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<th>Joint link-and-user scheduling for buffer-aided relaying system with adaptive rate transmission</th>
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Abstract—In this paper, we consider a relaying system which consists of a multiple-antenna source node (SN), $M$ single-antenna destination nodes (DNs) and a multiple-antenna relay node (RN). The RN possesses a buffer and it is able to store the decoded message before retransmitting the message to the DNs. We use the joint link-and-user scheduling method to maximize the long term average achievable rate of a multiple-input multiple-output (MIMO) relaying system. The optimal scheduling criteria is obtained and a two-step approach is proposed to implement it. In addition, we propose a rate allocation scheme of the source-relay (S-R) link to preserve the flow conservation constraint of each individual user. Furthermore, two reduced complexity joint link-and-user scheduling methods which use the zero forcing (ZF) beamforming in the relay-destination (R-D) link are investigated. It is shown that joint link-and-user scheduling can significantly increase the average achievable rate of the system.

I. INTRODUCTION

Cooperative relaying technique has attracted a lot of attentions in the last decades as it can increase the system capacity and improve the system reliability [1], [2], [3]. In traditional relaying systems, the relay node (RN) receives and transmits packets in two successive slots. As a result, the capacity of the relaying system is restricted by either the source-relay (S-R) link or the relay-destination (R-D) link with poor channel quality.

Recently, a buffer-aided relaying scheme has been proposed in [4] and [5]. With the help of a buffer, the RN can temporarily store the received packets before retransmitting them to the destination node (DN). Thus the RN with a buffer can adaptively choose to transmit or to receive based on the channel quality of the S-R link and the R-D link. Compared with the conventional relaying technique, the buffer-aided relaying system can benefit from the multihop diversity [6]. In [4], the optimal link adaptation scheme and the achievable diversity order of a three node buffer-aided relaying system were investigated. In [7], the buffer-aided relaying scheme is combined with the relay selection method. The authors proposed the max-max relay selection (MMRS) scheme and showed that the MMRS scheme could achieve some coding gains as compared with the traditional relay selection scheme. However, no extra diversity gain can be achieved by the MMRS scheme as it uses the fixed reception and transmission method. The max-link relay selection method in which the strongest link is chosen to receive or transmit in a given slot was proposed in [8]. It was shown that the max-link relay selection method could achieve the diversity order twice of that of the MMRS scheme. To improve the spectral efficiency of the relaying system, the buffer-aided successive relaying scheme were investigated in [9] and [10]. By assuming that there is no inter relay interference, it has been shown in [9] that the buffer-aided successive relaying scheme can achieve the same diversity order as that of the MMRS scheme. In [11], a buffer-aided two-way relaying system was studied. To improve the long term average capacity of the buffer-aided two-way relaying system, different transmission modes were defined and the system switched adaptively between these modes based on the quality of the channel response. In all the above works, it is assumed that all nodes in the relaying system have single antenna.

In this paper, we consider a relaying system which includes a multiple-antenna source node (SN), a multiple-antenna RN with buffer and $M$ single-antenna DNs. It can be considered as the typical down link of the LTE-Advanced relaying system where a fixed access-point relay is deployed. We aim to maximize the long term average sum rate of the system and use the joint link-and-user scheduling method to decide which link to be active and which subset of users to be served in each time slot. The optimal scheduling criterion is first obtained and a two-step method is proposed to implement it. The joint link-and-user scheduling method was also investigated in [12], but our work is different from theirs mainly from the following aspects: 1) In [12], all nodes are assumed to have single antenna, however, in our system, multiple-antenna SN and RN are used. This extension makes the joint link-and-user scheduling problem more complicated. 2) In [12], the flow conservation constraint is only satisfied for the sum rates of the S-R link and R-D link. In our model, we assume that the buffer of the RN is split into $M$ logical parts and each part stores the information of one DN. The flow conservation constraint should be satisfied for each individual DN. To satisfy this requirement, a rate allocation method of the S-R link is proposed. In addition, two zero forcing (ZF) beamforming based user selection methods which are performed in the first step of the joint link-and-user scheduling scheme are investigated. Furthermore, a real-time based approach which is similar to the one used in [12] is proposed to optimize the scheduling scheme and find the optimal rate allocation factors of the S-R link. Numerical results show that the proposed joint link-and-user scheduling approach can significantly increase the average achievable rate of the system.

The rest of the paper is organized as follows. Section
II describes the system model and formulates the problem. In Section III, the optimal joint link-and-user scheduling criteria is derived and a rate allocation method of the S-R link is proposed. Two reduced-complexity joint link-and-user scheduling methods which use ZF beamforming in the R-D link are also investigated in this section. Numerical results are presented in Section IV and conclusion is drawn in Section V.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

As shown in Fig. 1, the system consists of an SN with \( N_s \) antennas, a \( N_r \)-antenna RN and \( M \) (\( M > N_r \)) single-antenna DN s. We assume that there is no direct link between the SN and the DN s. The messages are delivered from the SN to the DN s with the help of the RN. After receiving the messages from the SN, the RN decodes them and stores them temporarily in the buffer before forwarding the messages to the respective DN s. As there are \( M \) DN s, the buffer of the RN is split into \( M \) logical parts and each part stores the messages of one DN. It is assumed that the RN can store infinite number of messages. In each time slot, the RN decides to receive or to transmit based on the channel qualities of the S-R link and the R-D link. If the SN is scheduled to transmit, it encodes all the messages which are to be delivered to all the DN s into one code word and transmits it to the RN through the point-to-point MIMO channel formed by the SN and the RN. After successfully decoding the code word, the RN stores the messages into the corresponding logical parts of the buffer. If the RN is scheduled to transmit, the RN finds out the particular DN s that are scheduled based on certain scheduling criterion and then the information from the corresponding parts of the buffer is extracted and delivered to these DN s. Moreover, we assume that the RN works in a half-duplex mode.

We assume a block flat fading channel model in which the channel impulse response is constant during one time slot and changes independently from one time slot to another. All the channel gains are independent Rayleigh distributed random variables with unit variance. In a given time slot, the RN first acquires the channel state information of the S-R link and the R-D link by using the reference signals transmitted by the SN and the RNs. Based on this channel state information, the RN makes the link and user scheduling decisions. Then these scheduling decisions are fed back to the SN and all DN s. The channels are assumed to be reciprocal as a time-division-duplexing method is used. Thus the SN can use these feedbacks to acquire the channel state information of the S-R link and therefore the SN and the RN can adjust their transmitting rates to the channel achievable rates. Without loss of generality, we assume that the additive white Gaussian noise received at each receive antenna has zero mean and unit variance.

B. Problem Formulation

For a particular time slot \( k \), we denote \( R_{sr}[k] \) as the achievable rate of the S-R link. The achievable rate of the \( i \)th (\( i = 1, 2, \ldots, M \)) user of the R-D link is denoted as \( R_{ri}[k] \). Note that \( R_{sr}[k] \) depends on the specific signal transmitting method (ZF beamforming etc.) used at the RN. In each time slot, the RN is responsible to select the active link and users. We use \( \xi[k] \) to denote the set of node(s) (SN or DN s) scheduled in the \( k \)th time slot. For instance, if the RN decides to transmit information to user 1 and user 2, we have \( \xi[k] = \{D1, D2\} \); if the SN is chosen to transmit, \( \xi[k] = \{SN\} \). We use \( R_{sr}[k] \) and \( R_{ri}[k] \) to denote the actual rates transmitted to the RN and user \( i \), respectively. Therefore, \( R_{sr}[k] = R_{ri}[k] \) if user \( i \) is scheduled in the \( k \)th time slot, otherwise it is set to be 0.

As the SN needs to send information to all the DN s, the rate transmitted in the S-R link should be properly allocated to the DN s. We use \( \alpha_i R_{sr}[k] \), \( \sum_{i=1}^{M} \alpha_i = 1 \), to denote the fraction of information sent to user \( i \) when the SN is scheduled to transmit. Our objective function for joint link-and-user scheduling is to maximize the average sum rate of the relaying system. Thus we can form the following optimization problem:

\[
\begin{align*}
\max_{\xi[k], \alpha_i, \forall k, i} & \sum_{i=1}^{M} R_{ri} \\
\text{subject to} & \quad R_{sr} \leq \alpha_i R_{sr}, \quad i = 1, 2, \ldots, M, \quad (1)
\end{align*}
\]

where \( R_{sr} = \frac{1}{N} \sum_{k=1}^{N} R_{sr}[k] \), \( R_{sr} \) are the average rates of user \( i \) and the S-R link, respectively, and \( N \) is the total number of time slots used. The constraints of (1) result from the flow conservation law which guarantees that the long-term average rate of each DN input to the corresponding parts of buffer matches the average rate output from these parts.

III. JOINT LINK-AND-USER SCHEDULING

A. Optimal Scheduling Criterion

To solve problem (1), a proposition is first presented in the following. Based on this proposition, we simplify the optimization problem and derive the optimal joint link-and-user scheduling criterion.

**Proposition 1:** If the condition \( \sum_{i=1}^{M} R_{ri} \leq R_{sr} \) is satisfied, we can always find a sequence of \( \alpha_i, i = 1, 2, \ldots, M \) which satisfies all the constraints of (1).

**Proof:** Let \( \alpha_i = R_{ri} / \sum_{i=1}^{M} R_{ri} \). If it satisfies \( \sum_{i=1}^{M} \alpha_i R_{ri} \leq R_{sr} \), then we always have \( R_{ri} \leq \alpha_i \sum_{i=1}^{M} R_{ri} \).
Following that, it can be easily checked that the value of $\alpha_i$, for $i = 1, 2, \cdots, M$, satisfies $\sum_{i=1}^{M} \alpha_i = 1$.

Remark 1: Proposition 1 implies that if the average sum rate of the R-D link and the average sum rate of the S-R link satisfy the sum flow conservation constraint ($\sum_{i=1}^{M} \bar{R}_{ri} \leq \bar{R}_{sr}$), we can always find a set of $\alpha_i$, for $i = 1, 2, \cdots, M$, which satisfies the individual flow conservation requirement. Hence, the SN can adjust the fractions of information allocated to each DN to satisfy the flow conservation requirements of all the DNs.

Based on Proposition 1, (1) can be rewritten as the following equivalent problem:

$$\max_{\xi[k], \nu[k]} \bar{R}_{rd}$$

s.t. $\bar{R}_{rd} \leq \bar{R}_{sr}$,

where $\bar{R}_{rd} = \sum_{i=1}^{M} \bar{R}_{ri}$ is the sum rate of the R-D link. In the following, we solve the equivalent simplified optimization problem in (2) by using the Lagrangian dual method, which yields:

$$\min_{\lambda} Q(\lambda),$$

where $\lambda \geq 0$ is the Lagrange multiplier and

$$Q(\lambda) = \max_{\xi[k], \nu[k]} \bar{R}_{rd} - \lambda(\bar{R}_{rd} - \bar{R}_{sr}).$$

By using the facts that $\bar{R}_{ri} = \frac{1}{N} \sum_{k=1}^{N} \bar{R}_{ri}[k]$, $\bar{R}_{sr} = \frac{1}{N} \sum_{k=1}^{N} \bar{R}_{sr}[k]$, (4) can be rewritten as

$$Q(\lambda) = \max_{\xi[k], \nu[k]} \frac{1}{N} \sum_{k=1}^{N} (\bar{R}_{rd}[k] - \lambda(\bar{R}_{rd}[k] - \bar{R}_{sr}[k])),$$

where $\bar{R}_{rd}[k] = \sum_{i=1}^{M} \bar{R}_{ri}[k]$. It can be observed that problem (5) can be divided into $N$ independent subproblems with the same structure and each subproblem corresponds to one time slot. As the subproblems have the same structure, it can be concluded that the optimal joint link-and-user scheduling strategy of one time slot is also the optimal scheduling method of the system. Therefore, the problem in (5) is equivalent to the following problem:

$$\max_{\xi[k]} (1 - \lambda)\bar{R}_{rd}[k] + \lambda\bar{R}_{sr}[k].$$

As only one of the two links (the S-R link and the R-D link) can be scheduled in each time slot, the solution of (6) which also provides the optimal joint link-and-user scheduling criterion for a given $\lambda$ can be expressed as

$$\xi[k] = \arg \max_{\xi[k]} \left( (1 - \lambda)\bar{R}_{rd}[k], \lambda\bar{R}_{sr}[k] \right).$$

Note that in (7), $\bar{R}_{rd}[k]$ denotes the sum rate of the set of scheduled DNs of the R-D link. Based on the scheduling criteria presented in (7), the following two steps can be applied by the RN to implement the optimal joint link-and-user scheduling:

1) **User scheduling step**: In the $k$th slot, the RN chooses the set of users that maximize the sum rate of the R-D link ($\bar{R}_{rd}[k] = \sum_{i=1}^{M} \bar{R}_{ri}[k]$).

2) **Link scheduling step**: If it satisfies $\bar{R}_{rd}[k] > \frac{\lambda}{1-\lambda} \bar{R}_{sr}[k]$, then the R-D link is scheduled and the RN transmits messages to the set of scheduled DNs; otherwise, the S-R link is scheduled and the SN transmits messages to the RN.

To solve the problem in (7), the optimal value of $\lambda$ needs to be found. It has been shown in [4] that there always exists a unique optimal $\lambda$ which makes $\bar{R}_{rd} = \bar{R}_{sr}$. We denote $f_{R_{ri}}(r)$ and $f_{R_{rd}}(r)$ as the probability density functions (PDFs) of the average sum rates of the S-R link and the R-D link, respectively. Let $F_{R_{ri}}(r)$, $F_{R_{rd}}(r)$ be the corresponding cumulative distribution functions (CDFs), the average achievable rates of the S-R link and the R-D link can be expressed as

$$\bar{R}_{sr}(\lambda) = \int_{0}^{\infty} f_{R_{sr}}(r) F_{R_{rd}} \left( \frac{\lambda f_{R_{rd}}}{1-\lambda} \right) dr$$

$$\bar{R}_{rd}(\lambda) = \int_{0}^{\infty} f_{R_{rd}}(r) F_{R_{sr}} \left( \frac{1-\lambda f_{R_{sr}}}{\lambda} \right) dr.$$  

The optimal $\lambda$, denoted as $\lambda^*$, should satisfy $\bar{R}_{sr}(\lambda^*) = \bar{R}_{rd}(\lambda^*)$. If $f_{R_{ri}}(r)$ and $f_{R_{rd}}(r)$ are available, the bisection searching method can be applied to find $\lambda^*$.

As the condition $\bar{R}_{sr} = \bar{R}_{rd}$ is always satisfied under the optimal scheduling scheme, the optimal value of $\alpha_i$ can thus be obtained as

$$\alpha_i^* = \frac{\bar{R}_{ri}(\lambda^*)}{\bar{R}_{sr}(\lambda^*)}, \forall i = 1, \cdots, M,$$

in which the long term average achievable rate of the $i$th user can be written as

$$\bar{R}_{ri} = \int_{0}^{\infty} r f_{R_{ri}}(r) \text{Prob}(\text{User } i \text{ is scheduled}|\bar{R}_{ri} = r) dr.$$  

(10)

Under this rate allocating method, it can be checked that the constraints of (1) are all satisfied with equality $\bar{R}_{ri} = \alpha_i^* \bar{R}_{sr}, \ i = 1, 2, \cdots, M$. Thus the flow conservation constraints of all the DNs are satisfied.

For the S-R link, the SN and the RN form a point-to-point MIMO channel which is denoted as a $N_s$ by $N_r$ matrix $H_{sr}$. Denote the transmit power of SN as $P_s$, then the achievable rate of the S-R link in slot $k$ can be expressed as

$$R_{sr}[k] = \sum_{j=1}^{m_{sr}} \log_2 \left( 1 + \frac{P_s}{m_{sr}} \gamma_{sr}[k] \right),$$

where $m_{sr} = \min\{N_s, N_r\}$. Note that $\gamma_{sr}[k]$ denotes the $j$th eigenvalue of $H_{sr}[k] H_{sr}^H[k]$, where $(\cdot)^H$ stands for conjugate transpose of a matrix. To make the problem more trackable and to focus on the joint link-and-user scheduling method, we do not consider the power allocation problem (water-filling) of the S-R link in this paper. As we assume that the channel responses are identically independent Rayleigh distributed variables, according to the results of [13], $f_{R_{sr}}(r)$ can be closely approximated as Gaussian distribution with
mean value $\mu_{sr}$ and variance $\sigma_{sr}^2$ given by
\[
\mu_{sr} = \sum_{j=1}^{m_{sr}} \frac{(j-1)!}{(j-1+|N_r-N_s|)!} \int_0^\infty \log_2 (1 + \rho_{sr} z) \int_0^\infty \log_2 (1 + \rho_{sr} z) L_{j-1}^{N_r-N_s}(z)^2 z^{N_r-N_s} e^{-z} dz dz
\]
\[
\sigma_{sr}^2 = \sum_{j=1}^{m_{sr}} \frac{(j-1)!}{(j-1+|N_r-N_s|)!} \int_0^\infty \log_2 (1 + \rho_{sr} z) \int_0^\infty \log_2 (1 + \rho_{sr} z) L_{j-1}^{N_r-N_s}(z)^2 z^{N_r-N_s} e^{-z} dz dz
\]
\[
- \sum_{l=1}^{\infty} \sum_{j=1}^{m_{sr}} \frac{(j-1)!}{(j-1+|N_r-N_s|)!} \int_0^\infty \log_2 (1 + \rho_{sr} z) L_{j-1}^{N_r-N_s} \frac{z^{N_r-N_s}}{e^z} dz dz
\]
\[
\int_0^\infty \log_2 (1 + \rho_{sr} z) L_{j-1}^{N_r-N_s}(z)^2 z^{N_r-N_s} e^{-z} dz dz
\]
(12)

where $\rho_{sr} = \frac{P_r}{m_{sr}}$ is the average signal-to-noise ratio (SNR) of each substream of the MIMO channel, $|N_r-N_s|$ denotes the absolute value of $N_r-N_s$, $L_{j-1}^{N_r-N_s}(z)$ is the Laguerre polynomial of order $j-1$.

For the R-D link, the achievable sum rate $R_{rd}[k]$ depends on the user scheduling method used in step one. For different user scheduling approaches, $f_{R_{rd}}(r)$ has different forms. In the following, we will investigate two ZF beamforming-based user scheduling approaches which have low computational complexity.

B. ZF Beamforming-Based Joint Link-and-User Scheduling

In the first step of obtaining the optimal scheduling criterion, the RN needs to find out the subset of DNs that maximize the sum rate of R-D link. However, to maximize the sum rate of R-D link, the dirty paper coding method and the exhaustive searching method should be performed for the broadcast channel formed by the R-D link [14], which makes the system to be extremely complicated. Thus in the following, we investigate two ZF beamforming-based user scheduling approaches, namely the ZF-based round robin (RR) user group scheduling and the ZF-based selective user group scheduling, which have low implementing complexity.

1) RR User Group Scheduling scheme: At the RN, ZF beamforming is applied to avoid interference between the DNs. As the RN has $N_r$ antennas, it can transmit information to at most $N_r$ DNs simultaneously using the ZF beamforming method. Thus the DNs are divided into groups with $N_r$ users in each group and these groups are in turn served by the RN in the time slots that the R-D link is scheduled. Note that although the system can not benefit from the multiuser diversity by using the RR user group scheduling, it can still achieve the multi-hop diversity through the link adaption process of step two. During a specific slot, the received signal of user $i$ can be expressed as
\[
y_i = \sqrt{\frac{P_r}{N_r}} w_i^H h_{ri} x_i + n_i,
\]
where $w_i$ is the beamforming vector for user $i$, which has a Frobenius norm $\|w_i\| = 1$, $h_{ri}$ is a column vector which stands for the channel response between user $i$ and the RN, $n_i$ is the additive Gaussian noise received by user $i$ which has unit variance, $x_i$ is the signal transmitted to user $i$ which satisfies $E[\|x_i\|^2] = 1$ and $P_r$ is the transmitting power of the RN. We assume that equal power $\frac{P_r}{N_r}$ is allocated to the $N_r$ users of one group.

The beamforming vector $w_i$ must satisfy $w_i \in \text{null}(H_i)$, where $\text{null}(H_i)$ denotes the null space of the matrix $H_i$, which is formed by the channel responses of the rest $N_r-1$ users of the group. As $H_i$ is a $N_r \times N_r - 1$ matrix, there is only one beamforming vector $w_i$ for user $i$ which lies in the null space of $H_i$. Thus the equivalent channel response $h_{ri} = w_i^H h_{ri}$ of user $i$ is an complex Gaussian random variable with zero mean and unit variance. The received SNR of the $i$th user $\rho_i$ is an exponential distributed random variable with mean value $\gamma = \frac{P_r}{P_s}$. As the achievable rate of user $i$ is $R_{i} = \log_2(1 + \rho_i)$, the CDF and the PDF of the achievable rate of user $i$ can be expressed as:
\[
F_{R_i}(r) = 1 - \exp(-\frac{2r - 1}{\gamma}),
\]
\[
f_{R_i}(r) = \ln2 \frac{2r - 1}{\gamma} \exp(-\frac{2r - 1}{\gamma}).
\]
(14)

As the sum rate of the R-D link is $R_{rd}[k] = \sum_{i=1}^{M} R_{i}[k]$, we have
\[
F_{R_{rd}}(r) = \prod_{i \in \Omega} f_{R_i}(r_i) dr_i \ldots dr_{N_r},
\]
(15)

where $\Omega$ is the set of users in one group, $A = \{ \sum_{i \in \Omega} r_i \leq r \}$ is the integrating area. The PDF can be obtained as $f_{R_{rd}}(r) = df_{R_{rd}}(r) / df_{R_i}(r_i)$. By combining these results with (8) and applying the bisection searching approach, $\lambda^*$ can be obtained numerically.

Remark 2: Under the assumption that all channel gains of the R-D link are identically independent distributed, $\alpha^* = \frac{1}{M}$ for the RR user group scheduling scheme. This can be explained as follows: for the RR user group scheduling scheme, the users are served in turn in the time slots the R-D link is scheduled, thus we can conclude that all the DNs have equal probability to be scheduled. Furthermore, as the channel responses of all DNs have the same distribution, according to (10), all the DNs have identical long term average achievable rates.

Although the RR user group scheduling scheme has low complexity, the system can not benefit from the multi-user diversity. In the following, the selective user group scheduling scheme which can benefit from the multiuser diversity is investigated.

2) Selective User Group Scheduling scheme: For the ZF beamforming-based transmitting scheme, the optimal user scheduling approach for the RN is to exhaustively search all possible user subsets to find the specific user set which maximizes the sum rate of the R-D link. However, this method still has high complexity if the number of users $M$ is large.
To reduce the complexity, an ZF beamforming-based selective user group scheduling method which was proposed in [14] will be investigated. Denotes $U = \{1, 2, \cdots, M\}$ as the set of indices of all $M$ users, and let $S_n = \{s_1, \cdots, s_n\} \subset U$ denote the set of $n \leq N_r$ selected users. The selective user group scheduling method can be described as follows:

1) Initialization:
- Set $n = 1$.
- Find user $s_1$ as $s_1 = \arg\max_{i \in U} h_i^H h_{s_1}$.
- Set $S_1 = \{s_1\}$ and denote $R_{rd}(S_1) = \sum_{i \in S_1} R_{ri}$.

2) while $n < N_r$:
- Set $n = n + 1$.
- Find user $s_n$ as $s_n = \arg\max_{i \in U \setminus S_{n-1}} R_{rd}(S_{n-1} \cup \{i\})$.
- Set $S_n = S_{n-1} \cup \{i\}$ and denote $R_{rd}(S_n) = \sum_{i \in S_n} R_{ri}$.
- If $R_{rd}(S_n) \leq R_{rd}(S_{n-1})$, break, decrease $n$ by 1.

3) The set of scheduled users are $S_n$.

Note that to find $s_n$, for $n \geq 2$, the ZF beamforming is performed in each iteration. Compared with the RR user group scheduling, the selective user group scheduling method can benefit from the multi-user diversity while it still has relatively low complexity. However, for the selective user group scheduling method, it is difficult to find the distribution of the sum rate $R_{rd}$ of the relay-destination link. As a result, we cannot use (8) and the bisection searching method to find the optimal $\lambda$. Actually, it is not easy to calculate (15) when $N_r \geq 3$. Thus in the following, we use an alternative algorithm to find $\lambda^*$ and $\alpha^*_i$ iteratively.

Algorithm 1 Find $\lambda^*$ and $\alpha^*_i$

Initialization:
- Initialize $\bar{R}_{sr}[0] = 0$, $\bar{R}_{rd}[0] = 0$, $\lambda[0] = \lambda_0$;

Iteration:
- for $k = 1$ to $N$ do
  - Generate random channel realizations;
  - Find the set of DNs using selective user group scheduling approach and find the beamforming vectors;
  - Calculate the achievable rates $\bar{R}_{sr}[k]$ and $\bar{R}_{rd}[k]$;
  - if $\bar{R}_{rd}[k] < \frac{\lambda[k-1]}{\lambda[k-1]} \bar{R}_{sr}[k]$ then
    - $\bar{R}_{sr}[k] = \bar{R}_{sr}[k-1] + (\bar{R}_{sr}[k] - \bar{R}_{sr}[k-1])/k$;
  - else
    - $\bar{R}_{rd}[k] = \bar{R}_{rd}[k-1] + (\bar{R}_{rd}[k] - \bar{R}_{rd}[k-1])/k$;
    - $\bar{R}_{ri}[k] = \bar{R}_{ri}[k-1] + (\bar{R}_{ri}[k] - \bar{R}_{ri}[k-1])/k$, $\forall i$;
  - end if
  - Let $\lambda[k] = \lambda[k-1] + \delta(\bar{R}_{rd}[k] - \bar{R}_{sr}[k])$;
  - end for
- $\lambda^* = \lambda[N]$, $\alpha^*_i = \bar{R}_{ri}[N]/\bar{R}_{sr}[N]$, $\forall i$;
  - return $\lambda^*$ and $\alpha^*_i$.

In Algorithm 1, $\delta$ is the step size used to control the convergence speed of the algorithm. In Fig. 2, the recursively updated values of $\lambda[k]$ using algorithm 1 are shown. We generate $N = 5 \times 10^5$ channel realizations randomly. It can be observed that $\lambda[k]$ converges as $k$ increases. As the long average sum rate of the S-R link and the R-D link satisfy $\bar{R}_{rd} = \bar{R}_{sr}$, the optimality of the obtained $\lambda[N]$ is demonstrated.

Remark 3: In practical systems where the PDFs of the channel impulse responses are not perfectly known, Algorithm 1 can serve as an effective method to find $\lambda^*$ and $\alpha^*_i$.

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, some numerical results are provided to evaluate the performance of the joint link- and user scheduling method of the buffer-aided relaying system. We set $N_s = 3$, $N_r = 3$ and $M = 6$. In the ZF-based RR user group scheduling, the users are divided into two groups and each group has 3 users. The average achievable rate is obtained through $10^6$ channel realizations and we assume that the SN and RN use the same transmit power $P$ and $SNR = \frac{P}{N_0}$. In addition, to reduce the influence of the case that the buffer is empty, we assume that there is an initialization process to input enough data into the buffer (for instance, half of the buffer). This assumption is also adopted in [12].

In Fig. 3, the long term average sum rate of the joint link- and user scheduling relaying system is presented. The performance of the relaying system without buffer at the RN is also presented as a comparison. For the relaying system without buffer at the RN node, the long term achievable rate can be expressed as $\frac{1}{2} E(\min\{R_{sr}, R_{rd}\})$. To evaluate the performance gain of user scheduling, the performance of the ZF-based selective user group scheduling method and the ZF-based RR user group scheduling is compared. It can be observed from Fig. 3 that with buffering at the RN, the achievable rate of the system can be increased. For the RR user scheduling method, the achievable rate can be increased by 50% for $SNR = 15dB$ with the help of the buffer of the RN; for the selective user group scheduling, a 16% performance gain can be achieved. Furthermore, by comparing the performance of the ZF-based exhaustive searching user
scheduling method and the selective user group scheduling method, it can be observed that the performance loss between them is negligible. Thus the selective user scheduling method can achieve a good tradeoff between complexity and system performance. The selective user group scheduling outperforms the RR user group scheduling scheme as it can benefit from the multiuser diversity.

It can be observed from Fig. 4 that as the buffer size increases, the achievable rate of the finite buffer size system approaches the achievable rate of the system with infinite buffer. When the buffer size is greater than 300, similar performance can be achieved for the finite and infinite buffer-aided relaying system.

V. CONCLUSION

In this paper, we considered a buffer-aided relaying system which consists of a multiple-antenna source, a multiple-antenna relay node with buffer and multiple destinations. We adapted the joint link-and-user scheduling approach to maximize the long term average rate of the system. The optimal scheduling criteria was obtained and a two-step method was proposed to implement it. A rate allocation method was also proposed to make all the buffers of the relay node satisfy the flow conservation constraint. Two zero forcing beamforming based user scheduling method which have reduced complexity were investigated. It was shown that the joint link-and-user scheduling approach can effectively improve the achievable rate of the MIMO relaying system by making use of the multiuser diversity and the multi-hop diversity.

REFERENCES