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Abstract—We study the capacity performance of signal-to-leakage-and-noise ratio precoding scheme (SLNR-PS) for large-scale multiuser multiple-input multiple-output systems. We derive capacity bounds of SLNR-PS when the number of base station antennas $N$ is large. A trade-off between energy efficiency (EE) and spectral efficiency (SE) is defined for the case of equal power allocation. Based on this trade-off, the closed-form expressions of optimal SE and optimal number of BS antennas for maximizing EE are derived, respectively. In addition, we consider the EE optimization problem under both maximum transmit power and quality of service constraints. The energy-efficient power allocation scheme is obtained to solve this optimization problem. Numerical simulations verify the benefits of the proposed scheme.

Index Terms—Massive MIMO system, SLNR-based scheme.

I. INTRODUCTION

LINEAR precoding techniques including zero-forcing (ZF), minimum mean square error, block diagonalization and maximizing signal-to-leakage-and-noise ratio (SLNR) have been considered as practical solutions for multiuser multiple-input multiple-output (MU-MIMO) communication systems [1]. The SLNR-based scheme does not require any dimension condition on the base station (BS) antennas. Moreover, it also takes into account the influence of noise when determining the beamforming vectors for all users. Recently, it has been shown that the large-scale MIMO (LS-MIMO) systems can greatly improve system performance and reliability [2]. Motivated by these advantages, we analyze the energy efficiency (EE) and spectral efficiency (SE) of the SLNR precoding scheme (SLNR-PS) for LS-MIMO systems.

In [3]–[5], the energy efficiencies of the uplink and downlink LS-MIMO systems have been studied based on simulated results only. Thus, the trade-off between SE and EE has not been analyzed. In [3] and [4], the authors adopted a power consumption model (PCM) that only considers the emitted power consumption. However, for LS-MIMO systems, the emitted power consumption, the radio frequency circuit power consumption, as well as the power consumption for performing digital signal processing (DSP) should be considered [5], [6]. In this paper, we first derive capacity bounds of the SLNR-PS when the number of BS antennas is large. Applying the realistic PCM, we analyze the trade-off between SE and EE based on the upper bound (UB) on the system capacity at high signal-to-noise ratio (SNR) region for the case of equal power allocation (EPA). Based on this trade-off, the optimal value of SE and the optimal number of BS antennas that can maximize the overall system EE are also presented, respectively. We obtain energy-efficient power allocation scheme to maximize the EE under the constraints of the transmit power (TP) and quality-of-service (QoS). In addition, the optimal rates of individual users are defined by solving the rate profile optimization problem under the constraints of the TP, QoS, and EE target.

II. SYSTEM MODEL

We consider a downlink MU-MIMO system which has one BS equipped with $N$ transmit antennas and $K$ single-antenna users. The received signal at user $k$ is given by [7]

$$y_k = \sqrt{p_k} g_k^H w_k s_k + \sum_{i=1, i \neq k}^{K} \sqrt{p_i} g_i^H w_i s_i + n_k,$$

where $s_k$ is the data symbol, $p_k$ is the transmit power for user $k$ with $\sum_k p_k = P$, and $n_k$ is the noise term with $n_k \sim \mathcal{CN}(0, \sigma^2)$. The channel vector of user $k$ is given by $g_k = h_k \sqrt{\beta_k}$, where $h_k$ is the fast fading vector of user $k$ with $h_k \sim \mathcal{CN}(0,1)$; $\beta_k$ is the large-scale fading coefficient. The beamforming vector $w_k$ is defined as [1]

$$w_k \propto \text{max.eigenvector}(A_k),$$

where $\|w_k\|^2 = 1$, $A_k = \left(G_k \tilde{G}_k^H + \frac{\sigma^2}{p_k} I \right)^{-1} g_k g_k^H$, and $\tilde{G}_k = [g_1, \ldots, g_{k-1}, g_{k+1}, \ldots, g_K]$, respectively [1]. The resulting maximum SLNR value $\varphi_k$ is the largest eigenvalue $\lambda_k$ of $A_k$, which is obtained by solving the following characteristic equation [1], [7]

$$\det \left( G_k G_k^H + \frac{\sigma^2}{p_k} I - \left( \frac{\lambda + 1}{\lambda} \right) g_k g_k^H \right) = 0,$$

where $G_k = [g_1, \ldots, g_K]$. Let us define $\Phi^{-1} = \left(G_k G_k^H + \frac{\sigma^2}{p_k} I\right)$, $c = -\frac{\lambda + 1}{\lambda} g_k$ and $d^H = g_k^H$. Applying the matrix determinant lemma, i.e., $\det(\Phi^{-1} + cd^H) = (1 + d^H \Phi c) \det(\Phi^{-1})$, $\varphi_k$ is defined as

$$\varphi_k = \lambda_k = \frac{1}{1 - \frac{d^H (G_k G_k^H + \frac{\sigma^2}{p_k} I)^{-1} g_k}{1}},$$

According to [7], the power of the interference plus noise can be estimated from the power of the leakage plus noise, i.e., $\sum_{\ell \neq k} p_{\ell} \|g_{\ell}^H w_{\ell}\|^2 + \sigma^2 \approx \sum_{\ell \neq k} p_{\ell} \|g_{\ell}^H w_{k}\|^2 + \sigma^2$. This estimation is generally tight when the leakage power is small compared with the noise power. Moreover, as $K < N$, the leakage power converges to zero at low and high SNR regions [7, Eq. (19)]. Hence, the signal-to-interference-plus-noise ratio (SINR) of the $k$th user can be approximated by the corresponding SLNR.
III. ERGODICAchievable Capacity

From (4) the ergodic capacity of the $k$th user is defined as

$$R_k = \mathbb{E} \left\{ \log_2 \left( \frac{1}{\sigma^2 \mathbf{G}_k^H \mathbf{G}_k + 1} \right) \right\}. \tag{5}$$

The lower bound (LB) on the ergodic capacity for the $k$th user can be obtained by applying Jensen’s inequality

$$R_k \geq \log_2 \left( 1 + (\mathbb{E}[1/\alpha_k])^{-1} \right), \tag{6}$$

where $\alpha_k = \frac{1}{\mathbb{E} \left[ (\mathbf{G}_k^H \mathbf{G}_k + 1)^{-1} \right]} - 1$. Following [8], the probability density function (pdf) of $\alpha_k$ is approximated by a Gamma function, i.e., $\alpha_k \sim \Gamma(\psi_k, \hat{\theta}_k)$ with shape parameter $\psi_k = (N - K + 1 + (K - 1))\gamma$ and scale parameter $\hat{\theta}_k = (N - K + 1 + (K - 1))\rho_k \beta_k$, respectively. Therefore, the LB on the ergodic capacity can be approximated as

$$\hat{R}_k \approx \log_2 \left( 1 + (\psi_k - 1)\hat{\theta}_k \right). \tag{7}$$

The values of parameters $\mu$ and $\theta$ are defined by solving the following equations

$$\begin{align*}
\mu &= \frac{1}{K-1} \sum_{i=1, i \neq k}^{K} \frac{1}{\bar{\alpha}_i} \\
\theta (1 + \sum_{i=1, i \neq k}^{K} \frac{\rho_i \beta_i}{\psi_i}) &= \frac{1}{K-1} \sum_{i=1, i \neq k}^{K} \frac{m_i}{\psi_i}
\end{align*} \tag{8}$$

where $\phi_k = N \rho_k \beta_k \left( 1 - \frac{K-1}{N} + \frac{K-1}{N} \mu \right) + 1$ and $m_k = \rho_k \beta_k \mu (K - 1) + 1$, respectively.

In addition, (4) can also be rewritten as [7], [9]

$$\psi_k = \frac{p_k}{\sigma^2} \mathbf{G}_k^H \mathbf{Y}_k \mathbf{g}_k + \mathbf{g}_k^H \mathbf{Z}_k \mathbf{g}_k, \tag{9}$$

where $\mathbf{Y}_k = \mathbf{I} - \mathbf{G}_k \left( \mathbf{G}_k^H \mathbf{G}_k \right)^{-1} \mathbf{G}_k^H$ and $\mathbf{Z}_k = \mathbf{G}_k \left( \mathbf{G}_k^H \mathbf{G}_k + \sigma^2 \rho_k \mathbf{I} \right)^{-1} \mathbf{G}_k^H$. Following [9], we have $\rho_{f,k} = \frac{p_k}{\sigma^2} \mathbf{g}_k^H \mathbf{Y}_k \mathbf{g}_k = \frac{(\rho_k \mathbf{G}_k^H \mathbf{G}_k + 1)^{-1}}{(\mathbf{G}_k^H \mathbf{G}_k)^{-1}} \mathbf{G}_k^H$, hence $\mathbb{E} \left[ \rho_{f,k} \right] = \frac{p_k}{\sigma^2} (N - K + 1)$ due to $1/ \left( (\mathbf{H}_k^H \mathbf{H}_k)^{-1} \right)_{kk} \sim \mathcal{G}(N - K + 1)$, where $\mathbf{H}_k = \{ h_1, \ldots, h_K \}$. We define $\rho_{ad,k} = \mathbf{g}_k^H \mathbf{Z}_k \mathbf{g}_k$; $\rho_{ad,k}$ and $\rho_{f,k}$ are statistically independent, and $\rho_{ad,k}$ is a nondecreasing function of $p_k/\sigma^2$ [9]. We denote $q_{ad,k} = \rho_{ad,k} \mathbf{g}_k \mathbf{g}_k^H \in C^{N \times (K-1)}$, where $U_k$ is obtained from the singular value decomposition of $\mathbf{G}_k = U_k \mathbf{A}_k \mathbf{V}_k^H$. We consider a matrix $\mathbf{Q}_k \in C^{N \times (K-1)}$ that has the same distribution of $\mathbf{G}_k$, and it is independent of $\mathbf{G}_k$. According to [9, Eq. (32)], at high SNR region, $\rho_{ad,k}$ has an identical distribution of $\|q_{ad,k}\|^2 \left( (\mathbf{Q}_k^H \mathbf{Q}_k)^{-1} \right)_{kk}$, hence $\mathbb{E} \left[ \rho_{ad,k} \right] \simeq \mathbb{E} \left[ \|q_{ad,k}\|^2 \right] \left( \frac{K-1}{N-1} \right) K.\text{Therefore, the UB on the capacity at high SNR region is obtained as}

$$R_k \leq \hat{R}_k = \log_2 \left( 1 + \frac{p_k \beta_k}{\sigma^2} (N - K + 1) + v \right), \tag{10}$$

where $v = \frac{K-1}{N-1}$. In the case of EPA, since $K$ and $N$ grow large without bound, i.e., $N, K \to \infty$, but $x = \frac{N}{K} \to 0$, then the UB expression in (10) is converged to

$$R_k \approx \log_2 \left( 1 + (1 - \sigma_{feed})(N - K + 1) \right). \tag{11}$$

This result is similar to the one which is obtained by using ZF-PS in [2]. Thus, the capacity of the SLNR-PS converges to that of the ZF-PS when SNR is high. Moreover, it is more convenient to obtain (10) than (7), hence the closed-form UB (10) is used for further analysis due to its low complexity.

IV. ENERGYEfficiency Optimization

We now study the optimal SE, optimal $N$, as well as optimal power allocation (OPA) for maximizing EE performance.

A. Optimum SE and Optimum N for Maximizing EE

We assume that the system has unit bandwidth, $p_k = P/K$ and $\sigma^2 = 1$. Thus, the system EE is given by [5], [6]

$$\eta = \frac{R_s}{\nu} \left( \frac{P}{\gamma (1 - \sigma_{feed}) + P_c + P_{sp}} \right), \tag{12}$$

where $\omega = (1 - \sigma_{PS})(1 - \sigma_{feed})$, $\gamma$ is the power amplifier, and $R_s = \sum_k R_k$ is the SE. The parameters $\sigma_{feed}, \sigma_{PS}, \sigma_{MS}, \sigma_{cool}$ are the loss factors of antenna feeder, direct current - direct current power supply, main power supply and active cooling system, respectively. $P_{sp}$ is the power consumption for performing DSP, $P_c$ is the circuit power consumption, i.e., $P_c = N (p_{dac} + p_{mix} + p_{filt} + p_{syn})$ where $p_{dac}, p_{mix}, p_{filt}, p_{syn}$ denote the power consumptions from a digital-to-analog converter, a mixer, a filter, and a frequency synthesizer, respectively. Note that $P$ can be defined by obtaining the inverse function of $R_s$. Based on (10), the solution of $f^{-1}(R_s)$ can be obtained as

$$P = f^{-1}(R_s) = \frac{K^2 R_s/K - K(1 + v)}{\beta_k (N - K + 1)}. \tag{13}$$

Substituting (13) into (12), the trade-off function between the SE and EE can be rewritten as

$$\eta = \frac{R_s b}{K^2 R_s/K - K(1 + v) + a}, \tag{14}$$

where $a = (P_c + P_{sp}) \gamma \beta_k (1 - \sigma_{feed})(N - K + 1)$ and $b = \alpha \gamma \beta_k (1 - \sigma_{feed})(N - K + 1)$. Since $\frac{d \eta}{d R_s} < 0$, the system EE in (14) is a concave function. By letting $\frac{d \eta}{d R_s} = 0$, the optimum value of SE for maximizing the EE is obtained as

$$R_s^{opt} = \frac{K}{\ln 2} \left( 1 + \Psi \left( \frac{a (K - v - 1)}{e} \right) \right), \tag{15}$$

where $\Psi(.)$ is the Lambert function, i.e., $\tilde{x} = \Psi(\tilde{x}) e^{\Psi(\tilde{x})}$, and $e$ is Euler’s number.

For a MIMO system with a specific $R_s$ requirement, we can obtain the optimum number of BS antennas for maximizing EE. Let us define $\tau = N - K + 1$, (14) can be rewritten as

$$\eta = \frac{R_c c t^2}{\nu^2 c_2 c_4 + \nu^2 c_4 c_6 + c_5 t - K (K - 1)}. \tag{16}$$
where \( c_1 = \omega y (1 - \sigma_{\text{feed}}) \beta_k, c_2 = P_d + p_{\text{mix}} + p_{\text{fil}}, c_3 = P_{\text{syn}} + P_{\text{sp}}, c_4 = \gamma (1 - \sigma_{\text{feed}}) \beta_k, c_5 = K (2 \tilde{R}_k - K - 1), \) and \( c_6 = (K - 1) c_2 + c_3. \) By letting \( \frac{\partial \eta}{\partial p_k} = 0, \) we have
\[
c_2 \alpha t^3 - c s + 2K(K - 1) = 0. \tag{17}
\]

The solutions of (17) are defined as \( t_j = -\frac{1}{3c_2 c_4} \left( u_j \tilde{Z} + \frac{\delta_0}{u_j \tilde{Z}} \right) \) with \( u_1 = 1, u_2 = 1 + \sqrt{\frac{3}{2}}, u_3 = 1 - \sqrt{\frac{3}{2}}, \) and \( \tilde{Z} = \sqrt{\delta_1 + \delta_2^2 - 4 \delta_3^2} / 2 \) with \( \delta_0 = 3c_2 c_4 c_3 \) and \( \delta_1 = 54(c_2 c_4)^2 K(K - 1). \) If \( t_j \) is real and \( t_j > K - 1, \) the optimal number of BS antennas is obtained as \( N_{\text{opt}} = \min \{ \lfloor |t_j| + K - 1 \rfloor \}, \) where \( \lfloor . \rfloor \) denotes the round number.

B. Optimal Power Allocation for Maximizing EE

The optimization problem for maximizing EE under both transmit power and QoS constraints is formulated as
\[
(\tilde{Q}) \quad \max_{p_k} \frac{\sum_k R_k(p_k)}{\xi \sum_k p_k + u}, \quad \text{s.t. } (C_1) : R_k(p_k) \geq c_{\text{min}}^k, (C_2) : \sum_k R_k \leq P, \tag{18}
\]
where \( \xi = \frac{1}{\omega y (1 - \sigma_{\text{feed}})} \) and \( u = (P_c + P_{\text{sp}}) / \omega. \) The objective function \( Q \) is a ratio of two functions of \( p_k. \) The result in (18) is a fractional programming (FP) problem which is nonconvex. By applying the properties of FP, the objective function is equivalent to \( \sum_k R_k(p_k) - \eta^* (\xi \sum_k p_k + u), \) where \( \eta^* \) is the EE when \( p_k \) is equal to the optimal value \( p_k^*, i.e., \eta^* = \frac{\sum_k R_k(p_k^*)}{\xi \sum_k p_k^* + u} \) [10]. Thus, (18) can be rewritten as
\[
\max_{p_k} \sum_k R_k(p_k) - \eta^* \left( \xi \sum_k p_k + u \right), \quad \text{s.t. } C_1 \text{ and } C_2. \tag{19}
\]

The optimization in (19) has been transformed into a convex optimization problem. The Lagrange function is obtained as
\[
\mathcal{L}(p_k, \nu, \tau) = \sum_k R_k(p_k) - \eta^* \left( \xi \sum_k p_k + u \right) + \sum_k \nu_k (R_k(p_k) - C_{\text{min}}^k) - \tau \left( \sum_k R_k - P \right). \tag{20}
\]

Based on the Karush-Kuhn-Tucker (KKT) conditions, the OPA for the \( k \)th user is obtained as \( p_k^* = \max \left\{ 0, \frac{1 + \nu_k}{\tau \eta^* \Gamma k K} \right\}, \) where \( \nu \) and \( \tau \) are Lagrange multipliers, respectively. They are updated by the gradient method which can be defined as \( \nu_k(i + 1) = \max \{0, \nu_k(i) - \Delta \nu_k (R_k(p_k^*) - C_{\text{min}}^k) \} \) and \( \tau(i + 1) = \max \{0, \tau(i) - \Delta \tau (P - \sum_k p_k) \}, \) where \( \Delta \nu_k \) and \( \Delta \tau \) are the positive iteration steps.

We consider the rate optimization problem for individual users based on \( C_1 \) and \( C_2, \) and the EE target \( \eta, i.e., (C_3) : \sum_k R_k \geq \eta \left( \xi \sum_k p_k + u \right). \) Let us define the achievable rate region \( \mathcal{R} \) as a set which composes of rate-tuples for \( K \) users, i.e., \( \mathcal{R} = \{ (r_1, r_2, \ldots, r_K) \in \mathbb{R}^K \mid r_k = R_k(p_k) \} \) with \( C_1, C_2, \) and \( C_3 \} \) [11]. We need to obtain the Pareto boundary (PB) of \( \mathcal{R}, \) at which it is impossible to improve a user’s rate without degrading others’ rate. Any rate-tuple on the PB of \( \mathcal{R} \) can be achieved as a solution of the rate profile optimization problem, i.e.,
\[
\max_{\{p_k\}_k} R_\Sigma, \text{ s.t. } R_k(p_k) \geq z_k R_\Sigma, C_1, C_2, \text{ and } C_3, \quad \forall k. \tag{21}
\]

V. NUMERICAL RESULTS AND DISCUSSIONS

The active users are assumed to be uniformly distributed in the cell with radius \( R = 800 \) m, the large-scale fading is \( \beta_k = \frac{u_k}{\left( r_k / r_{\text{avg}} \right)^{\alpha}}, \) where \( u_k \) is a log-normal random variable with standard deviation \( \sigma_{\text{ln}} = 8 \) dB, and \( r_k = 100 \) m [2]. We set \( K = 10 \) users, \( P_{\text{diam}} = 15.6 \) mW, \( P_{\text{mix}} = 30.3 \) mW, \( P_{\text{fil}} = 20 \) mW, \( P_{\text{syn}} = 50 \) mW, \( P_{\text{sp}} = 25 \) mW, \( \sigma_{\text{DC}} = 7.5 \%, \sigma_{\text{MS}} = 9 \%, \sigma_{\text{cool}} = 10 \%, \) and \( \sigma_{\text{feed}} = -3 \) dB, respectively [5], [6].

Firstly, we examine the accuracy of the derived capacity bounds of SLNR-PS for both cases of EPA and OPA. For the case of OPA, we use the well-known water-filling algorithm (WFA) to allocate power for each user while the QoS requirement is satisfied. We set \( N = 30 \) and \( \sigma^2 = 1. \) In Fig. 1, the SLNR-PS outperforms the ZF-PS. The LB on the capacity is very close to simulated results and the UB on the capacity is very tight to the simulation at high SNR region. This fact can be shown more evidently by observing Fig. 2 where \( N \) is increased. Clearly, all bounds are very tight when \( N > 50. \) These results validate our theoretical analysis. The WFA provides an improvement in capacity at low SNR region, but there is no difference in system performance between two cases of EPA and OPA at high SNR region or \( N \) is large.

In Fig. 3, we investigate the trade-offs between EE and SE, and between EE and \( N \) based on both the UB and the LB on the capacity via numerical results. The y-axis on the left and...
The capacity bounds of the SLNR-PS and the trade-off between SE and EE have been derived and validated by numerical results when $N$ is large. The closed-form expressions of the optimal SE and optimal $N$ corresponding to maximum EEs have also been derived based on this trade-off for the case of EPA. By applying the energy-efficient power allocation, our results have shown that the SLNR-PS with OPA can improve the system EE as compared to that of EPA at low SNR region.

VI. CONCLUSION

The capacity bounds of the SLNR-PS and the trade-off between SE and EE have been derived and validated by numerical results when $N$ is large. The closed-form expressions of the optimal SE and optimal $N$ corresponding to maximum EEs have also been derived based on this trade-off for the case of EPA. By applying the energy-efficient power allocation, our results have shown that the SLNR-PS with OPA can improve the system EE as compared to that of EPA at low SNR region.

REFERENCES


