Estimate of sediment pickup rate with densimetric Froude number

Nian-Sheng Cheng¹ and Adel Emadzadeh²

¹School of Civil and Environmental Engineering, Nanyang Technological University
Nanyang Avenue, Singapore, 639798. Email: cnscheng@ntu.edu.sg

²School of Civil and Environmental Engineering, Nanyang Technological University
Nanyang Avenue, Singapore, 639798. Email: aemadzadeh@ntu.edu.sg

Abstract

Experimental data collected in open channel flows show that the sediment pickup rate is better correlated with the densimetric Froude number than the Shields number. An empirical formula is then proposed for estimating the pickup rate. It does not involve the critical condition for the incipient sediment motion, and therefore can be applied to sediment entrainment of different stages including weak sediment motion. The result obtained in the present study is also supported by several series of data reported in the literature.
**Introduction**

The pickup rate $E$ is defined as the volume of the sediment entrained by flow over a unit bed area and has a unit of m/s. Its dimensionless form can be defined as (van Rijn 1984)

$$E_* = \frac{E}{\sqrt{\Delta g D}}$$ \hspace{1cm} (1)

where $\Delta = (\rho_s - \rho)/\rho$ with $\rho_s$ and $\rho$ denoting the grain and fluid density, respectively, $g$ is the gravitational acceleration and $D$ is the grain diameter. A pickup function that describes sediment entrainment rate can be integrated as part of a bedload theory. For example, Einstein (1950) proposed a probability-based pickup function in the derivation of bedload formula. The bedload theory as a whole has been verified with various laboratory and field data; however the pickup function included in the theory has been seldom calibrated independently.

Laboratory pickup experiments have been reported only in a few publications. Van Rijn (1984) designed a sediment lift and conducted a series of pickup experiments in open channel flows. Based on the data collected, he proposed the following pickup function

$$E_* = 0.00033D_*^{0.3}T^{1.5}$$ \hspace{1cm} (2)

where $D_*$ is the dimensionless grain diameter defined as $(\Delta g/\nu^2)^{1/3}D$ with $\nu$ being the kinematic viscosity of fluid, and $T = (\tau_*/\tau_{*c}) - 1$, with $\tau_*$ and $\tau_{*c}$ denoting the Shields number and its critical value for incipient sediment motion, respectively. By noting that $\tau_{*c}$ can be expressed as a function of $D_*$, Eq. (2) shows that $E_*$ generally varies with $\tau_*$ and $D_*$. Van Rijn (1984) also reported that the prediction using the pickup function proposed by Einstein (1950) is very poor, and some other formulas apply only for either small or large sizes of sediment. Okayasu et al. (2010) observed pickup rates over a small area of mobile bed that was located at 10 cm downstream a fixed bed in open channel flows. They employed a laser distance
meter to measure the change in the bed elevation and thus the pickup rate. Other relevant studies include Damgaard et al. (1997) and Dey and Debnath (2001), who investigated pickup rates in steep channels. A recent theoretical attempt is due to Zhong et al. (2011), who derived a pickup function by integrating particle velocity over all possible upward motions. In addition, Fernandez Luque (1974) applied a photography technique to measure the deposition rate, which is equal to the pickup rate when bedload transport is in equilibrium.

In the present study, a sediment lift similar to van Rijn (1984) was designed to measure pickup rates of three sizes of uniform sediments in open channel flows. An empirical formula is then developed by correlating the dimensionless pickup rate with the densimetric Froude number and the dimensionless grain diameter. The result shows that a better correlation is associated with the densimetric Froude number rather than the Shields number.

Experiments

Experiments were conducted in a slope-adjustable flume, 14 m long, 0.6 m wide and 0.6 m deep. Flow rates were measured with an electro-magnetic current meter with precision of 0.3 l/s. The test section was 9 m downstream from the channel entrance (Fig. 1). The flow depth remained close to 20 cm and the water temperature varied in the range of 28±1.5°C for all experiments.

To measure sediment pickup rate, a sediment lift was designed and installed at the bottom of the flume, through a rectangular opening located in the centre of the channel bed, which measured 1.5 cm in the streamwise direction and 10 cm in the lateral direction (see Fig. 1). The system included a piston that was installed inside a rectangular cylinder and connected with an electro-motor. By varying the speed of the piston, one could control the
rate of sediment supply. To operate the lifting system at very low entrainment rates, a gear box was also employed so that the piston could move as low as 0.2 mm/min. The displacement of the piston was measured by a dial gage with a precision of 0.01 mm. The product of the measured displacement and the area of the bed opening $A$ yielded the bulk volume $V$ of sediment grains entrained by flow. The sediment pickup rate $E$ was then calculated as $V(1-\varepsilon)/(AT)$, where $\varepsilon$ is the porosity of sediment bed and $T$ is the time of observation.

Three sets of uniform sediments were tested, with $D=0.23, 0.44$ and $0.86$ mm, and $\rho_s = 2650$ kg/m$^3$. The corresponding bed porosity measured was 0.44, 0.44 and 0.41, respectively. To have consistent bed roughness through the channel, the same grains as filled inside the sediment lift were also glued onto the entire bottom of the flume. To load sediment into the lift, sediment grains was first submerged in water in a separate glass container and then stirred in order to release air bubbles. Next, the grains were poured inside the lift with a funnel beneath the water surface. Finally, the grains inside the slot were briefly stirred and smoothed flat.

During the measuring of the pickup rate, the speed of the piston was adjusted slowly to avoid any formation of hump or pit over the bed area of observation. It was considered acceptable only when a flat surface with moving grains was observed over the area of observation. It should be mentioned that the upward speed of the piston was much smaller than the fall velocity of the sediment particles being picked up, so the lift would not affect significantly the hydrodynamic force acting on the particles and thus the pickup rate. The time for pickup rate measurement was taken as long as possible to avoid significant temporal changes. For weak grain movement, the pickup rate was measured as long as 12 minutes and for high transport the measurement took less than a minute for all the grains in the lift to be entrained. Each run was repeated at least five times under the same flow condition to make
sure that reliable values were recorded. The results were considered acceptable only when the deviation of each reading from the average was less than 15%. The standard deviation of the measurements was computed and then normalized by their mean as an indicator of the variation in the pickup rate measurements. The normalized standard deviation had an average of 7% for all the experiments and a maximum of 22% for the very large pickup rates for which all the sediment loaded in the lift were entrained in less than a minute.

To perform each test, the flume was first filled with water from the downstream end of the flume. Then, the pump was turned on to increase the flow rate gradually. At the same time, the tailgate was also adjusted until approximately a uniform flow was observed. Finally, the sediment pickup rate and the flow velocity were measured separately. Flow velocity measurements were carried out prior to or after the pickup measurements using a 3D down-looking Acoustic Doppler Velocimeter (ADV), which was positioned at the centre of the flume right above the lift slot. To provide sufficient seeding material inside the flow, an air bubble generator was positioned 1 m upstream the ADV location. It consisted of a mesh made from 0.2 mm stainless steel wire, which was connected to a direct current to generate tiny bubbles by means of electrolysis. For each velocity profile, 18-20 points were sampled from the bed towards the water surface at a rate of 50 Hz, each for 3 minutes. The raw data of flow velocity were processed using software WinADV. The shear velocity \( u_\tau \) was estimated by fitting the velocity profile measured in the near-bed zone (about 25% of the flow depth) to the logarithmic law. The boundary Reynolds number \( Re_\tau = u_\tau D / \nu \) varied from 2.4 to 38.7, showing that the bed condition encountered in the experiments was either hydraulically smooth or transitional.

In total, 61 experiments were completed in the present study for a range of low and high sediment pickup rates. The pickup rate \( E \) varied from \( 4.7 \times 10^{-7} \) to \( 7 \times 10^{-4} \) m/s, the
Reynolds number $Re$ from $4.7 \times 10^4 - 2 \times 10^5$ and the Froude number $Fr$ from 0.21-0.66. A summary of the experimental data is presented in Table 1.

Comparison with previous studies

The measured pickup rates are compared with those predicted using van Rijn’s (1984) formula [i.e. Eq. (2)], as shown in Fig. 2. It can be observed that the experimental measurements are not well predicted by van Rijn’s formula. The predictions generally agree with the measurements for $D = 0.44$ mm, but the predicted pickup rates become much greater than (up to ten times) the measurements for $D = 0.23$ mm and $0.86$ mm. In addition, there are five cases that cannot be predicted using van Rijn’s formula because their bed shear stresses are lower than the critical value for incipient sediment motion. Significant discrepancies were also observed when comparing the data with the formulas by Fernandez Luque (1974) and Nakagawa and Tsujimoto (1980).

Correlation based on densimetric Froude number

In the following, two kinds of correlations are performed, one based on the dimensionless grain diameter $D_*$ and the Shields number $\tau_*$ and the other based on $D_*$ and the densimetric Froude number $F_*$ defined as

$$ F_* = \frac{U}{\sqrt{\Delta g D}} $$

(3)
where $U$ is the depth-averaged flow velocity. The densimetric Froude number describes the bulk fluid force relative to the submerged grain weight. In comparison, the Shields number quantifies the bed shear force relative to the submerged grain weight. It is noted that $F_*$ has been often used in the prediction of local scour (Hager 2007; Hong et al. 2013). When correlating with the Shields number $\tau_*$ and the dimensionless sediment diameter $D_*$, the dimensionless pickup rate $E_*$ can be expressed in the power form,

$$ E_* = a_1 D_*^{b_1} \tau_*^{c_1} $$

(4)

where $a_1$, $b_1$ and $c_1$ are constants. It is noted that van Rijn (1984) presented a similar $\tau_*$-based correlation, but involving the critical shear stress. Similarly, for the $F_*$-based correlation, $E_*$ can be expressed as

$$ E_* = a_2 D_*^{b_2} F_*^{c_2} $$

(5)

where $a_2$, $b_2$ and $c_2$ are constants.

To quantify the goodness of fit of the proposed power functions, the Pearson product-moment correlation coefficients (i.e. Pearson’s $R$) were calculated. Fig. 3 presents the variation of $R_1^2$ with $b_1$, where $R_1$ is the correlation coefficient of log($E_*/D_*^{b_1}$) and log($\tau_*$). It can be seen that $R_1^2$ achieves its maximum 0.82 at $b_1 \approx 1.8$. Also plotted in Fig. 3 is the variation of $R_2^2$ with $b_2$, where $R_2$ is the correlation coefficient of log($E_*/D_*^{b_2}$) and log($F_*$). By using $F_*$, the $R_2^2$-value increases by 15% and $R_2^2$ achieves its maximum 0.94 at $b_2 \approx 2.5$. The improvement in the correlation can be further appreciated by comparing the relation of $E_*/D_*^{1.8}$ against $\tau_*$ (see Fig. 4) and that of $E_*/D_*^{2.5}$ against $F_*$ (see Fig. 5). Furthermore, the relation presented in Fig. 5 can be described using the following empirical equation,

$$ E_* = 0.0001 D_*^{2.5} F_* \exp\left(-\frac{40}{F_*}\right) $$

(6)
It is noted that Eq. (6) does not involve any critical condition for incipient sediment motion, and thus is applicable for low pickup rate.

Finally, the data collected in the present study [and thus Eq. (6)] are further compared with the experimental results reported in the previous studies by Fernandez Luque (1974), van Rijn (1984) and Okayasu et al. (2010). Fernandez Luque (1974) conducted experiments using natural and artificial grains with $D = 0.9-3.3$ mm and $\rho_s = 1340-4580$ kg/m$^3$. Van Rijn’s (1984) experiments involved five sizes of sand grains with $D = 0.13-1.3$ mm, while Okayasu et al.’s (2010) study was limited to a single size of sand with $D = 0.31$ mm. All these data are presented in Fig. 6, in the form of $E_*/D_*^{2.5}$ against $F_*$. It shows that almost all the data points follow the same trend as given by Eq. (6).

**Summary**

A series of experiments were completed in the present study to measure sediment pickup rate in open channel flows. The measurements cannot be well predicted using the previous formulas. Correlation analyses suggest that the dimensionless pickup rate is better associated with the densimetric Froude number than the commonly-used Shields number. An empirical pickup function is finally proposed to associate the dimensionless pickup rate with the dimensionless grain diameter and the densimetric Froude number, which fit the data collected in the present study and also those reported in the literature.
References


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Fig. 1. Experimental setup
Fig. 2. Comparison of measured pickup rates with predictions by van Rijn’s formula
Fig. 3. Variations of $R^2$ with the power $b$
Fig. 4. Variation of $E_\ast / D_\ast^{1.8}$ with $\tau_\ast$.
Fig. 5. Variation of $\frac{E_*}{D_*^{2.5}}$ with $F_*$. 

$E_*$

$D_*^{2.5}$

$F_*$

D = 0.23 mm

0.44

0.86

Eq. (6)
Fig. 6. Comparison of the present study with experimental data reported in the literature.