The CIR can be discretized in the time domain as a vector $\mathbf{h}_{mn}\in\mathbb{C}^{L\times 1}$ in which $\mathrm{E}\left[h_{mn}[l]\right]=0$, $\mathrm{E}\left[\left|h_{mn}[l]\right|^2\right]=\mathrm{E}\left[\left|\alpha_{mn}^{(l)}\delta(t-\tau_{mn}^{(l)})\right|^2\right]=\sigma_{mn,l}^2$, and $\mathrm{E}\left[\cdot\right]$ represents the expectation operator. We can arrange the propagation channels in an $MN\times L$ matrix form as

$$\mathbf{H}=\left[\mathbf{h}_{11}\ \ldots\ \mathbf{h}_{M1}\ \ldots\ \mathbf{h}_{1n}\ \ldots\ \mathbf{h}_{Mn}\ldots\ \mathbf{h}_{1N}\ \ldots\ \mathbf{h}_{MN}\right]^T. \tag{2}$$

The transmit antenna and inter-user correlations should be taken into account because of the validation of our analysis in low scattering environments. The correlation can be included into the channel model by introducing transmit and receive correlation matrices following the well-known Kronecker model [17]. This correlation model is widely applied in the literature for correlated multi-antenna systems. In our system, we assume that the distances between the BS and users are large, and only antennas from the same equipment (either the transmitter or the receiver) are correlated, due to scattering and electromagnetic coupling. In other words, the correlation between transmit antennas and receive antennas can be omitted. We also assume that all channels convey the same average power. The Kronecker model is used to take into account the correlation in the channel matrix $\mathbf{H}$. The expression of $\mathbf{H}$ is therefore given by

$$\mathbf{H}=\left(\left(\mathbf{R}_U^{1/2}\right)^T\otimes\mathbf{R}_T^{1/2}\right)\mathbf{H}_w, \tag{3}$$

where the inter-user and transmit correlations are represented by the real positive-define matrix $\mathbf{R}_U\in\mathbb{R}^{N\times N}$ and the real positive-define matrix $\mathbf{R}_T\in\mathbb{R}^{M\times M}$, respectively. $\mathbf{H}_w\in\mathbb{C}^{MN\times L}$ is the channel matrix of the independent CIRs. Note that $\otimes$ denotes the Kronecker product, and the correlation matrix follows the general model with arbitrary positive coefficients (i.e. $\rho_{T,mm'} \in \mathbb{R}$) . We provide hereafter an example of transmit correlation matrix $\mathbf{R}_T$

$$\mathbf{R}_T = \begin{bmatrix} 1 & \rho_{T,12} & \cdots & \rho_{T,1M} \\ \rho_{T,21} & 1 & \cdots & \rho_{T,2M} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{T,M1} & \rho_{T,M2} & \cdots & 1 \end{bmatrix}. \quad (4)$$

Moreover, in practice, only imperfect estimates of the CIRs are available at the transmitter side. The true channel $\mathbf{h}_{mn}$ is an unknown parameter to the transmitter. We model the impact of channel estimation errors as follows

$$\hat{\mathbf{h}}_{mn} = \mathbf{h}_{mn} + \mathbf{e}_{mn}, \quad (5)$$

where $\hat{\mathbf{h}}_{mn} \in \mathbb{C}^{L\times 1}$ and $\mathbf{e}_{mn} \in \mathbb{C}^{L\times 1}$ are the vectors of independently and identically distributed (i.i.d) variables denoting the estimated channel and error vectors respectively. With a nonnegative factor ψ, we define

$$\mathrm{E}\left[\left|e_{mn}[l]\right|^2\right] = \psi \mathrm{E}\left[\left|h_{mn}[l]\right|^2\right], \quad (6)$$

$$\mathrm{E}\left[\left|\hat{h}_{mn}[l]\right|^2\right] = \hat{\sigma}^2_{mn,l}, \quad (7)$$

Based on the TR scheme [2]-[5], we define $\hat{\mathbf{g}}_{mn} \in \mathbb{C}^{L\times 1}$ as the pre-filtering vector for the message-bearing signal with the transmit power p_n

$$\hat{g}_{mn}[l] = \sqrt{p_n}\,\hat{h}^*_{mn}[L+1-l] \Big/ \sqrt{\sum_{m=1}^{M} \mathrm{E}\left[\left\|\hat{\mathbf{h}}_{mn}\right\|^2\right]}. \quad (8)$$

Thus, we have the received signal vector $\mathbf{y}_n \in \mathbb{C}^{(2L-1)\times 1}$ at the n-th user as

$$\mathbf{y}_n = \sum_{m=1}^{M} s_n \hat{\mathbf{g}}_{mn} * \mathbf{h}_{mn} + \sum_{\substack{n'=1 \\ n'\neq n}}^{N} \sum_{m=1}^{M} s_{n'} \hat{\mathbf{g}}_{mn'} * \mathbf{h}_{mn} + \mathbf{n}, \quad (9)$$

in which the first, second and third terms are the received signal, the IUI and the additive white Gaussian noise, respectively. $s_n \in \mathbb{C}$ and $s_{n'} \in \mathbb{C}$ are the transmit signals for the n-th and n'-th users respectively, and the symbol $*$ represents the discrete convolution operator.

We denote $n[L] = \tilde{n}$, and $P_{sig}^{(n)}$, $P_{isi}^{(n)}$ and $P_{iui}^{(n)}$ as the power of the signal, the ISI and the IUI terms respectively. Using these notations, we propose the following closed-form expression of the SINR

$$\mathrm{E}[\gamma_n] = \mathrm{E}\left[\frac{\underbrace{\left| \sum_{m=1}^{M} (\hat{g}_{mn} * h_{mn})[L] \right|^2}_{P_{sig}^{(n)}}}{\underbrace{\sum_{\substack{k=1 \\ k\neq L}}^{2L-1} \left| \sum_{m=1}^{M} (\hat{g}_{mn} * h_{mn})[k] \right|^2}_{P_{isi}^{(n)}} + \underbrace{\sum_{\substack{n'=1 \\ n'\neq n}}^{N} \left\| \sum_{m=1}^{M} \hat{g}_{mn'} * h_{mn} \right\|^2}_{P_{iui}^{(n)}} + |\tilde{n}|^2} \right], \quad (10)$$

III. Average SINR Analysis

In this section, we analyze the SINR performance metric. The expression (10) can be derived as in

[18], [19],

$$\mathrm{E}[\gamma_n] = \frac{\mathrm{E}\left[P_{sig}^{(n)}\right]}{\mathrm{E}\left[P_{isi}^{(n)}\right]+\mathrm{E}\left[P_{iui}^{(n)}\right]+\sigma_{Gauss}^2} + \frac{\sum_{i=1}^{\infty}(-1)^i \mathrm{E}\left[P_{sig}^{(n)}\right]\left\langle {}^i\left(P_{isi}^{(n)}+P_{iui}^{(n)}+|n|^2\right)\right\rangle + \left\langle P_{sig}^{(n)},{}^i\left(P_{isi}^{(n)}+P_{iui}^{(n)}+|\tilde{n}|^2\right)\right\rangle}{\mathrm{E}\left[P_{isi}^{(n)}+P_{iui}^{(n)}+|n|^2\right]^{i+1}}. \tag{11}$$

where $\mathrm{E}\left[|\tilde{n}|^2\right]=\sigma_{Gauss}^2$, $\left\langle {}^i\left(P_{isi}^{(n)}+P_{iui}^{(n)}+|\tilde{n}|^2\right)\right\rangle = \mathrm{E}\left[\left(P_{isi}^{(n)}+P_{iui}^{(n)}+|\tilde{n}|^2-\mathrm{E}\left[P_{isi}^{(n)}+P_{iui}^{(n)}+|\tilde{n}|^2\right]\right)^i\right]$ is the i-th central moment of $\left(P_{isi}^{(n)}+P_{iui}^{(n)}+|\tilde{n}|^2\right)$, and $\left\langle P_{sig}^{(n)},{}^i\left(P_{isi}^{(n)}+P_{iui}^{(n)}+|\tilde{n}|^2\right)\right\rangle = \mathrm{E}\left[\left(P_{sig}^{(n)}-\mathrm{E}\left[P_{sig}^{(n)}\right]\right)\left(P_{isi}^{(n)}+P_{iui}^{(n)}+|\tilde{n}|^2-\mathrm{E}\left[P_{isi}^{(n)}+P_{iui}^{(n)}+|\tilde{n}|^2\right]\right)^i\right]$ is the i-th mixed central moment of $\left(P_{isi}^{(n)}+P_{iui}^{(n)}+|\tilde{n}|^2\right)$ and $P_{sig}^{(n)}$.

Since the second term on the right side of (10) is too complex, it is very difficult (if not impossible) to directly tackle (11). Therefore, the expression of the SINR can be well approximated by

$$\mathrm{E}[\gamma_n] \approx \frac{\mathrm{E}\left[P_{sig}^{(n)}\right]}{\mathrm{E}\left[P_{isi}^{(n)}\right]+\mathrm{E}\left[P_{iui}^{(n)}\right]+\sigma_{Gauss}^2}. \tag{12}$$

In fact, this approximation has been widely used in many distinct models [5], [19]-[22]. Particularly, the reported contribution [5] has verified the validity of this approximation in term of "*Average Effective SINR*". The work [19] indicates that the correctness of the above approximation increases in the case of a larger delay spread, a smaller tap separation, or a stronger scattering. In section IV, the validity of approximation (12) is evaluated taking into account correlation and CEE. Proposition 1 below is used to start our analysis of (12).

Proposition 1. *Under the assumptions of multipath channels, error vectors and the correlation given in section II, we obtain the following analytic expressions*

$$\xi_{mm'}^n(l) = \mathrm{E}\left[e_{mn}[l]e_{m'n}^*[l]\hat{h}_{mn}^*[l]\hat{h}_{m'n}[l]\right] = \left(\hat{\sigma}_{mn,l}\hat{\sigma}_{m'n,l}\hat{\sigma}_{mn,l}\hat{\sigma}_{m'n,l}\right)\frac{\psi(\mathbf{R}_T)_{mm'}^2}{1+\psi}. \tag{13}$$

$$\Theta_{mm'}^{nn'}(l) = \mathrm{E}\left[\hat{h}_{mn'}^*[l]h_{mn}[l]\hat{h}_{m'n'}[l]h_{m'n}^*[l]\right] = (\hat{\sigma}_{mn,l}\hat{\sigma}_{mn',l}\hat{\sigma}_{m'n,l}\hat{\sigma}_{m'n',l})\left(\frac{(\mathbf{R}_U)_{nn'}^2}{(1+\psi)^2}+\frac{(\mathbf{R}_T)_{mm'}^2}{1+\psi}\right). \tag{14}$$

$$\Lambda_{mm'}^{nn'}(l,l') = \mathrm{E}\left[\hat{h}_{mn'}^*[l]h_{mn}[l']\hat{h}_{m'n'}[l]h_{m'n}^*[l']\right] = \left(\hat{\sigma}_{mn,l}\hat{\sigma}_{mn',l'}\hat{\sigma}_{m'n,l}\hat{\sigma}_{m'n',l'}\right)\frac{(\mathbf{R}_T)_{mm'}^2}{1+\psi}. \tag{15}$$

$$\Omega_{mm'}^{nn'}(l,l') = \mathrm{E}\left[\hat{h}_{mn'}^*[l]h_{mn}[l]\hat{h}_{m'n'}[l']h_{m'n}^*[l']\right] = \left(\hat{\sigma}_{mn,l}\hat{\sigma}_{mn',l}\hat{\sigma}_{m'n,l'}\hat{\sigma}_{m'n',l'}\right)\frac{(\mathbf{R}_U)_{nn'}^2}{(1+\psi)^2}. \tag{16}$$

Proof: We first obtain the following expression of $\mathrm{E}\left[\hat{h}_{mn}[l]\hat{h}_{m'n'}^*[l']\right]$, using equations (3) to (7), and taking into account that it is the expectation of the product of two random variables:

$$\mathrm{E}\left[\hat{h}_{mn}[l]\hat{h}^{*}_{m'n'}[l']\right]=\hat{\sigma}_{mn,l}\hat{\sigma}_{m'n',l'}\left(\mathbf{R}_T\right)_{mm'}\left(\mathbf{R}_U\right)_{nn'}. \tag{17}$$

We then recall that the expectation of the product of four jointly Gaussian random variables X_1, X_2, X_3 and X_4, can be expressed as follows [23]

$$\begin{aligned}\mathrm{E}[X_1X_2X_3X_4]=&\mathrm{E}[X_1X_2]\mathrm{E}[X_3X_4]+\mathrm{E}[X_1X_3]\mathrm{E}[X_2X_4]\\&+\mathrm{E}[X_1X_4]\mathrm{E}[X_2X_3]-2\mathrm{E}[X_1]\mathrm{E}[X_2]\mathrm{E}[X_3]\mathrm{E}[X_4].\end{aligned} \tag{18}$$

We then apply the rule (18) to every product of four real or complex random variables defined in (13) to (16), in order to obtain expectations of products of only two random variables (similar to the one in (17)) instead of four variables. We finally use (17) to replace each expectation of the product of two random variables by expressions which only depend on the correlation matrix, the parameter ψ and the powers of the estimated channel coefficients. ■

In multi-user networks, the interference between the users greatly affects the performance, and its analysis is therefore very important. For the IUI component, we discover differences between the correlation effects on the central tap and the other taps. In correlated channels, the power of central tap at the unintended user depends on the correlation between the intended user and unintended users whereas the other taps do not. To evaluate the power of IUI component, we thus decompose it into the power of the central tap, namely $P^{(n)}_{iui\_central}$, and the power of the other taps, namely $P^{(n)}_{iui\_others}$.

$$P^{(n)}_{iui}=\underbrace{\sum_{\substack{n'=1\\n'\neq n}}^{N}\left|\sum_{m=1}^{M}\left(\hat{g}_{mn'}*h_{mn}\right)[L]\right|^2}_{P^{(n)}_{iui\_central}}+\underbrace{\sum_{\substack{n'=1\\n'\neq n}}^{N}\sum_{\substack{l=1\\l\neq L}}^{2L-1}\left|\sum_{m=1}^{M}\left(\hat{g}_{mn'}*h_{mn}\right)[l]\right|^2}_{P^{(n)}_{iui\_others}}. \tag{19}$$

In fact, the $P^{(n)}_{iui\_central}$ can be expressed as a function of $\left(\Omega^{nn'}_{mm'}(l,l'),\Lambda^{nn'}_{mm'}(l,l')\right)$, whereas the $P^{(n)}_{iui\_others}$ denoting the power of other taps is a function of $\left(\Lambda^{nn'}_{mm'}(l,l')\right)$ only. Detailed derivations are provided in *Theorem 1*.

Following *Proposition 1* and (19), we can determine the expectation of the SINR by *Theorem 1* below.

Theorem 1. *In the equation (12),* $\mathrm{E}\left[P^{(n)}_{sig}\right]$, $\mathrm{E}\left[P^{(n)}_{isi}\right]$, $\mathrm{E}\left[P^{(n)}_{iui\_central}\right]$ and $\mathrm{E}\left[P^{(n)}_{iui\_others}\right]$ *can be replaced by the following expressions*

$$\begin{aligned}\mathrm{E}\left[P^{(n)}_{sig}\right]=&\frac{p_n\sum_{m=1}^{M}\left(\sum_{l=1}^{L}\hat{\sigma}^4_{mn,l}+\left(\sum_{l=1}^{L}\hat{\sigma}^2_{mn,l}\right)^2\Big/(1+\psi)\right)}{(1+\psi)\sum_{m=1}^{M}\sum_{l=1}^{L}\hat{\sigma}^2_{mn,l}}\\&+\frac{p_n\sum_{\substack{m'=1\\m'=m}}^{M}\sum_{m=1}^{M}\left(\sum_{l'=1}^{L}\sum_{l=1}^{L}\hat{\sigma}^2_{mn,l}\hat{\sigma}^2_{m'n,l}+\sum_{l=1}^{L}\hat{\sigma}^2_{mn,l}\hat{\sigma}^2_{m'n,l}\left(\mathbf{R}_T\right)^2_{mm'}+\sum_{l=1}^{L}\xi^{n}_{mm'}(l)(1+\psi)\right)}{(1+\psi)^2\sum_{m=1}^{M}\sum_{l=1}^{L}\hat{\sigma}^2_{mn,l}}.\end{aligned} \tag{20}$$

$$\mathrm{E}\left[P^{(n)}_{isi}\right]=2p_n\sum_{k=1}^{L-1}\left(\frac{\sum_{m=1}^{M}\left(\sum_{l=1}^{k}\hat{\sigma}^2_{mn,k+1-l}\hat{\sigma}^2_{mn,L+1-l}\right)}{(1+\psi)\sum_{m=1}^{M}\sum_{l=1}^{L}\hat{\sigma}^2_{mn,l}}+\frac{\sum_{\substack{m'=1\\m'\neq m}}^{M}\sum_{m=1}^{M}\left(\sum_{l=1}^{k}\Lambda^{nn}_{mm'}(k+1-l,L+1-l)(1+\psi)\right)}{(1+\psi)\sum_{m=1}^{M}\sum_{l=1}^{L}\hat{\sigma}^2_{mn,l}}\right). \tag{21}$$

$$\mathrm{E}\left[P_{iui\_central}^{(n)}\right]=p_n\sum_{\substack{n'=1\\n'\neq n}}^{N}\left(\frac{\sum_{m=1}^{M}\left(\left(1+\frac{\left(\mathbf{R}_U\right)_{nn'}^2}{1+\psi}\right)\sum_{l=1}^{L}\hat{\sigma}_{mn,l}^2\hat{\sigma}_{mn',l}^2+\sum_{\substack{l'=1\\l'\neq l}}^{L}\sum_{l=1}^{L}\Omega_{mm}^{nn'}(l,l')(1+\psi)\right)}{(1+\psi)\sum_{m=1}^{M}\sum_{l=1}^{L}\hat{\sigma}_{mn,l}^2}+\frac{\sum_{\substack{m'=1\\m'\neq m}}^{M}\sum_{m=1}^{M}\left(\sum_{\substack{l'=1\\l'\neq l}}^{L}\sum_{l=1}^{L}\Omega_{mm'}^{nn'}(l,l')+\sum_{l=1}^{L}\Theta_{mm'}^{nn'}(l)\right)}{\sum_{m=1}^{M}\sum_{l=1}^{L}\hat{\sigma}_{mn,l}^2}\right) \tag{22}$$

$$\begin{aligned}\mathrm{E}\left[P_{iui\_others}^{(n)}\right]&=2p_n\sum_{\substack{n'=1\\n'\neq n}}^{N}\left(\sum_{k=1}^{L-1}\left(\frac{\sum_{m=1}^{M}\sum_{l=1}^{k}\hat{\sigma}_{mn,k+1-l}^2\hat{\sigma}_{mn',L+1-l}^2}{(1+\psi)\sum_{m=1}^{M}\sum_{l=1}^{L}\hat{\sigma}_{mn',l}^2}\right)\right)\\&+2p_n\sum_{\substack{n'=1\\n'\neq n}}^{N}\left(\sum_{k=1}^{L-1}\left(\frac{\sum_{\substack{m'=1\\m'\neq m}}^{M}\sum_{m=1}^{M}\sum_{l=1}^{k}\Lambda_{mm'}^{nn'}(k+1-l,L+1-l)(1+\psi)}{(1+\psi)\sum_{m=1}^{M}\sum_{l=1}^{L}\hat{\sigma}_{mn,l}^2}\right)\right).\end{aligned} \tag{23}$$

Proof: The demonstration for each of the four components of *Theorem 1*, one by one, can be tedious. However, one can observe that it is only sufficient to determine the expression of $\left|\sum_{m=1}^{M}(\hat{\mathbf{g}}_{mn}*\mathbf{h}_{m'n'})[k]\right|^2$. Indeed $\mathrm{E}\left[P_{sig}^{(n)}\right]$, $\mathrm{E}\left[P_{isi}^{(n)}\right]$, $\mathrm{E}\left[P_{iui\_central}^{(n)}\right]$ and $\mathrm{E}\left[P_{iui\_others}^{(n)}\right]$ can all be expressed based on variants of $\left|\sum_{m=1}^{M}(\hat{g}_{mn}*h_{m'n'})[k]\right|^2$. The term $\left|\sum_{m=1}^{M}(\hat{g}_{mn}*h_{m'n'})[k]\right|^2$ is equal to

$$\left|\sum_{m=1}^{M}(\hat{g}_{mn}*h_{m'n'})[k]\right|^2=\mathrm{E}\left[\sum_{m=1}^{M}\left|\left(\hat{g}_{mn}*h_{m'n'}\right)[k]\right|^2+\mathrm{Re}\left\{\sum_{\substack{m'=1\\m'\neq m}}^{M}\sum_{m=1}^{M}\left(\hat{g}_{mn}*h_{m'n'}\right)[k]\left(\hat{g}_{mn}*h_{m'n'}\right)^*[k]\right\}\right]. \tag{24}$$

We first consider the first term on the right side of the equality symbol, in (24). Each tap of $\hat{\mathbf{g}}_{mn}*\mathbf{h}_{m'n'}$ can be written as

$$\mathrm{E}\left[\left|\hat{g}_{mn}*h_{m'n'}[k]\right|^2\right]=p_n\mathrm{E}\left[\frac{\left|\sum_{l=1}^{k}\hat{h}_{mn}^*[L+1-l]h_{m'n'}[k+1-l]\right|^2}{\sum_{m=1}^{M}\mathrm{E}\left[\left\|\hat{\mathbf{h}}_{mn}\right\|^2\right]}\right]. \tag{25}$$

From (5) and (6), we also have

$$\begin{aligned}\mathrm{E}\left[\left|\sum_{l=1}^{k}\hat{h}_{mn}^*[L+1-l]h_{m'n'}[k+1-l]\right|^2\right]&=\sum_{l=1}^{k}\left|\hat{h}_{mn}^*[L+1-l]h_{m'n'}[k+1-l]\right|^2\\&+\sum_{\substack{l'=1\\l'\neq l}}^{k}\sum_{l=1}^{k}\mathrm{E}\left[\hat{h}_{mn}^*[L+1-l]h_{m'n'}[k+1-l]\hat{h}_{mn}^*[L+1-l']h_{m'n'}[k+1-l']\right]\end{aligned} \tag{26}$$

Parallely, we also have

$$\mathrm{E}\left[\left|\hat{h}_{mn}^*[L+1-l]h_{m'n'}[k+1-l]\right|^2\right]=\begin{cases}\dfrac{\left(2\hat{\sigma}_{mn,L+1-l}^4+\psi\hat{\sigma}_{mn,L+1-l}^4\right)}{(1+\psi)^2}, & k=L,m=m',n=n'\\[2ex]\dfrac{\hat{\sigma}_{mn,L+1-l}^2\hat{\sigma}_{m'n',k+1-l}^2}{(1+\psi)}, & \text{otherwise},\end{cases} \tag{27}$$

and

$$\mathrm{E}\left[\hat{h}_{mn}^{*}[L+1-l]h_{m'n'}[k+1-l]\hat{h}_{mn}^{*}[L+1-l']h_{m'n'}[k+1-l']\right] = \begin{cases} \dfrac{\hat{\sigma}_{mn,L+1-l}^{2}\hat{\sigma}_{mn,L+1-l'}^{2}}{(1+\psi)^{2}}, & k=L, m=m', n=n' \\ 0, & \text{otherwise}. \end{cases} \quad (28)$$

By combining (26) and (27), and by substituting the result of the combination in (25), we obtain the intended results as follows

$$\mathrm{E}\left[\left|\hat{g}_{mn} * h_{m'n'}[k]\right|^{2}\right] = \begin{cases} \dfrac{p_n \sum_{m=1}^{M}\left(\sum_{l=1}^{L}\hat{\sigma}_{mn,l}^{4} + \left(\sum_{l=1}^{L}\hat{\sigma}_{mn,l}^{2}\right)^{2} \Big/ (1+\psi)\right)}{(1+\psi)\sum_{m=1}^{M}\sum_{l=1}^{L}\hat{\sigma}_{mn,l}^{2}}, & k=L, m=m', n=n' \\ \dfrac{p_n \sum_{m=1}^{M}\left(\sum_{l=1}^{k}\hat{\sigma}_{mn,k+1-l}^{2}\hat{\sigma}_{m'n',L+1-l}^{2}\right)}{(1+\psi)\sum_{m=1}^{M}\sum_{l=1}^{L}\hat{\sigma}_{mn,l}^{2}}, & \text{otherwise}. \end{cases} \quad (29)$$

We then consider the second term on the right side of the equality symbol, in (24). $(\hat{g}_{mn} * h_{m'n'})[k](\hat{g}_{mn} * h_{m'n'})^{*}[k]$ can be further derived as

$$\begin{aligned} &\mathrm{E}\left[\mathrm{Re}\left\{(\hat{g}_{mn} * h_{m'n'})[k](\hat{g}_{mn} * h_{m'n'})^{*}[k]\right\}\right] \\ &= \frac{p_n \mathrm{E}\left[\left(\hat{h}_{mn} * h_{m'n'}\right)[k]\left(\hat{h}_{mn} * h_{m'n'}\right)^{*}[k]\right]}{\sum_{m=1}^{M}\mathrm{E}\left[\left\|\hat{\mathbf{h}}_{mn}\right\|^{2}\right]} \\ &= \frac{p_n \sum_{l=1}^{k}\mathrm{E}\left[\hat{h}_{mn}^{*}[L+1-l]h_{mn}[k+1-l]\hat{h}_{m'n'}[L+1-l]h_{m'n'}^{*}[k+1-l]\right]}{\sum_{m=1}^{M}\mathrm{E}\left[\left\|\hat{\mathbf{h}}_{mn}\right\|^{2}\right]}. \end{aligned} \quad (30)$$

Theorem 1 is then obtained in two steps. First, in the expressions $\mathrm{E}\left[P_{sig}^{(n)}\right]$, $\mathrm{E}\left[P_{isi}^{(n)}\right]$, $\mathrm{E}\left[P_{iui\_central}^{(n)}\right]$ and $\mathrm{E}\left[P_{iui\_others}^{(n)}\right]$, each term which is a variant of (24) is replaced by the sum of the corresponding variant of (29) and the corresponding variant of (30). Secondly, the notations defined in *Proposition 1* are used. ■

From (20), the signal power is inversely proportional to ψ, the assumed error factor. It is obvious that the focalization property of TR is reduced by the error in estimated channels. Consequently, we consider the average power of ISI and IUI.

Corollary 1. *Based on (20)-(23), in TR-based systems, the average power of ISI and IUI excluding the central tap are independent from the effects of CEE and inter-user correlation, whereas the average desired signal power is a decreasing function of CEE.*

In order to prove for Corollary 1, we re-write (21) and (23) as functions of the true components, instead of estimated components, as follows

$$\mathrm{E}\left[P_{isi}^{(n)}\right]=2p_n\sum_{k=1}^{L-1}\left(\frac{\sum_{m=1}^{M}\left(\sum_{l=1}^{k}\sigma_{mn,k+1-l}^{2}\sigma_{mn,L+1-l}^{2}\right)}{\sum_{m=1}^{M}\sum_{l=1}^{L}\sigma_{mn,l}^{2}}+\frac{\sum_{\substack{m'=1\\ m'\neq m}}^{M}\sum_{m=1}^{M}\left(\sum_{l=1}^{k}\left(\sigma_{mn,k+1-l}\sigma_{mn',L+1-l}\sigma_{m'n,k+1-l}\sigma_{m'n',L+1-l}\right)\left(\mathbf{R}_T\right)_{mm'}^{2}\right)}{\sum_{m=1}^{M}\sum_{l=1}^{L}\sigma_{mn,l}^{2}}\right). \quad (31)$$

$$\begin{aligned}\mathrm{E}\left[P_{iui\_others}^{(n)}\right]=&\,2p_n\sum_{\substack{n'=1\\ n'\neq n}}^{N}\left(\sum_{k=1}^{L-1}\left(\frac{\sum_{m=1}^{M}\sum_{l=1}^{k}\sigma_{mn,k+1-l}^{2}\sigma_{mn',L+1-l}^{2}}{\sum_{m=1}^{M}\sum_{l=1}^{L}\sigma_{mn',l}^{2}}\right)\right)\\ &+2p_n\sum_{\substack{n'=1\\ n'\neq n}}^{N}\left(\sum_{k=1}^{L-1}\left(\frac{\sum_{\substack{m'=1\\ m'\neq m}}^{M}\sum_{m=1}^{M}\sum_{l=1}^{k}\left(\sigma_{mn,k+1-l}\sigma_{mn',L+1-l}\sigma_{m'n,k+1-l}\sigma_{m'n',L+1-l}\right)\left(\mathbf{R}_T\right)_{mm'}^{2}}{\sum_{m=1}^{M}\sum_{l=1}^{L}\sigma_{mn,l}^{2}}\right)\right).\end{aligned} \quad (32)$$

From (31) and (32), it can be seen that the CEE and inter-user correlation have no impact on the average power of the *ISI* and *IUI excluding the main tap. These are important properties of Time Reversal which have not been revealed by any previous paper.*

IV. Numerical Results

In order to validate our analysis, a Monte Carlo simulation is carried out. We remark that the simulation results are obtained with the original expression (10), whereas the analytical results are achieved with the approximation (12).

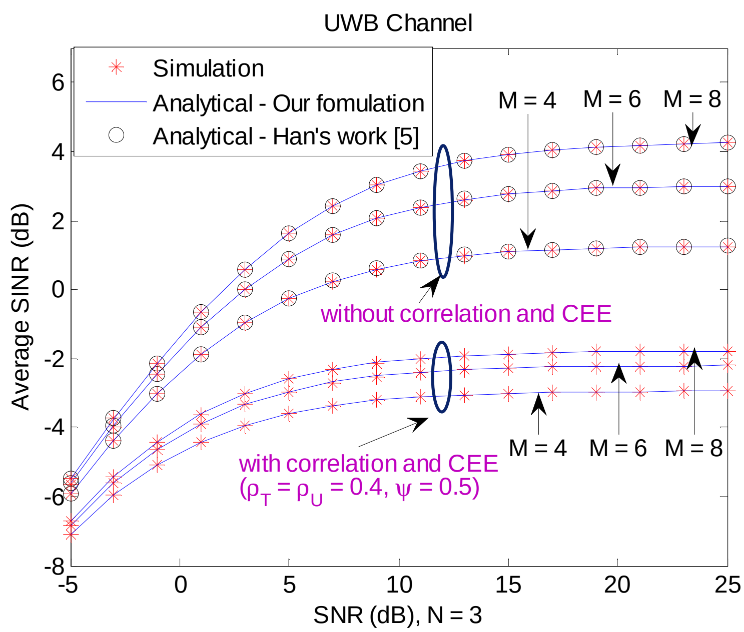

Fig. 1. Average SINR with/without correlation and CEE.

In our simulation, we use the simplified correlation model which is defined as follows: $(\mathbf{R}_T)_{mm'} = 1$ if $m = m'$, $(\mathbf{R}_T)_{mm'} = \rho_T$ otherwise, and the same expression for $(\mathbf{R}_U)_{nn'}$ with the correlation factor ρ_U.

First, we compare numerically our analysis with the prominent work [5] to show the benefit of generalized formulations. For UWB channel model, we use a channel model with bandwidth *(B)* = 500MHz, L = 110 and root mean square delay spread (rmsds) = 100/B. For a fair comparison with cases without correlation and CEE, the channel model is chosen strictly similar to Han *et al.*'s analysis [5] and the rate back-off is set to 1. Under the assumptions of i.i.d multipath channel and perfect CSI, Fig. 1 shows that the well-matched analytical results can be achieved using both *Theorem 1* and Han *et al.*'s analysis [5]. When the CEE and correlation are considered, Han's formula cannot be applied, but our analytical results show the good agreement with the simulation results.

Note here that our analysis may be flexibly adapted to multipath channels with arbitrary average power of each tap, while Han's work is suitable for exponential decay only. Therefore, the proposed analysis possesses more mathematical conveniences than prior work in realistic models. Moreover, because we consider in this letter the general MU MISO TR-based system, the integration of back-off module is then not taken into account. Nevertheless, since each tap of ISI and IUI can be independently calculated by *Theorem 1*, we can conveniently omit specific taps depending on the back-off rate factor [5] to obtain the outcomes that are equivalent to the results of the system with the installation of back-off module.

The verification of our analysis is consequently performed in conventional broadband channels. Under correlation and CEE assumptions, we consider the wireless local area network (WLAN) channel model (i.e. model B) [24] that is compatible with the system operating with 20 MHz bandwidth around a central frequency of 2.4GHz. In Fig. 2, we may evaluate the variation of SINR at p_n=1dBm for all n and σ^2_{Gauss} = 0dBm. Fig. 2 indicates a good agreement between the analytical and simulation results for the two kinds of channels. Moreover, we can observe the variation of SINR as a function of the estimated error and correlation. The growth of IUI induces a reduction in the SINR performance. Furthermore, it can be seen that the average SINR performance monotonically degrades when the error increases due to the loss of the converging property.

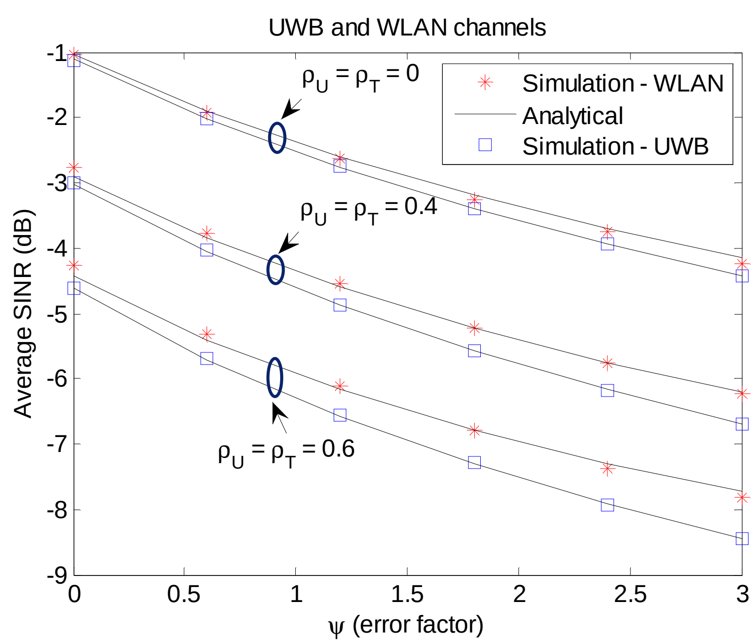

Fig. 2. Effects of the correlation and CEE.

Relying on Figs.1 and 2, the approximation (12) provides a greater performance when the channel which has a larger bandwidth is considered. This is in complete agreement with the conclusion in [19]. Moreover, we can evaluate that (12) well approximates (10) even in the presence of correlation and CEE. These observations have validated the expression in (12).

In general, based on our analysis, we can evaluate the average SINR performance efficiently for conventional broadband and UWB channels in the presence of correlation and CEE suppositions. The after-effects of the correlation and imperfect CIR lead the performance to be significantly scaled down due to the scaling up of IUI and the loss of signal focalization, respectively.

V. Conclusion

The transmission performance of general MU MISO TR-based system has been analyzed. The closed-form SINR expression of the considered system is derived. Our study demonstrates that CEE causes a reduction in the average desired signal power whereas CEE and inter-correlation do not affect the average power of ISI. In the estimation of IUI component, there are differences between the effects of correlation on the central tap and the other taps. Surprisingly, the average power of the main tap of IUI depends on the variations of inter-user correlation and CEE, whereas the average power of the others does not.

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References

[1] J. Wu and Alcatel-Lucent, “Green wireless communications: from concept to reality [industry perspectives],” *IEEE Wireless Communications*, vol. 19, no. 4, pp. 4–5, 2012.

[2] B. Wang et al., “Green wireless communications: A time-reversal paradigm,” *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 8, pp. 1698–1710, 2011.

[3] H. Nguyen, Z. Zhao, F. Zheng, and T. Kaiser, “Preequalizer design for spatial multiplexing SIMO-UWB TR systems,” *IEEE Transactions on Vehicular Communications*, vol. 59, no. 8, pp. 3798–3805, 2010.

[4] V. Tran-Ha et al., “Time reversal-based transmissions with distributed power allocation for two-tier networks”, *in Proc. of 29-th Advanced Information Networking and Applications Workshops*, pp. 181 – 186, 2015

[5] Feng Han et al., “Time-reversal division multiple access over multi-path channels,” *IEEE Transactions on Communications*, vol. 60, no. 7, pp. 1953–1965, 2012.

[6] K. Popovski et al., “Closed-form derivations of ISI and MUI for time-reversed ultra wideband,” *in Proceedings of the International Conference on Signal Processing and Communication Systems*, 2007

[7] Bruno A. Angelico et al., “Performance analysis of a single-user MISO ultra-wideband time reversal system with DFE,” *Telecommunication System*, vol. 46, pp. 333-342, 2010.

[8] S. Alizadeh et al., “Effect of channel estimation error on performance of time reversal-UWB communication system and its compensation by pre-filter,” *IET Communications*, vol. 6, issue 12, pp. 1781-1794, 2012.

[9] D. Abbasi-Moghadam and V. T. Vakili, “Effect of channel estimation error on Time Reversal UWB communication system,” *Wireless Personal Communication*, vol. 68, issue. 2, pp. 433-439, 2013.

[10] G. C. Ferrante, J. Fiorina and M.G. Di Benedetto , “Statistical analysis of the SNR loss due to imperfect time reversal,” *in Proc. of Int. Conf. on Ultra-wideband (ICUWB)*, pp. 36-40, 2014.

[11] H. T. Nguyen, J. B. A. G. F. Pedersen, P. Kyritsi, and P. C. F. Eggers, “Time reversal in wireless communications: a measurement based investigation,” *IEEE Transactions on Wireless Communications*, vol. 5, no. 8, pp. 2242–2252, 2006.

[12] M.-A. Bouzigues, I. Siaud, M. Helard, and A.-M. Ulmer-Moll, “Turn back the clock: Time reversal for green radio communications,” *IEEE Vehicular Technology Magazine*, vol. 8, no. 1, pp. 49–56, 2013.

[13] I. H. Naqvi et al., “Robustness of a time reversal system: From ultra-wide to narrow bandwidths,” *in Proc. of IEEE Wireless Communications and Networking Conference (WCNC)*, pp. 37–41, 2012.

[14] A. Meijerink; A.F. Molisch, "On the Physical Interpretation of the Saleh-Valenzuela Model and the Definition of Its Power Delay Profiles," *IEEE Transactions on Antennas and Propagation*, vol.62, no.9, Sept. 2014.

[15] R. Müller, “A random matrix model of communication via antenna arrays,” *IEEE Trans. Inform. Theory*, vol. 48, pp. 2495–2506, Sept. 2002.

[16] H. Bölcskei, D. Gesbert, and A. J. Paulraj, "On the capacity of OFDMbased spatial multiplexing systems," IEEE Trans. Commun., vol. 50, pp. 225–234, Feb. 2002.

[17] C. Oestges, "Validity of the Kronecker Model for MIMO Correlated Channels," *Vehicular Technology Conference*, pp. 7-10, May 2006.

[18] S. H. Rice , "The expected value of the ratio of correlated random variables," November 2009.

[19] C. A. Viteri-Mera; F. L. Teixeira, "Equalized Time Reversal Beamforming for Indoor Wireless Communications," arXiv, abs/1411.6897, 2014. [Online]. Available: http://arxiv.org/abs/1411.6897

[20] J. Lee, H.-ling Lou, D. Toumpakaris, and J. M. Cioffi, "SNR Analysis of OFDM Systems in the Presence of Carrier Frequency Offset for Fading Channels," *IEEE Transactions on Wireless Communications*, vol. 5, No. 12, pp. 3360-3364, Dec. 2006.

[21] M. Emami, M. Vu, J. Hansen, A.J. Paulraj, and G. Papanicolaou "Matched filtering with rate back-off for low complexity communications in very large delay spread channels," *Proc. 38th Asilomar Conf. Signals, Syst. Comput.*, vol. 1, pp. 218-222, Nov. 2004.

[22] Tao Jia and Dong In Kim, "Analysis of channel-averaged SINR for indoor UWB rake and transmitted reference systems," *IEEE Transactions on Communications*, vol. 55, no. 10, pp. 2022-2032, October 2007.

[23] P. H. M. Janssen and P. Stoica, "On the expectation of the product of four matrix-valued Gaussian random variables", *IEEE Transaction on Automatic Control*, vol. 33, no. 9, Sep. 1988.

[24] IEEE Technical Report, 2012, "*Channel models for TG8*," [Online] Available: https://mentor.ieee.org/802.15/dcn/12/15-12-0459-06-0008- tg8-channel-models.doc