Error Analysis of NC-BFSK for Cooperative Diversity with Correlated Links

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***Abstract*—Cooperative communication plays a vital role in improving the error performance and throughput of a communication system. In this paper, we consider a cooperative diversity system with a source (S), a relay (R) and a destination (D). The major contribution of this paper is two fold: (1) closed form symbol error probability (SEP) expression has been derived considering noncoherent binary frequency-shift keying (NC-BFSK) modulation scheme for cooperative selection diversity system over dependent and identical Nakagami-m fading channels assuming correlation between all possible fading factors and (2) based on the derived end-to-end average SEP expression, performance analysis has been carried out for various special cases assuming decode and forward (DF) protocol at the relay node. Monte Carlo simulations are also performed to validate the derived SEP expression.**

***Index Terms*—noncoherent binary frequency-shift keying (NC-BFSK), decode and forward (DF) protocol, Nakagami-m fading, symbol error probability (SEP).**

I. INTRODUCTION

Small scale fading is one of the major impairment of wireless channel encountered by radio signals. Diversity is a promising solution to mitigate the adverse effects of small scale fading. A new form of diversity called cooperative diversity, where distributed relays in different geographical locations will cooperate with the transmitter in processing and forwarding the received signal to the destination node, with commonly used relay protocols such as amplify and forward (AF), decode and forward (DF), etc are proposed and its performance is analysed in [1]-[3]. Till now most of the wireless communication literatures assume source-to-relay (SR), relay-to-destination (RD), and source-to-destination (SD) links to be statistically independent. However, there are some scenarios where the relay nodes may come closer to source or destination node [4]-[5] and all the nodes may be surrounded by very less local scatterers. In such scenarios, most likely certain amount of correlation exists between any of the given links (SR, SD, and RD). Performance analysis of cooperative diversity system over correlated Nakagami-m fading channels is less dealt in the literatures. So comprehensive performance analysis of noncoherent binary frequency-shift keying (NC-BFSK) scheme has been carried out assuming correlation between all possible fading factors over identical Nakagami-m fading channels.

II. SYSTEM MODEL

We consider a cooperative diversity system comprising a source, a relay, and a destination. All the fading channels are modelled as $2m$ dimensional column vectors following zero-mean Gaussian distribution. Meanwhile, the norm of fading channels follow Nakagami-m distribution. Similarly, all the additive white Gaussian noises are modelled as zero-mean complex Gaussian random variable with variance $2N_0$. The complex information-bearing symbol transmitted from the source node and detected at the relay node are denoted by s and $\hat{s}$ respectively and it is one of the two constellation points in the signal space S. The complex baseband signal received at the relay and destination is modelled respectively as

$$r_{sr} = h_{sr}s + n_{sr}, \;\; r_{sd} = h_{sd}s + n_{sd}, \tag{1}$$

The decision rule for detecting the complex baseband signal at the relay node is $\hat{s} = arg\left\{\max_{s\epsilon S} Re(s^*\, h_{sr}^*\, r_{sr})\right\}$.

Now the received signal at the destination node can be modelled as

$$r_{rd} = h_{rd}\hat{s} + n_{rd}, \tag{2}$$

The decision rule at the destination node can be framed as

$$\hat{\hat{s}} = \begin{cases} arg\left\{\max_{s\epsilon S} Re(s^*\, h_{rd}^*\, r_{rd})\right\} & \text{if } \gamma_{rd} > \gamma_{sd}, \\ arg\left\{\max_{s\epsilon S} Re(s^*\, h_{sd}^*\, r_{sd})\right\} & \text{if } \gamma_{sd} > \gamma_{rd}, \end{cases} \tag{3}$$

where γ_{sr} , γ_{rd} , and γ_{sd} are the instantaneous signal-to-noise ratio (SNR) values which follow gamma distribution with mean values equal to their corresponding average SNR values (Γ).

III. PERFORMANCE ANALYSIS

To derive the end-to-end SEP expression, we need to use trivariate gamma distribution which can be obtained from trivariate Nakagami-m distribution [6, eq.3] and is given by

$$f_{\gamma_{sd},\gamma_{rd},\gamma_{sr}}(t_1,t_2,t_3) \; = \; \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} \frac{|W|^m\, |p_{1,2}|^{2k}\, |p_{2,3}|^{2l}}{\Gamma(m) \quad k!\;\, l!}$$
$$\times \frac{m^B\, t_1^{v'-1}\, t_2^{v-1}\, t_3^{v''-1} e^{\left(-\frac{m}{\Gamma}(p_{1,1}t_1+p_{2,2}t_2+p_{3,3}t_3)\right)}}{\Gamma^B\,\Gamma(v')\,\Gamma(v'')}, \tag{4}$$

where W is the tridiagonal matrix which is given in [6, eq.(2)] and it is the inverse of Green's matrix whose elements are having closest possible values to an arbitrary correlation matrix, $p_{1,1}, p_{2,2}, p_{3,3}, p_{1,2}$, and $p_{2,3}$ are the elements of W, $B = 3m + 2k + 2l, v' = m + k, v'' = m + l, v = m + l + k$, and $\Gamma(.)$ indicates the gamma integral function.

Now the conditional error probability conditioned on instantaneous SNR γ (in general) for NC-BFSK scheme is $P_e(\gamma) = \frac{1}{2}e^{-\frac{\gamma}{2}}$.

The average SEP expression of SD link considering $\gamma_{sd} > \gamma_{rd}$ can be obtained by averaging $P_e(\gamma)$ over trivariate gamma statistics given in (4). After simplification using [7, eq.(6.455)], the average SEP expression is given by

$$P_{eSD} = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \frac{|W|^m \, |p_{1,2}|^{2k} \, |p_{2,3}|^{2l} \, \Gamma(A)}{2 \, \Gamma(m) \, k! \, l! \, \Gamma(v')} \times \left(\frac{m^A \, \chi(v, \alpha, \beta)}{v \, \Gamma^A \, (p_{3,3})^{v''}} \right), \quad (5)$$

where $\chi(v, \alpha, \beta) = \frac{{}_2F_1(1, A; v+1; \frac{\alpha}{\alpha+\beta})}{(\alpha+\beta)^A}$, ${}_2F_1(.)$ indicates the Gauss hypergeometric function[7, eq.(9.100)], $\alpha = \frac{p_{2,2}m}{\Gamma}, A = 2m + 2k + l, \ \beta = a + \frac{p_{1,1}m}{\Gamma}$, and $a = \frac{1}{2}$.

Now the conditional error probability of source-to-relay-to-destination (SRD) link is $P_{eSRD}(\gamma_{sr}, \gamma_{rd}) = P_e(\gamma_{sr}) \, (1 - P_e(\gamma_{rd})) + (1 - P_e(\gamma_{sr}))P_e(\gamma_{rd})$. By averaging the conditional error probability of SRD link considering $\gamma_{rd} > \gamma_{sd}$ over trivariate gamma statistics given in (4), the average SEP expression of SRD link can be obtained and is given by

$$P_{eSRD} = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \frac{|W|^m \, |p_{1,2}|^{2k} \, |p_{2,3}|^{2l} \, m^B}{2 \, \Gamma^B \, \Gamma(m) \, k! \, l! \, v'!} \times \left(\left[\left(\frac{\Gamma^{v''}}{(p_{3,3}m)^{v''}} - \frac{1}{2 \, \left[a + \frac{p_{3,3}m}{\Gamma}\right]^{v''}} \right) \times \chi(v', \alpha_1, \beta_{fsk}) \right] + \left[\left(\frac{1}{\left[a + \frac{p_{3,3}m}{\Gamma}\right]^{v''}} \right) \times \left(\chi(v', \alpha_1, \beta'_{fsk}) - \frac{\chi(v', \alpha_1, \beta_{fsk})}{2} \right) \right] \right), \quad (6)$$

where $\alpha_1 = \frac{p_{1,1}m}{\Gamma}$, $\beta_{fsk} = a + \frac{p_{2,2}m}{\Gamma}$, and $\beta'_{fsk} = \frac{p_{2,2}m}{\Gamma}$. Now the end-to-end average SEP expression for NC-BFSK scheme from (5) and (6) is given by

$$P_e = P_{eSD} + P_{eSRD}. \quad (7)$$

IV. Numerical Results and Discussions

Fig 1. shows the SEP versus average SNR plots of NC-BFSK modulation scheme computed using (7) over correlated Nakagami-m fading channels for various special cases and different values of m. The special cases considered are strong correlation between SD and RD links alone, strong correlation between SR and RD links alone, and strong correlation among all the links. It is assumed that the correlation values denote the correlation coefficients of the underlying Gaussian process. From the figure it has been inferred that there is a substantial increase in the SEP performance as the fading severity decreases (or the value of m increases) as expected. Moreover, it has also been inferred that there is a substantial improvement

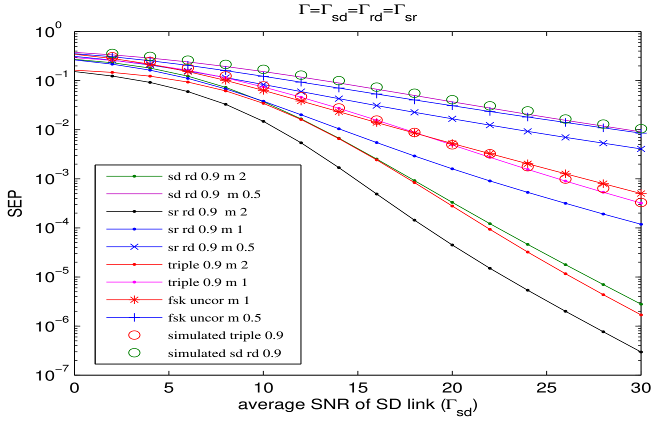

Fig. 1. Performance comparison of triple correlated selection combining technique for NC-BFSK modulation scheme

in the SEP performance over correlated Nakagami-m fading channels assuming correlation between SR and RD links, when compared to uncorrelated case. The improvement in the SEP performance is due to the performance improvement of individual SR, SD, and RD links over correlated SR and RD fading channels. However, when there is a strong correlation exists between SD and RD links, there is a slight degradation in the SEP performance which is due to the performance degradation of SD link alone over correlated SD and RD fading channels in the lower SNR regime. But in the higher SNR regime, the SEP performance is independent of the correlation effects due to the improved SEP performance and major impact of SRD link on the end-to-end SEP compared to SD link. When all the links are strongly correlated, the SEP performance is most likely independent of the correlation effects in the lower SNR regime and in the higher SNR regime there is a significant performance improvement. Finally, Monte Carlo simulation analysis well agree with the theoretical results, which validate our analysis.

References

[1] A.Sendonaris, E.Erkip, and B.Aazhang, "User cooperation diversity - part I: system description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1938, November 2003.

[2] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity - part II: implementation aspects and performance analysis," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1939–1948, November 2003.

[3] M.D.Selvaraj and R.K.Mallik, "Error analysis of decode-and-forward protocol with selection combining," *IEEE Trans. Wireless Commun*, vol. 8, no.6, pp. 3086-3094, June 2009.

[4] Y.Lee and M-H.Tsai,"Performance of decode-and-forward cooperative communications over nakagami-m fading channels," *IEEE Trans. Vehic. Tech*, vol.58, no.3, pp. 1218-1228, March 2009.

[5] G.-C.Zhang, X.-H.Peng, and X.-Y.Gu, "On the capacity of cooperative communication in correlated fading channels," *in the conference record of IEEE WiCom*, pp. 894-897, September 2007.

[6] D.A.Zogas, G.K.Karagiannidis, and S.A.Kotsoupoulos, "On the average output SNR in selection combining with three correlated branches over Nakagami-m fading channels, " *IEEE Trans. on Wireless Commun.*, vol. 3, no.1, pp. 25-28, January 2004.

[7] I.S.Gradshteyn and I.M.Ryzhik, *Table of Integrals, Series, and Products*, 7^{th} ed. San Diego, CA:Academic, 2007.