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<td>Author(s)</td>
<td>Tjahjowidodo, Tegoeh; Zhu, Ke; Dailey, Wayne; Burdet, Etienne; Campolo, Domenico</td>
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<td>[<a href="http://hdl.handle.net/10220/42439">http://hdl.handle.net/10220/42439</a>]</td>
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<td>Rights</td>
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</table>
Title: Multi-source micro friction identification for a class of cable-driven robots with passive backbone

Article Type: Full Length Article

Keywords: transparent haptic interface; cable driven robot with passive backbone; friction; hysteresis; Coulomb; LuGre

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Order of Authors: Tegoeh Tjahjowidodo, Ph.D.; Ke Zhu; Wayne Dailey; Etienne Burdet, PhD; Domenico Campolo, PhD
Dear Editor,

Please find enclosed a revised manuscript of MSSP16-47R1 following your recommendations.

We have revised the paper taking into account all the comments of the reviewers, as you can find delineated in the accompanying detailed revision note. We believe that we have paid due attention to the reviewer comments, with most of which we are in full agreement. I trust that the manuscript will now need no significant changes.

The paper grew out of a research programme on the dynamics of mechanical systems for haptic interface. It reports the dynamic modelling of a planar cable differential transmission robot, which was designed for haptic interaction applications.

Looking forward to hearing from you.

Yours Sincerely,

Tegoeh Tjahjowidodo, Ph.D.
Asst./Professor
e-mail: ttegoeh@ntu.edu.sg
Revision Notes

Comments from the reviewer and author responses are listed below and the revisions on the manuscript are marked in red fonts.

Reviewer#1

The paper deals with the identification of the mechanical behavior of a cable-driven 2-dof robot, called H-Man. The latter is constituted by two rotary DC motors actuating a Cartesian double sliding mechanism. The focus of the paper consists in the investigation of the frictional forces arising in the devices whose identification is fundamental to compensate the commanded actuation. The sources of friction considered are the actuators and the sliders.

Different friction models are employed for identifying the mechanical response. The considered models are: Coulomb model, Stribeck model and LuGre model. For the Coulomb model the parameters are identified by means of a linear regression while for the Stribeck and LuGre models a Genetic Algorithm is employed.

The content of the document is clearly exposed and the results are of practical and scientific interest. The paper is recommended for publication after a minor revision.

1) The number of degrees of freedom is N for the actuators space and M for the backbone space. It should be better to use a different index (not j for two both) for denoting the dofs in the two spaces.

The indices for actuators space and backbone space are now indicated as n and m, respectively.

2) In Eq. (3), it is correct the subscript tau of the last term on the right-hand side? Is it r instead of t?

The manuscript has been revised accordingly. We thank to the reviewers for their thorough review.

3) Check carefully the text. Correct the sentence "...presented in Table 1, which are derive based on variation..." on page 5. Correct "(Eq. 9)" on page 7.

We thank the reviewer for this comment. We have revised the text accordingly.

4) On page 8 provide the Units for alpha and gamma parameters.

The units are now indicated in the revised manuscript.
5) **Provide the type and material of the cable operating in the device.**
We are using stainless steel cables ASAHI INTECC (NB45-61, 0.61 mm, nylon-coated with 7 wirerope cores). We have added the information on the revised manuscript.

6) **The final Discussion can be shorter because some contents are already given in the introduction.**
The concluding section has been rewritten to be more concise. A few repetitive statements have been omitted.
Reviewer #2

In this paper, the application of different well-known friction models for a cable-driven robot is analysed and experimentally investigated. The purpose of the friction model is to compensate for the friction occurring in the actuator and task space. Coulomb, Striebeck and LuGre models are compared to model the friction of the robot. The evaluation is divided into the actuator and task space. The identification and verification with an additional force sensor at the end-effector are presented. The conclusions are straightforward and are not surprising as the most complex model delivers the best approximation and the Striebeck model already proofs a good modelling accuracy.

Using the MSE as metric the influence of the hysteresis depends strongly on the number of changes in direction of movements, as also stated in the paper. The reviewer is wondering if there is maybe a better variable to evaluate the accuracy of the friction model especially the accuracy of hysteresis.

MSE is the common practical quantifier to express the accuracy of a model. It is used in some papers, e.g. [7], [19], [22], [30] [33]. In particular, [12] discusses the analysis of an advanced friction model, the so-called GMS model that demonstrates its ability in capturing the frictional effect during reversal points. The model is shown to be capable to minimize the spiky errors that are commonly occurs in any friction models. In order to quantify the superiority of the model, maximum error values are also presented besides the MSEs. However, in our case, the significance of the dynamics at the presliding regimes is less-significant. Therefore, MSE is considered to be sufficient to quantify the model performance.

Major issues:

* It is unclear, why you introduce first the summary of the modelling approaches in section 3 and then present the stiction models in section 4. It should be vice versa. The main objective of the paper is to characterize mechanical systems with multi-friction source, illustrated on the H-Man system. As such, we feel that it is necessary to describe the mechanism and the origin for the friction sources on the system before we present the stiction models. In addition a brief discussion on the mechanism preceding the stiction models will provide a quick take for the readers to understand our incremental modelling depth, which not only covers stiction model selection but also multiple sources of friction.

* Furthermore, it should be explained why you chose Coulomb, Striebeck and LuGre models and neglect other models like Dahl. The motivation for selecting the models has been added to the last part of section 3. Coulomb and Striebeck models were chosen to represent the conventional class of stiction model, while LuGre was selected to represent the advanced class. In particular, the LuGre model is selected because of its simplicity and the ability to capture sliding-stiction transition.
The significance of the frictional effect on the system is confirmed: I cannot see in the paper, if the friction has an important effect as the nominal force of the system is missing. So it cannot be concluded if 3 N friction is much or negligible.

The H-Man robot is a haptic device designed to generate relatively low forces. A typical application of the H-Man is described in [9], where users train under the influence of disturbances implemented as curl-type force fields, i.e. force fields always proportional to the magnitude of the handle velocity but always perpendicular to its motion. Forces, therefore, depend on the subject motor strategies but, as show in [9], maximum forces generated by the robot in typical applications hardly exceeds 10 N (in most cases is confined below 5 N). This confirms the significance of the observed friction force in Figure 4, where it reaches maximum force of 3 N.

The manuscript has been revised to clarify this issue.


The paper needs a major revision regarding linguistic errors and format. Please see the following comments.

We have let a native English speaker to edit the final version of the manuscript.

Minor issues:

* serial or parallelogram mechanisms -> parallel mechanisms?
We thank the reviewer for noticing this, and have revised the text accordingly.

* Figure 1: You say the figure is reproduced from [14] (Agrawal et al.), shouldn't it be [6]?
We have revised the text accordingly. Thank you.

* How do you measure the command force? By current measurement or direct force/torque measurement at the motor?
For identification purpose, the forces are taken from the reference forces prescribed to the system as it is a force controlled system. Later on, the dynamic models are validated using the force readings obtained from force sensors that are applied mainly for this validation purpose. The manuscript has been revised and the corresponding information has been appended on the text.

* Which frictions are modeled by the viscous friction term (actuator or end-effector?)
As presented in Eq. [2] and Eq. [4], all friction models comprise two components, i.e. viscous and stiction. Both spaces (actuator and backbone) are modelled considering the two friction components, except for M1, which only consider viscous component on the frictional model.
* **M3-M5:** Do you apply one model for motor and one model for the slider or is it one combined model for both?
Based on the assumption that the two motors are identical, we used the same friction model from both motors, but we model each of the sliders separately. This means we have two identical models for the motors and two different models for different sliders. The text has been revised to clarify this issue.

* **Figure 1 suffers from a too small font size and poor contrast**
We have revised the figure accordingly.

* **Section 5: simple asymmetric Coulomb model: Eq. 9 -> Eq. 9?**
We have revised the text accordingly.

* **Eq. (9) represents the symmetric Coulomb model. How do you incorporate the asymmetry with the tanh-function?**
We made a mistake in referencing the equation. The asymmetric Coulomb is not formulated based on the tanh (sigmoid) function, but it is written as an asymmetric signum function as presented in Section 5. Eq. 9 should refer to the equation preceding to the (previously) Eq. 9:

\[
F_{asym}(\dot{x}) = \begin{cases} 
F_{c}^+, & \text{for } \dot{x} \geq 0 \\
F_{c}^-, & \text{for } \dot{x} < 0 
\end{cases}
\]

..........................(9)

* **Table 1: Here you denote the sources as motor and sliders, in the text you use actuator and backbone**
We have revised the table accordingly to be more consistent with the text.

* **Figure 4: legend hides part of the diagram**
We have revised the figure.

* **Equation (9): reference for the equation using the tanh-function.**
Equation 9 now is referred to the preceding equation as explained in the previous 3 points above.

* **Equation (11): why there is a dot for multiplication?**
We have revised the text accordingly.

* **In this purpose -> For this purpose**
We have revised the text accordingly.

* **Simulated system dynamics: units for gamma and tau are missing**
The corresponding units have been added in the text.
* **Table 3: Kg -> kg**
We have revised the text accordingly.

* **Eq. 12: variable z is not introduced**
We have now described the variable z in the text.

* **Section 5: what is theta axis?**
Theta (θ) indicates any axis in the system. It can be from actuator space that takes in rotational direction or backbone space that is in translational system. We have revised the text to clarify this.

* **Figure 5: the kinematics reaches 0.6 m/s on a distance of only approximately 0.08m. Is this correct? It seems to be a high acceleration.**
The prescribed excitation kinematics shown in Fig.5 is also given in its analytical form in the text as
\[ y = 0.01\sin(2\pi t) + \sin(2.26\pi t) + \sin(3\pi t) + \sin(3.46\pi t) + \sin(4\pi t) + \sin(4.5\pi t) \]
it is possible to evaluate the acceleration and verify that it never exceeds 6m/s^2.
Even considering a maximum directional inertia of 1.694 Kg (as per Table 3), this peak of acceleration corresponds to a peak force of 6*1.694 = 10.1 N, which is again within the intended range of operation.
We have added a statement clarifying this matter.

* **Excitation trajectory: why squared brackets [ ]?**
We have changed the equation on the text accordingly.

* **Consistent terminology should be improved: guidance space = backbone-space?**
Backbone space: with or without a hyphen? dof or DOF (furthermore, the shortcut is not introduced), human motor interaction vs. human-robot interaction, end-effector vs. end effector, lightweight vs light-weight, and further more...
We have revised the text accordingly to keep its consistency.

* **References [4], [5]: it seems as there is a justification. Please check the formatting**
We have revised the text accordingly.
Highlights

> Dynamics of cable-driven robots for haptic interface is modelled
> The significance of the frictional effect on the system is confirmed
> We evaluate the required complexity of the friction model
> The optimized models are validated experimentally with satisfactory results
> Suitable models for haptic interface are identified
Multi-source micro friction identification for a class of
cable-driven robots with passive backbone

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Abstract
This paper analyses the dynamics of cable-driven robots with a passive backbone and develops techniques for their dynamic identification, which are tested on the H-Man, a planar cable differential transmission robot for haptic interaction. The mechanism is optimized for human-robot interaction by accounting for the cost-benefit-ratio of the system, specifically by eliminating the necessity of an external force sensor to reduce the overall cost. As a consequence, this requires an effective dynamic model for accurate force feedback applications which include friction behavior in the system. We first consider the significance of friction in both the actuator and backbone spaces. Subsequently, we study the required complexity of the stiction model for the application. Different models representing different levels of complexity are investigated, ranging from the conventional approach of Coulomb to an advanced model which includes hysteresis. The results demonstrate each model’s ability to capture the dynamic behavior of the system. In general, it is concluded that there is a trade-off between model accuracy and the model cost.

Keywords: transparent haptic interface, cable-driven robot with passive backbone, friction, hysteresis, Coulomb, LuGre

1. Introduction

Human-Robot Interaction has received increasing attention in the last three decades. New robots have been specifically designed to work in direct physical contact with humans. In contrast to industrial robots, which are designed to be stiff and accurate, robots designed to interact with humans often need to be transparent to the user. When physically interacting with humans, robots are used to assist/resist the human operator in specific maneuvers but are otherwise expected to offer minimal resistance to natural motions. For this purpose, these robots are designed to be intrinsically lightweight in terms of the inertia perceived by the human during interaction, e.g. at the handle. Ideally, the perceived inertia should also be homogeneous (the perceived inertia is the same across the whole workspace) and isotropic (the perceived inertia is the same along all possible directions of motion). H-Man, a planar robot, which exemplifies these properties, serves as the test platform for this study.

Most existing robotic manipulanda involve planar linkages based on serial or parallel mechanisms ([2]-[6]). Cable-driven robots are intrinsically lightweight and also represent one possible solution to satisfy the need for a better cost-to-benefit ratio. In particular, this paper considers the class of cable-driven robots with a passive backbone, supporting an end-effector (or handle), which is driven remotely by cabled actuators. Two common sources of complexity in robots are backlash ([7],[8]) on the joints and friction between the moveable parts. In this class of robot, however, the latter one is playing more significant role. The friction forces from the passive backbone and from the actuation system are reflected onto the handle through different kinematic maps (i.e. Jacobian matrices). This differs from the case of serial manipulators where friction arises only at the joint.

While equations are presented in the general case, we also show experimental results relative to a novel planar cable differential transmission robot, H-Man, which resulted from efforts to develop a practical, low-cost platform for human-robot interaction studies and robot-assisted neurorehabilitation.
H-Man’s mechanical simplicity allows for easy implementation of haptic channels, force fields, and a variety of impedance and position control paradigms, with a high degree of backdriveability and near perfect isotropy [9]. H-Man’s hardware is low-cost relative to other functionally similar robotic systems which are commercially available; it is mechanically simple, and therefore easy to control, and intrinsically safe for humans. However, the cost of commercial load cells, which are typically necessary to accurately quantify haptic interaction, is a significant factor in the cost of the robot itself. Therefore, being able to accurately estimate haptic interaction through system characterization rather than direct measurement of force would significantly impact practicality of the device.

Additionally, it is possible to improve H-Man’s backdriveability by compensating for the robot’s dynamics. This is desirable in many situations such as implementing sensitive haptics or in neurorehabilitation applications where weak subjects may not be able to move a robot with a high degree of intrinsic resistance. In particular, friction typically plays a critical role in the resistance offered by the system during low acceleration movements carried out by humans. In order to accurately characterize the dynamics of the system, friction must be accurately identified. There exist a variety of techniques for parameter identification, ranging from the simplest model to the highly complex ([10]-[13]). Unlike fine and precise robots, such as those for micro-positioning systems, applications for human-robot interaction might not require extremely precise dynamic models. It may be possible to balance the cost of external sensing hardware with model complexity to reduce system cost without sacrificing performance for a given application.

In this paper, we develop techniques for dynamic identification of friction in cable-driven robots with a passive backbone, and apply and test these techniques on H-Man. We investigate, in particular, the effectiveness of different types of friction models ranging from the Coulomb model to the advanced LuGre model. The objectives of this study are: i) to analyze the phenomenon of multi-source friction, in particular static friction; and ii) to investigate the effectiveness of friction models with varying degrees of complexity.

2. Dynamics of Robots with Cable Actuation and Passive Backbone

When designing a cable-driven system for interacting with humans through a handle, the starting point is the definition of a desired configuration space for handle itself. Depending on the application, a user might be required to grasp a handle and move it in one degree of freedom (1 DOF, not necessarily linear), on a plane (2 DOF) or more degrees of freedom. Considering that a handle, as a rigid body in free space, has at most 6 DOF of motion, a passive mechanical support, or backbone, with M passive degrees of freedom can be designed to constrain unwanted motions along the remaining 6-M DOF. The advantage of such a class of robots, especially for haptic interaction, is that the passive backbone can be designed to be stiff, and thus reliably support the user’s hand, and lightweight, while the cable-driven actuation can significantly reduce the inertia as the motors do not move with the manipulandum [14].

For this class of robots, system dynamics naturally occur in two different spaces, namely the actuator space and the backbone space, in contrast to others, e.g. serial manipulators. In the latter, the dynamics of serial linkages and the dynamics of motors act concurrently in the so-called joint space and their effects simply sum up, i.e. one could write equations for the serial linkages and simply add additional parameters to include inertia and friction due to motors. As we show next, this is not the case for cable-driven robots with passive mechanical supports. This is especially important when taking into account static friction, or stiction (pre-sliding form of friction) which plays a significant role in many robotic systems [12].

We assume that the desired motions of the handle are guided by an M DOF (M ≤ 6) passive mechanical support, or backbone. The handle and the backbone can, in this sense, be identified as their dynamics take place in an M-dimensional space backbone space. At the same time, we can consider N cables, where N ≥ M, acting on the handles and transmitting forces produced by N motors, each with its own 1-dimensional dynamics. These motor dynamics can be combined in an N-dimensional space, called the actuator space.
2.1 Actuator Space

The actuator space is given by coordinates \{q_1, ..., q_n\}. In this space, the actuators produce force commands \{u_1, ..., u_n\}, which account for cable tensions \(\tau_1, ..., \tau_n\), friction \(\tau_m(q_n)\) and inertia \(I_n\). The cable tensions produced by the actuators are thus characterized by the dynamics:

\[
\tau = [\tau_1, ..., \tau_n, ..., \tau_m]^T \quad \text{and}
\]

\[
\tau_nR_n = I_n\ddot{q}_n + \tau_m(q_n, \dot{q}_n) + u_n \tag{1}
\]

where \(R_n\) is the effective motor radius, while the actuators' friction forces'\footnote{By nature, interactions between two sliding surfaces will also exhibit the viscous effect. In this paper we consider friction as a combination of a viscous term and a state-rate dependent effect, which we refer to as a stiction effect. In some other literature, the viscous effect is considered separately from the state-rate dependent effect, where the latter effect is referred to as the frictional effect.} comprise a viscous term and stiction effect that appears as a state-rate dependent function of \(q\):

\[
\tau_r, n = \beta_nq_n + \tau_{st, n}(q_n, \dot{q}_n) \tag{2}
\]

2.2 Backbone Space

The backbone is in general a non-redundant (to avoid non-repeatability and drift issues \footnote{By nature, interactions between two sliding surfaces will also exhibit the viscous effect. In this paper we consider friction as a combination of a viscous term and a state-rate dependent effect, which we refer to as a stiction effect. In some other literature, the viscous effect is considered separately from the state-rate dependent effect, where the latter effect is referred to as the frictional effect.}) mechanism designed to constrain the \(M\) DOF of the handle whose dynamics can then be represented in task space coordinates \(\{x_1, ..., x_m, ..., x_M\}\), which constitute the backbone space. These task space dynamics are described by the generalized force:

\[
F = M(x)\ddot{x} + C(x, \dot{x})\dot{x} + G(x) + F_r(x) \tag{3}
\]

where \(M(x)\ddot{x}\) is the inertial term with mass matrix \(M\), \(C(x, \dot{x})\) combines the Coriolis and centrifugal forces, and \(G(x)\) is the gravitational force where each term is restricted to the directions of the workspace. The friction along the mechanical guidance is, again, composed of state-rate dependent function (stiction) and a viscous term:

\[
F_{r, m} = B_m\dot{x}_m + F_{st, m}(x_m, \dot{x}_m) \tag{4}
\]

Backbone space and actuator space velocities and kinetics are related to each other via the structure matrix \(A\) (see \footnote{By nature, interactions between two sliding surfaces will also exhibit the viscous effect. In this paper we consider friction as a combination of a viscous term and a state-rate dependent effect, which we refer to as a stiction effect. In some other literature, the viscous effect is considered separately from the state-rate dependent effect, where the latter effect is referred to as the frictional effect.}):

\[
F \equiv A^T(x)\tau \tag{5}
\]

where the equivalence between these relationships can be shown from the invariance of power in the two different spaces.

It is important to note that the dynamics of Eqs. (1) and (3) contain friction terms in both actuator and guidance spaces. Note that in the representation of (3) the cable’s inertia is ascribed to the respective actuators’ inertia. Furthermore, we neglect the cables’ visco-elasticity, i.e. we assume that the cables are rigid.

Assuming that the actuators’ dynamics are linearly dependent on the parameters, the dynamics of the complete robot system can be written as

\[
\tau = \Psi(u, \dot{q}, \dot{x}, \ddot{x})p \tag{6}
\]

where \(p\) is a parameter vector. Using this linear representation of the dynamics enables identification using (e.g. iterative) least minimization of the model error or nonlinear adaptive control techniques minimizing the linear feedback of a feedforward controller ([10],[11]).

3. Application to H-Man Dynamics

H-Man is a two degrees-of-freedom robot constrained by perpendicular Cartesian sliding mechanisms. These sliders are actuated remotely by two rotary DC motors (Faulhaber Series 3863 024C) through stainless steel cables ASAHI INTECC (NB45-61, 0.61 mm, nylon-coated with 7 wirerope cores). An H-shaped cable-differential transmits force from the two motors to the end-effector as depicted in Figure 1. This mechanical configuration is isotropic and the configuration is independent with a conditioning number of 1. H-Man’s operation spans the actuator space of both motors and the task space of both sliders. A thorough description of the H-Man robot is provided in [9]. This section describes the kinematics and dynamics of a simple model of the H-Man.

The Jacobian matrix \(J (= A^T)\) maps the end-effector’s linear velocities \(v_x\) and \(v_y\) to the angular motor velocities, \(\omega_L\) and \(\omega_R\).
i.e. the Jacobian matrix depends only on the radius \( \rho \) of the driving pulleys.

\[
\begin{bmatrix}
    \omega_L \\
    \omega_R
\end{bmatrix} = J \begin{bmatrix} v_x \\
    v_y
\end{bmatrix}, \quad J = \begin{bmatrix} 1 & -1 \\
    \rho & -1
\end{bmatrix}
\]

Figure 1. (A) Kinematics of the H-Man robot. Counter-clockwise rotation of the motors is considered to be positive. (B) Isometric view of the mechanical assembly. Motor velocities and Cartesian coordinates are indicated on the rendering. Image reproduced from [9].

An accurate dynamic model provides a reliable mapping between the kinematics of the end-effector and the total force exerted both on and by the robot. The position, and through offline filtering and differentiation, the velocity, are available via the motor encoders and the force field is commanded by the real-time controller. In this application, the command force is already known based on the controller employed in a given task. Thus, any perturbation or interaction from a human user is simply the difference between the command force and the total force estimated by the model.

Knowledge of this interaction force is vital for many applications and human-robot interaction studies. Many existing manipulanda utilize expensive force sensing instrumentation which further complicates the system and introduces hardware costs that may limit the system's practicality. With relatively simple system dynamics, it can be practical to identify the force through accurate characterization rather a direct measurement. Furthermore, knowledge of the interaction force can be used to compensate for these dynamics and thus make the interface ‘transparent.’

The total force at the H-Man end-effector is defined as \( \tilde{F} = \begin{bmatrix} F_x, F_y \end{bmatrix}^T \). There is no Coriolis or centrifugal force (\( C \equiv 0 \)), gravity can be neglected (\( G \equiv 0 \)), and the inertia tensor \( H \) is diagonal:

\[
H = \begin{bmatrix}
    H_{xx} & 0 \\
    0 & H_{yy}
\end{bmatrix} \text{kg, while the viscous friction tensor } B \text{ is given by}
\]

\[
B = \begin{bmatrix}
    B_{xx} & B_{xy} \\
    B_{yx} & B_{yy}
\end{bmatrix} \text{Nm/s}.
\]

The general model including inertia, viscous friction and two friction terms is given by:

\[
\tilde{F} = \tilde{F}_I + \tilde{F}_B + \tilde{F}_{st\text{motors}} + \tilde{F}_{st\text{sliders}} \]

where the force of inertia is described by

\[
\tilde{F}_I = H \begin{bmatrix} \dot{x} \\
    \dot{y}
\end{bmatrix}, \text{the viscous friction forces by}
\]

\[
\tilde{F}_B = B \begin{bmatrix} v_x \\
    v_y
\end{bmatrix}
\]

and the friction of the motors and linear sliders by

\[
\tilde{F}_{st\text{motors}}(\omega_L, \omega_R) = \begin{bmatrix} \tau_{st}(\omega_L, \omega_R)
\end{bmatrix}
\]

\[
\tilde{F}_{st\text{sliders}}(v_x, v_y) = \begin{bmatrix} \tau_{fl}(v_x) \\
    \tau_{fr}(v_y)
\end{bmatrix}
\]

where \( \tau_{fl}(v_x) \) is the friction force from the x-slider and \( \tau_{fr}(v_y) \) is that from the y-slider.

In general, based on the complexity, stiction models can be grouped into two different classes. Coulomb and Stribeck models represent the first category of the conventional class. These models
only capture the sliding forces and ignore effects during the transition from sliding to presliding\(^2\). A few models fall into a more advanced class; Dahl’s [17] and LuGre [18] models are examples from the advanced class that are able to capture effects arising from a smooth transition between sliding and presliding modes. The two models were later improved by Leuven ([19],[20]) and GMS [21] models that incorporate a non-local memory effect during the transition as observed by many researchers. The last two models are proven to be capable of capturing stiction characteristics accurately; however, these approaches require many parameters and complex mathematical implementation. On the other hand, Dahl’s and LuGre models are shown to be more economical in implementation. Dahl’s has a better shape factor to capture the presliding effect, while LuGre offers better performance in capturing the transition between the two regimes [22].

In this study, the Coulomb and Stribeck models are selected to represent the conventional class of stiction model, while LuGre is chosen to represent the advanced class due to its relative simplicity and its performance in the transition regime, as the application of the robot will operate in both sliding and presliding regimes.

Each model will be evaluated through different approaches considering the origins of the friction in the system. This results in the system models presented in Table 1 which are derived from variations of Eq. (8). The models are briefly summarized as follows. The first model, \(M1\), only considers inertial force, \(\vec{F}_I\), and viscous effect, \(\vec{F}_v\), while \(M2\) comprises the third term of Eq. (8), the frictional forces of the motors that are captured using the Coulomb model. The remaining three models, \(M3-M5\), consider all friction sources on the system, including those from the sliders, and capture the friction with three different modelling approaches, i.e. Coulomb, Stribeck and LuGre, respectively. In the following section, the friction models are described and discussed.

<table>
<thead>
<tr>
<th>Model</th>
<th>Friction and dynamics</th>
<th>Sources</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Coulomb (asymmetric)</td>
<td>Actuators (Motors)</td>
</tr>
<tr>
<td></td>
<td>Stribeck</td>
<td>M1</td>
</tr>
<tr>
<td>(M1)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(M2)</td>
<td>✓</td>
<td>-</td>
</tr>
<tr>
<td>(M3)</td>
<td>✓</td>
<td>-</td>
</tr>
<tr>
<td>(M4)</td>
<td>-</td>
<td>✓</td>
</tr>
<tr>
<td>(M5)</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

| 4. Stiction Models |

Stiction is the result of interaction between two sliding surfaces and is dependent on many parameters including contact geometry, topography, surface materials, and the presence and type of lubrication. The interaction causes a complex dynamic behavior that involves a non-local memory property, which is actually observed in micro-sliding displacements ([23],[24],[25]).

The behavior can be described by resolving the stiction effect into two different regimes, i.e. the presliding and sliding regimes [25]. When a mass resting on a frictional surface is sliding and moving away from a reversal point exceeding a certain limit, the stiction force predominantly appears as a function of the sliding velocity. This stage is referred to as the sliding regime. In contrast, when the motion of the mass is reversed, within a certain limit from the reversal point, the system enters a presliding regime until it exceeds the limit (the presliding limit). Within this regime, the frictional effects of the mechanism are determined not only by the velocity, but also by displacement. In particular, the relation between stiction and displacement is characterized by non-local memory points and demonstrates hysteresis [26].

The complex hysteresis behavior of friction manifests itself in many mechanical elements that comprise sliding elements, such pneumatic artificial muscles ([27],[28]), piezoelectric materials

---

\(^2\) Presliding regime refers to the condition when the object is about to slide. At this stage the stiction force is characterized by relative displacement between the two contact surfaces.
high precision transmission systems ([31],[32]) and tendon-sheath mechanisms ([33],[34]).

Many different models of stiction have been proposed ranging from simple, capturing only the sliding property, to more advanced models that capture the hysteresis behavior. With respect to our concern in capturing the frictional property in a multi-friction-source system, several models of stiction behavior are discussed below.

4.1 Coulomb Model

The classical Coulomb model defines the stiction force for non-zero relative velocity in the sliding regime and when the velocity is zero, the force is set below the static force value. This model describes the stiction force as a discontinuous function of the sliding velocity at zero value (left panel of Figure 2 illustrates the Coulomb model of stiction force together with typical forces at the vicinity of zero velocity). The model is simple to identify:

\[ F_s(\dot{x}) = F_C \text{sign}(\dot{x}) \]

where \( F_C \) is a constant Coulomb force. Extending the flexibility of the model to capture asymmetric frictional behavior, the model can be presented as:

\[ F_s(\dot{x}) = \begin{cases} F_C^+, & \text{for } \dot{x} \geq 0 \\ F_C^-, & \text{for } \dot{x} < 0 \end{cases} \]

\[ (9) \]

In some cases, in order to avoid discontinuity at the vicinity of zero velocity, a sigmoid function is normally used to replace the signum function:

\[ F_s(\dot{x}) = F_C \tanh(\alpha \dot{x}) \]

where \( \alpha \) is a variable to control the function shape at the vicinity of zero velocity.

4.2 Stribeck Model

The Stribeck function, \( s(\dot{x}) \), is decreases with velocity and is bounded by an upper limit at zero velocity that is equal to the static friction force, \( F_S \) and a lower limit equal to the Coulomb force, \( F_C \) (see the illustration on the right panel of Figure 2). This approach is able to capture the stick-slip phenomenon that commonly occurs in sliding elements. One form of the Stribeck function is presented by the following equation:

\[ s(\dot{x}) = F_C + (F_S - F_C) e^{-\frac{V_s}{\delta}} \]

\[ (10) \]

\[ F_s(\dot{x}) = s(\dot{x}) \cdot \text{sign}(\dot{x}) \]

\[ (11) \]

where \( V_s \) and \( \delta \) are the shape factors for the function.

Figure 2. Grey solid lines in both panels illustrate typical friction forces that involve hysteresis effect and smooth transition around zero sliding velocities. The bold dashed lines illustrate the simplified Coulomb friction model (left panel) and the Stribeck friction approach (right panel).
4.3 LuGre Model

The LuGre model [18] was among the first formulations to offer a smooth transition between sliding and presliding regimes. It is also able to capture other friction characteristics such as breakaway force and friction lag. This model introduces a state variable that represents the deflection of surface asperities on the contact surfaces due to the acting tangential force. The average deflection of the asperities is presented in a state equation:

\[
\frac{dz}{dt} = \dot{x} - \sigma_0 \frac{|\dot{x}|}{s(\dot{x})} \cdot z \tag{12}
\]

while the stiction force is represented as a combination of asperity deflection and micro-viscous forces:

\[
F_s = \sigma_0 z + \sigma_i \frac{dz}{dt} \tag{13}
\]

where \(\sigma_0\) is the viscous friction coefficient, \(\sigma_i\) refers to the micro-viscous friction coefficient, and \(z\) is an internal state variable that physically represents the average deflection of the frictional surface bristles.

The LuGre model has been proven capable of capturing the stick-slip phenomenon and friction lag property, including the hysteresis behavior in the pre-sliding regime, though it lacks the non-local memory effect. Some more advanced models ([20],[21]) are available in literature to mitigate the limitation on the LuGre model, but considering the simplicity of the models that require the fewest parameters, in this paper we will constrain our study to the aforementioned models.

Haptic interfaces may or may not require extremely precise dynamic models depending on the application. In some cases, simplicity and cost are important factors for a system, e.g. in neurorehabilitation equipment. Therefore, it is necessary to systematically investigate which friction model should applied to robots with cable actuation and a passive backbone. For this purpose, we address two questions through experiments carried out on H-Man:

(i) Whether or not it is necessary to consider friction both in the actuator and backbone spaces.

(ii) What is the required complexity of the stiction model.

These two questions are examined in the next two sections.

To address these factors, the force sensing instrumentation required by the haptic systems can be replaced by a sufficiently accurate dynamic model of the system. However, in cable-driven robots with a passive backbone, there exist multiple sources of friction in the system. This makes it more difficult to identify friction in the system.

5. Characterization and modelling of the multi-source-friction system

Considering the system presented in Figure 1 and the general dynamics model in Eq. (8), the system dynamics mainly originate from three sources: inertia, viscous effects, and stiction. This section examines identification resulting from the first three models in Table 1 \((M1, M2, M3)\) in order to understand the importance of considering friction in both the actuator and backbone spaces. For this purpose, we consider the simple asymmetric Coulomb model (Eq. 9):

\[
\tau_{\alpha, j} = \begin{cases} 
\alpha_j^+, & \text{for } \omega_j \geq 0 \\
\alpha_j^-, & \text{otherwise}
\end{cases}
\]

\[
F_{\gamma, j} = \begin{cases} 
\gamma_j^+, & \text{for } v_j \geq 0 \\
\gamma_j^-, & \text{otherwise}
\end{cases}
\]

where \(\tau_{\sigma, j}\) and \(F_{\gamma, j}\) are frictional torques and frictional forces from motors and sliders, respectively, and \(j (=1,2)\) indicates the motor/slider individual.

**Force-field Auto-excitation.** We consider an excitation signal consisting of a simple rotational force field about the approximate center of the H-Man workspace, described by
The Cartesian workspace location of the end-effector $\mathbf{p}$ is given by
\[
\mathbf{p} = \begin{bmatrix} x \\ y \end{bmatrix}
\]
and the radial vector by
\[
\mathbf{r} = \mathbf{p} - \mathbf{p}_0
\]
An elastic force with stiffness coefficient $K_e$ couples the end-effector to the origin:
\[
\mathbf{F}_e = K_e \mathbf{r}
\]
and a rotational force with coefficient $K_r$ excites the system into a rotating limit cycle about the origin $\mathbf{p}_0$.
\[
\mathbf{F}_r = K_r \begin{bmatrix} 0 \\ 1 \end{bmatrix} \mathbf{r}
\]
The total force field is simply the sum of Eq. (14) and (15):
\[
\mathbf{F} = \mathbf{F}_e + \mathbf{F}_r
\]

Linear Regression with Experimental Data. Force control and data collection were carried out on a Quanser QPIDe (Quanser Inc., Canada) real-time target operating at 1000 Hz. Motor position was recorded via two encoders (Avago HEDL-5540) with a resolution of 500 increments/revolution. The rate of change in angular position over time was transformed into Cartesian coordinates in real-time using Eq. (7). The forces prescribed to the system were assumed to be sufficiently low to neglect any cable slipping or stretching.

The position data were bandpass filtered using a 7th order zero delay Butterworth filter with a pass frequency of 20 Hz and a cutoff frequency of 100 Hz. Velocity and acceleration data were obtained by analytic differentiation of spline coefficients fit to the position time-series.

The MATLAB linear regression function `regress` was used to fit the relevant parameters for each model. The estimated system force for each model was compared to the force commanded via the H-Man control system. The estimated inertia values were used to isolate friction forces from both the command and the model estimation forces.

Results. Rotational force field excitation data was collected for approximately 10 s. Figure 3 shows the angular velocity of the left and right motors, $\omega_L$ and $\omega_R$, plotted against the end-effector’s linear velocities, $v_x$ and $v_y$, respectively. We observed that even during a continuous motion in the task space, one or both of the motors may stop momentarily due to the differential configuration. This is a unique phenomenon that is inherent in H-Man and other non-serial differential cable-driven robots, thus motivating the consideration of friction in the dynamic model.

Simulated System Dynamics. The three chosen friction models were simulated using the ODE45 solver function in MATLAB. The CAD inertia values $H_{xx} = 0.558$ kg and $H_{xx} = 1.694$ kg were used in the simulated model. The remaining parameters were chosen based on values observed in previous regression experiments: $B_{xx} = B_{xx} = 1$ Nm/s, $B_{xy} = B_{yx} = 0$, $\alpha^+_L = \alpha^+_R = \alpha^-_L = \alpha^- = 0.1$ Nm, $y^+_x = y^-_y = y^+_y = y^-_y = 5 \times 10^{-3}$ N.
Figure 3. Motor velocities against linear velocities as results of the presence of friction on the system. The angular velocity of the left and right motors plotted against the $x$- and $y$-axis linear velocities, respectively. When linear velocity is zero, one or both of the motors can still be in motion, demonstrating that friction effects can occur even when continuous motion is observed at the end-effector.

Figure 4. Fitting of commanded force with different frictions models. The left column shows the $x$-direction commanded force predicted from the models. The right column shows the force of $x$-direction friction forces plotted against $x$-velocity. The top row shows the result from $M1$ model, the second row from $M2$, while the third row is the result from $M3$.

Table 2. Normalized MSE error of the friction models implemented on H-Man.

<table>
<thead>
<tr>
<th>Model</th>
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<th>$MSE_{total}$</th>
<th>$H_{xx}$ (kg)</th>
<th>$H_{yy}$ (kg)</th>
</tr>
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<tbody>
<tr>
<td>M1</td>
<td>13.4%</td>
<td>7.5%</td>
<td>20.9%</td>
<td>0.279</td>
<td>1.770</td>
</tr>
<tr>
<td>M2</td>
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<td>13.0%</td>
<td>0.305</td>
<td>1.727</td>
</tr>
<tr>
<td>M3</td>
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<td>0.469</td>
<td>1.476</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>CAD estimated inertia value: 0.558</td>
<td>1.694</td>
</tr>
</tbody>
</table>

The performance of the three models is summarized in Figure 4. The force $F_x$ commanded in the $x$-direction was plotted alongside the corresponding model's estimate of the force. The $x$-direction
inertial forces were subtracted from the commanded and model-estimated force \( F_X \) using the estimated inertia tensor \( \mathbf{H} \) and plotted against the velocity in the \( x \)-direction \( v_X \). Table 2 summarizes the normalized mean square error (MSE) values for the force estimate produced by each model along with the estimated inertia values.

\[
MSE_{\theta} = \frac{1}{N \cdot \sigma^{2}_{\text{fit}}} \sum_{i=1}^{N} (F_{\text{fit},i} - F_{\text{meas},i})^2
\]

where \( F_{\text{meas},i} \) is the measured force and \( F_{\text{fit},i} \) is the estimated force from the model in any axis components \( \theta \), while \( \sigma^{2}_{\text{fit}} \) is the variance of \( F_{\text{fit},i} \) for \( N \) number of output.

Besides the MSE values of two different axes, Table 2 also presents the total MSE value that illustrates the overall performance measure:

\[
MSE_{\text{total}} = MSE_x + MSE_y
\]

Based on Figure 4, it is clear that the inclusion of friction in both the actuator and task spaces improved the model performance without adding significant complexity to the model or computational methods. The model force provided a closer fit to the command force with the addition of motor and slider friction. These effects are particularly evident in the friction force plots in the right column. Adding friction parameters greatly enhances the model’s accuracy when estimating the velocity-dependent friction forces considered in this study. This finding is confirmed quantitatively as shown in Table 2 where significant improvements in model accuracy are achieved via this method.

We note that a typical application of the H-Man (described in [9]) for physical training, where users train to perform movements in curl-type force fields with a force proportional to the magnitude of the handle velocity and perpendicular to its motion. As reported in [9], the maximum force generated by the robot in typical applications seldom exceeds 10 N, and in most cases is confined below 5 N. This confirms the significance of the observed friction force in Figure 4 which can be as high as 3 N.

6. Detailed friction characterization on the H-man

Having established the necessity of considering friction in both actuators and backbone spaces, let us study the degree of complexity that should be considered in the stiction model. The previous section presented a linear dynamic model which includes the classical Coulomb effect in both spaces identified using linear regression algorithm. However, the Coulomb model’s discontinuity during motion reversal and its failure to capture the Stribeck effect will introduce unavoidable difficulties in producing a precise description of friction and further affect haptic feedback performance at near zero velocities, especially when the trajectory covers multiple motion reversals at low velocities and low displacements, i.e. during the presliding regime. Thus, a more sophisticated dynamic model which utilizes more advanced friction components may be required to improve accuracy. This may increase the fidelity of H-Man over a wider range of operation, even in some extreme and uncommon scenarios. Here we compare Coulomb stiction to the Stribeck model (M4) and the advanced LuGre model (M5).

Identification method. The characterization of micro friction behavior was carried out by exciting each axis individually. Based on the fact that advanced friction models treat friction at presliding and sliding regimes separately, a specially designed command trajectory was utilized. The input signal actuates the end-effector linearly in the task space with multiple reversals to emphasize the presliding regime. In this section, a simplified dynamic model of H-Man for linear motion identification, along with the related experiment setup, the excitation signal, and the identification procedures are discussed.

Dynamics model of H-Man in linear motion. The linear motion model still follows the general form presented in Eq. (8). Assuming that the two motors are identical, the same friction model can be applied to both actuators. Further assuming that the cable is stiff, the model for linear motion in both the \( x \)- and \( y \)-axis is presented as:

i) \( F_X = H_{xx} \ddot{x} + B_{xx}v_X - \frac{2 \tau_M}{\rho} + F_{st,x} \), for linear motion in \( x \)-axis \((v_y = 0)\) when \( \omega_L = \omega_R = \omega \) and the frictional torque, \( \tau_M \), from both motors are identical, and
ii) \( F_y = H_{yy} \dot{y} + B_{yy} \dot{y} - \frac{2\pi \tau_m}{\rho} + \overline{F_{st,y}}, \) for linear motion in y-axis \((v_x = 0)\) when \( \omega_L = -\omega_R = \omega \) and the frictional torque, \( \tau_m \), from both motors are identical.

With this simplified model, the friction torque in motors \( \overline{\tau_{st}} \), the slider friction in x-axis, \( \overline{F_{st,x}} \), and slider friction in y-axis, \( \overline{F_{st,y}} \), will be modeled using the Striebeck and LuGre models introduced in Section 4. For simplicity, these models will only consider symmetric frictional forces in the identification process.

**Experimental Setup.** Using the same experimental setup presented in Section 3, the angular position and speed of motor were determined from the motor encoder measurements, while the Cartesian position \((x\) and \(y\)), speed \((v_x\) and \(v_y\)), and acceleration of end-effector \((\ddot{x}\) and \(\ddot{y}\)) were obtained through the Jacobian transformation of the motor encoder readings. A PID controller was used to track the desired trajectory for the experiment. The applied force is provided by the controller in response to a position error feedback signal. All the signals and PID controller design were processed or accomplished in MATLAB Simulink environment and realized in the Quanser real-time system.

**Excitation trajectory.** In order to effectively identify the nonlinear micro friction characteristics in both presliding and sliding regime, a special excitation trajectory was designed. The testing trajectory must include multiple velocity reversals at low velocities and low displacements to emphasize the presliding friction behavior, and contain relatively high velocity and displacement to characterize the sliding regime. A non-periodic trajectory was designed within the realistic dynamic range of operation in typical applications:

\[
y = 0.01(\sin(2\pi t) + \sin(2.26\pi t) + \sin(3.46\pi t) + \sin(3.46\pi t) + \sin(4\pi t) + \sin(4.5\pi t))
\]

The synthesized test trajectory signal along a single axis (in the backbones space) was subsequently commanded the H-Man control system under a closed loop PID environment. The PID controller’s parameters were heuristically optimized through minimization of the error between the command trajectory and the measured position of the end-effector. The error signal was filtered in a real-time using a first order low pass filter with a cutoff frequency of 20 Hz.

The end-effector trajectory was obtained from the encoders, and the corresponding velocity signal was estimated from numerical differentiation. The resulting data are illustrated in Figure 5. Due to the similarity of the signal between axes, only the position and velocity data for the x-axis are shown.

![Figure 5. Prescribed excitation kinematics in x-axis (left: position signal, right: velocity signal).](image)

**Identification procedure.** To identify dynamic parameters including inertia, viscous, and frictional parameters that account for advanced nonlinear friction features in the x- and y-axes, a Genetic Algorithm (GA) was applied to render with an initial guess of the parameter to be identified. Subsequently, the result was refined through the Nelder-Mead Simplex algorithm. The optimization process was performed upon the minimization of a cost function defined by the overall MSE value \( (MSE_{total}) \). It is commonly agreed that an MSE of less than 5.0% indicates a good fit, while less than 1.0% indicates an excellent fit [35]. Ten thousand data points sampled at 1000 Hz were collected for the identification and the process was conducted in the MATLAB Global Optimization toolbox environment.

**Identification performance comparison and discussion.** The optimized models \( (M4\) and \(M5) \) are quantitatively compared by the \( MSE\) value and shown in Table 3. Figure 6 illustrates the measured
force alongside the estimated force resulting from both models and provides a qualitative comparison between their performances.

Table 3. The quantitative measures for different models.

<table>
<thead>
<tr>
<th>Model</th>
<th>$MSE_x$</th>
<th>$MSE_y$</th>
<th>$MSE_{total}$</th>
<th>$H_{xx}$</th>
<th>$H_{yy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M4$</td>
<td>5.34%</td>
<td>0.92%</td>
<td>6.26%</td>
<td>0.49 kg</td>
<td>1.50 kg</td>
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<tr>
<td>$M5$</td>
<td>4.86%</td>
<td>0.63%</td>
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<td>1.56 kg</td>
</tr>
</tbody>
</table>

CAD estimated inertia value: 0.558 kg 1.694 kg

From Figure 6 we observe that the estimated forces from the Strubeck and LuGre models demonstrate reasonable agreement with the measured forces. In particular, the LuGre friction model offers a better estimation of force as indicated by the individual MSE values under 5%, and reasonable inertia values that resemble values reliably estimated from the CAD model of the system. Since both models neglect to account for asymmetric frictional forces, as was also the case in the Coulomb model ($M3$), the results indicate that frictional forces on the system are relatively symmetric.

Validation with a Force Sensor. While the parameter identification was carried out using the control input as the applied force, the performance of the models was subsequently assessed and validated by comparing the force estimation to the force measured directly from load-cell measurement. The experiment conducted in this validation step is similar to previous experiments, except that the measured force was taken directly by mounting a load-cell on the end-effector and rather than estimated from the controller command signal. The validation results shown in Figure 7 demonstrate the performance spectrum of the models. The LuGre model offers the best performance, but the Stribeck and LuGre models yield comparable performance as indicated by the quantitative measures. This leads us to conclude that the Strubeck effect in the system under study plays a more critical role than the hysteresis phenomenon of the frictional effect. It is important to note that the LuGre model is significantly more complex than the Stribeck model in terms of the state equation and the number of parameters.
7. Conclusion

This paper considered the dynamics of cable-driven robots for haptic applications with passive mechanical supports spanning two spaces, each with unique physical dynamics. This mechanism is effectively demonstrated on H-Man, a simple and relatively low-cost planar cable-driven robot. To function effectively as a haptic interface, an accurate dynamic model is required to deliver accurate force fields without using a force sensor.

We first carried out preliminary identification experiments using a simple Coulomb stiction model and viscous friction. The results indicated the need for friction identification for accurate positioning or force-field tasks and the necessity to consider friction sources in both spaces as each plays a significant role in the system’s dynamics.

Subsequently, a series of experiments investigated the dynamic identification performance with stiction models of varying complexity. Several models of stiction representing two different classes of complexity were considered. The Coulomb model represents the simplest and most economical model, the Stribeck model, which is able to capture the steady state frictional forces, and the LuGre model, which represents the advanced class because of its ability to capture the hysteresis phenomenon in friction.

The experimental results demonstrated that the best performances can be obtained using the LuGre model and that the Coulomb model falls short at estimating the frictional force, particularly during direction reversals. Although it demonstrates superior ability in modelling friction, the LuGre model is significantly more complex compared to the other models and requires the most parameters. On the other hand, the Stribeck model exhibited a comparable level of performance to the LuGre model with less complexity. These results suggest that in this case, the Stribeck effect (the steady state friction force in sliding velocity) is more dominant than the hysteresis effect of friction.

Therefore, for applications that do not require high modelling accuracy, such as for haptic interfaces used in rehabilitation systems, the Stribeck model provides a reasonable compromise between efficient dynamic compensation and relatively simple implementation. On the other hand, in a scenario where high fidelity haptic feedback is required, such interfacing with the microworld, it may be necessary to use an advanced friction model such as LuGre in order to ensure maximum possible transparency of the haptic interface and excellent estimation of the interaction force.
References


[23] F. Al-Bender, V. Lampaert, and J. Swevers, "Modeling of dry sliding friction dynamics: From


Table 1. Summary of modelling approaches

Table 2. Normalized MSE error of the friction models implemented on H-Man

Table 3. The quantitative measures for different models
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<table>
<thead>
<tr>
<th>Model</th>
<th>Friction and dynamics</th>
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<tbody>
<tr>
<td></td>
<td>Coulomb (asymmetric)</td>
<td>Actuators (Motors)</td>
</tr>
<tr>
<td></td>
<td>Stribeck</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LuGre</td>
<td></td>
</tr>
<tr>
<td>(M1)</td>
<td>-</td>
<td>-</td>
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</table>
| \(M2\) | ✓                    | -       | ✓                   | ✓
| \(M3\) | ✓                    | -       | ✓                   | ✓
| \(M4\) | -                    | ✓       | ✓                   | ✓
| \(M5\) | -                    | -       | ✓                   | ✓ |
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CAD estimated inertia value: 0.558 kg 1.694 kg
Figure 1. (A) Kinematics of the H-Man robot. Counter-clockwise rotation of the motors is considered to be positive. (B) Isometric view of the mechanical assembly. Motor velocities and Cartesian coordinates are indicated on the rendering. Image reproduced from [9].

Figure 2. Grey solid lines in both panels illustrate typical friction forces that involve hysteresis effect and smooth transition around zero sliding velocities. In the left panel, the bold dashed lines illustrate the simplified Coulomb friction model and in the right panel shows the Stieltjes friction approach.

Figure 3. Motor velocities against linear velocities as results of the presence of friction on the system. The angular velocity of the left and right motors plotted against the x- and y-axis linear velocities, respectively. When linear velocity is zero, one or both of the motors can still be in motion, demonstrating that friction effects can occur even when continuous motion is observed at the end-effector.

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Figure 5. Prescribed excitation kinematics in x-axis (left: position signal, right: velocity signal)

Figure 6. Fitting of experimental data using three different friction models

Figure 7. Comparison of experimental forces compared with forces estimated from the models
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