Decentralized output-feedback adaptive control for a class of interconnected nonlinear systems with unknown actuator failures ⋆

Chenliang Wang [a], Changyun Wen [b], Lei Guo [a]

[a] *School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, China*

[b] *School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798, Singapore*

Abstract

In this paper, a decentralized output-feedback adaptive backstepping control scheme is proposed for a class of interconnected nonlinear systems with unknown actuator failures. By introducing a kind of high-gain K-filters, a bound estimation approach and some smooth functions, the effect of actuator failures and interactions among subsystems is successfully compensated for and the actuators are allowed to change among the normal operation case and different failure cases infinitely many times. The proposed scheme is able to guarantee the global stability of the overall closed-loop system, regardless of the possibly infinite number of unknown actuator failures. An initialization technique is also introduced so that the $\mathcal{L}_\infty$ performance of tracking errors can be adjusted no matter if there exist unknown actuator failures. Simulation results performed on double inverted pendulums are presented to illustrate the effectiveness of the proposed scheme.

Key words: Interconnected systems; Actuator failures; Adaptive control; Decentralized control; Backstepping.

1 Introduction

Decentralized adaptive control for uncertain interconnected systems has long been an active issue in the control community. Different from centralized controllers, decentralized controllers are designed independently for subsystems and use only local signals for feedback, which brings challenge to the design and analysis in face of uncertain interactions among subsystems. With the development of backstepping design [1], the research has been accelerated and considerable achievements have been made over the past two decades; see, for instance, [2]-[4] and the references therein for more details.

On the other hand, in practical control systems, actuators may encounter abrupt failures during operation. For the sake of safety and reliability, the compensation of unknown actuator failures has received an increasing amount of attention. Roughly speaking, existing compensation schemes can be classified into two categories, i.e., passive and active ones. Using fixed controllers designed mainly by robust control theory, passive schemes aim at achieving insensitivity of the system to actuator failures and have relatively less computational burden [5], [6]. However, they can only handle some presumed failures and thus the fault-tolerant capability is limited. In contrast to passive schemes, active schemes react to failures actively by adjusting parameters and/or structure of controllers so that stability and acceptable performance can be maintained. Many active schemes have been developed based on various approaches such as multiple-model [7], eigenstructure assignment [8], sliding mode control [9], and so on.

Adaptive control is also an effective active way for actuator failure compensation, and is specially suitable for systems with uncertainties in both system dynamics and actuator failures. In [10]-[12], a class of direct adaptive compensation schemes was proposed for linear time-invariant systems. Different from many other active approaches, these schemes need neither explicit fault detection/diagnosis nor controller reconstruction, which sig-

⋆ This paper was not presented at any IFAC meeting. This work was supported by the National Natural Science Foundation of China under Grants 61304083 and 61127007, and the Doctoral Fund of Ministry of Education under Grant 20131102120007. Corresponding author: Chenliang Wang.

Email addresses: wangcl@buaa.edu.cn (Chenliang Wang), ecywen@ntu.edu.sg (Changyun Wen), lguo@buaa.edu.cn (Lei Guo).

nificantly simplifies the closed-loop system. Besides, the uncertainties caused by failures and system parameters can be handled simultaneously. With the aid of backstepping design, the results of [10]-[12] were extended in [13]-[16] to nonlinear systems via state-feedback. Employing K-filters to estimate unmeasured states, the output-feedback case was investigated in [17]-[20]. A common feature of [10]-[20] lies in that the Lyapunov functions constructed for the closed-loop system experience jumps when failures take place and, in order to ensure the stability, the total number of failures is restricted to be finite. In fact, it is assumed in these schemes that one actuator may fail only once and the failure mode does not change afterward. However, in practice an actuator may change among the normal operation case and different failure cases intermittently, and the number of failures may increase towards infinity with the passage of time. Compared with the finite case, the problem of adaptive compensation for a possibly infinite number of unknown actuator failures is much more challenging and it is only recently that some progress was made. In [21], a modular design method was developed to address this problem and global stability was ensured under the assumption that the bounds of all uncertainties are known.

In spite of the progress, it is noticed that all the above adaptive compensation schemes cannot be directly applied to decentralized control of interconnected systems. Aiming at a class of interconnected nonlinear systems with a possibly infinite number of unknown actuator failures, in our recent work [22], a decentralized adaptive backstepping control scheme was proposed, which, by introducing a bound estimation approach to handle the failure uncertainties, is able to guarantee global stability without the bound knowledge of uncertainties required in [21]. Nevertheless, the result in [22] is only applicable to output regulation and lacks transient performance analysis. When considering decentralized output tracking, the task becomes more complicated because the nonzero desired trajectories to be tracked will affect the dynamics of other subsystems through interactions. Besides, it is well known that the standard backstepping design provides a promising way to guarantee the transient performance in terms of $\mathcal{L}_\infty$ norms after incorporating certain initialization techniques [1]. A nature and interesting question is that whether it can be extended to adaptive systems with unknown actuator failures. However, to the best of our knowledge, there is still no such extension available in the literature, mainly because if the failure uncertainties are directly estimated as in [10]-[21], the jumps in Lyapunov functions may require repeated initialization at the time instants when failures occur. This is unrealistic since the occurrence time, value and pattern of failures are unknown, especially when the total number of failures is not restricted to be finite. Moreover, we point out that both the schemes in [21] and [22] require the measurement of full states and cannot be applied to the output-feedback case where only the system output is measured.

In this paper, within the framework of backstepping design, a decentralized output-feedback adaptive control scheme is proposed for a class of interconnected nonlinear systems with unknown actuator failures. The proposed scheme has the following features:

- By estimating the bounds of those uncertainties caused by actuator failures, the Lyapunov function constructed for the overall closed-loop system has no jump when failures take place and the total number of failures is allowed to be infinite. In contrast to our previous work [22], which is only applicable to decentralized output regulation via state-feedback, here the more challenging problem of decentralized output tracking via output-feedback is studied and the assumption on interactions is relaxed.
- The $\mathcal{L}_\infty$ tracking performance can be guaranteed. We show that by introducing an initialization technique and adjusting some design parameters, the $\mathcal{L}_\infty$ norms of tracking errors can be made arbitrarily small. The initialization technique is failure-independent and does not need to be performed repeatedly at the time instants when failures occur. To the best of our knowledge, this is the first adaptive scheme capable of guaranteeing the $\mathcal{L}_\infty$ tracking performance in the presence of unknown actuator failures.
- Instead of the traditional K-filters used in existing output-feedback adaptive compensation schemes [17]-[20], we construct a kind of high-gain K-filters to estimate the unmeasured states. Besides, some smooth functions are introduced to compensate for the interactions among subsystems and actuator failures. With the aid of such efforts, it is proved that all closed-loop signals are globally uniformly bounded.

The remainder of this paper is organized as follows. In Section 2, the control problem is introduced. In Section 3 the controller design is presented, followed by the stability and tracking performance analysis in Section 4. Simulation results are given in Section 5 to illustrate the effectiveness of the proposed scheme. Finally, we conclude in Section 6.

2 Problem formulation

Consider an interconnected nonlinear system consisting of N subsystems in output-feedback form, described by

$$
\begin{aligned}
&\dot{x}_i = A_i x_i + \varphi_i(y_i)\theta_i + b_i \sum_{j=1}^{\lambda_i} \eta_{i,j}(y_i) u_{i,j} + f_i(y_1, \cdots, y_N, t), \\
&y_i = x_{i,1}, \; i = 1, \cdots, N, \\
&A_i = \begin{bmatrix} 0 & \\ \vdots & I_{n_i-1} \\ 0 & \cdots 0 \end{bmatrix} \in \mathbb{R}^{n_i \times n_i}, \; b_i = [0 \cdots 0 \; \bar{b}_i^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{n_i}, \qquad (1)
\end{aligned}
$$

where $x_i = [x_{i,1}, \cdots, x_{i,n_i}]^{\mathrm{T}} \in \mathbb{R}^{n_i}$, $u_i = [u_{i,1}, \cdots, u_{i,\lambda_i}]^{\mathrm{T}} \in \mathbb{R}^{\lambda_i}$ (the outputs of actuators) and $y_i \in \mathbb{R}$ are the states, inputs and output of the ith subsystem, respectively; I_{n_i-1} is the $(n_i - 1) \times (n_i - 1)$ identity matrix; $\theta_i \in \mathbb{R}^{p_i}$ and $\bar{b}_i = [b_{i,m_i}, \cdots, b_{i,0}]^{\mathrm{T}} \in \mathbb{R}^{m_i+1}$ with $b_{i,m_i} \neq 0$ are unknown constants; $\varphi_i(y_i) = [\varphi_{i,1}(y_i), \cdots, \varphi_{i,n_i}(y_i)]^{\mathrm{T}} \in \mathbb{R}^{n_i \times p_i}$ with $\varphi_{i,q}(y_i) \in \mathbb{R}^{p_i}$ and $\eta_{i,j}(y_i) \in \mathbb{R}$ with $\eta_{i,j}(y_i) \neq 0$ are known smooth functions; $f_i = [f_{i,1}, \cdots, f_{i,n_i}]^{\mathrm{T}} \in \mathbb{R}^{n_i}$ are unknown interactions among subsystems, which are locally Lipschitz in $y_1, \cdots, y_N$ and piecewise continuous in t; and n_i, m_i, λ_i, p_i are known integers with $\rho_i := n_i - m_i > 1$. The states $x_{i,2}, \cdots, x_{i,n_i}$ are unmeasured.

Let the input of the (i, j)th actuator be denoted as $v_{i,j}$, which is to be designed. The failures that may occur on the (i, j)th actuator can be modeled as [21]

$$u_{i,j}(t) = g_{i,j,h} v_{i,j}(t) + \bar{u}_{i,j,h}(t), \ t \in [T^s_{i,j,h}, T^e_{i,j,h}),$$

$$g_{i,j,h} \bar{u}_{i,j,h}(t) = 0, \ h = 1, 2, 3, \cdots, \quad (2)$$

where $g_{i,j,h}$, $T^s_{i,j,h}$ and $T^e_{i,j,h}$ are all unknown constants with $0 \leq g_{i,j,h} < 1$ and $0 \leq T^s_{i,j,1} < T^e_{i,j,1} \leq T^s_{i,j,2} < T^e_{i,j,2} \leq \cdots \leq +\infty$, and $\bar{u}_{i,j,h}(t)$ are unknown, piecewise continuous and bounded signals. Equation (2) covers the following two types of failures:

- $0 < g_{i,j,h} < 1$ and $\bar{u}_{i,j,h}(t) = 0$. In this case, $u_{i,j}(t) = g_{i,j,h} v_{i,j}(t)$ and the actuator is called partial loss of effectiveness (PLOE).
- $g_{i,j,h} = 0$. In this case, $u_{i,j}(t) = \bar{u}_{i,j,h}(t)$ and the actuator is called total loss of effectiveness (TLOE).

In (2), $T^s_{i,j,h}$ and $T^e_{i,j,h}$ denote the time instants when the hth failure on the (i, j)th actuator starts and ends, respectively. If $T^s_{i,j,h+1} > T^e_{i,j,h}$, the actuator recovers its normal operation from time $T^e_{i,j,h}$ till $T^s_{i,j,h+1}$ when the next failure takes place. If $T^s_{i,j,h+1} = T^e_{i,j,h}$, it implies that the failure value $g_{i,j,h}$ changes to $g_{i,j,h+1}$ at $T^e_{i,j,h}$.

Remark 1: The actuator failure model (2) is much more general than those investigated in [10]-[20]. In (2), h is not restricted to be finite and the total number of failures is allowed to be infinite. In other words, each actuator is allowed to change among the normal operation case and different failure cases infinitely many times.

To unify the description of the actuator in the normal operation case and different failure cases, we introduce

$$g_{i,j}(t) = \begin{cases} g_{i,j,h}, & \text{if } t \in [T^s_{i,j,h}, T^e_{i,j,h}), \\ 1, & \text{if } t \in [T^e_{i,j,h}, T^s_{i,j,h+1}), \end{cases} \quad (3)$$

$$\bar{u}_{i,j}(t) = \begin{cases} \bar{u}_{i,j,h}(t), & \text{if } t \in [T^s_{i,j,h}, T^e_{i,j,h}), \\ 0, & \text{if } t \in [T^e_{i,j,h}, T^s_{i,j,h+1}). \end{cases} \quad (4)$$

Then, the (i, j)th actuator can be represented as

$$u_{i,j}(t) = g_{i,j}(t) v_{i,j}(t) + \bar{u}_{i,j}(t). \quad (5)$$

Define $\Pi_i = \mathrm{diag}\{g_{i,1}, \cdots, g_{i,\lambda_i}\}$, which is the actuator failure pattern indicator of the ith subsystem. We denote $T^*_{i,q}$ $(q = 1, 2, 3, \cdots)$ as the time instants when the failure pattern changes (i.e., Π_i changes) and let $T^*_{i,0} := 0$.

For the ith subsystem, the following assumptions are made.

Assumption 1: *At any time instant, up to $\lambda_i - 1$ actuators are at the state of TLOE.*

Assumption 2: *For the PLOE case, $g_{i,j,h} \geq a_{i,j} > 0$, where $a_{i,j}$ are unknown constants.*

Assumption 3: *There exits an unknown constant $\varsigma_i > 0$ such that $T^*_{i,q} - T^*_{i,q-1} \geq \varsigma_i$, $\forall T^*_{i,q}$.*

Assumption 4: *The sign of b_{i,m_i} is known and the polynomial $B_i(s) = b_{i,m_i} s^{m_i} + \cdots + b_{i,1} s + b_{i,0}$ is Hurwitz.*

Assumption 5: *The unknown interaction f_i satisfies* [1]

$$||f_i(y_1, \cdots, y_N, t)||^2 \leq \sum_{j=1}^{N} \varrho_{i,j} \phi_{i,j}(y_j), \quad (6)$$

where $\varrho_{i,j} \geq 0$ are unknown constants and $\phi_{i,j}(y_j) \geq 0$ are known smooth functions.

Remark 2: The same as existing adaptive compensation schemes, Assumption 1 is a basic condition to ensure the controllability. Assumption 2 is adapted from [21] with the relaxation that $a_{i,j}$ are no longer required to be known. Assumption 3 can also be found in [21]. It implies that, for each subsystem, the time interval between two successive changes of failure pattern is bounded below by a positive constant ς_i, where ς_i can be arbitrarily small. Under Assumption 3, $g_{i,j}$ and $\bar{u}_{i,j}$ in (5) are piecewise constant and piecewise continuous, respectively, satisfying the condition for the existence and uniqueness of solution of the system. If the total number of failures is finite as in [10]-[20], Assumptions 2 and 3 automatically hold. Assumption 4 implies that the system is minimum phase. Assumption 5 relaxes the restriction on interactions in [22], because the latter requires $\phi_{i,j}(y_j)$ in (6) to have the special form $\phi_{i,j}(y_j) = y_j^2 \phi^*_{i,j}(y_j)$ with $\phi^*_{i,j}(y_j)$ being known smooth functions.

The objective is to design a decentralized output-feedback adaptive control scheme such that all signals of

[1] In this paper, $\|\cdot\|$ denotes the Euclidian norm of a vector or the corresponding induced norm of a matrix.

the overall closed-loop system are bounded and the outputs $y_i(t),\ i=1,\cdots,N$, track desired trajectories $y_{ri}(t)$, where $y_{ri}(t)$ and their first ρ_i derivatives are known and bounded and $y_{ri}^{(\rho_i)}(t)$ are piecewise continuous.

The following lemma will be useful in our design.

Lemma 1 *[4]: For any scalars $\delta > 0$ and $z \in \mathbb{R}$, the following relationship holds: $0 \le |z| - \frac{z^2}{\sqrt{z^2+\delta^2}} < \delta$.*

3 Decentralized adaptive controllers design

In this paper, the control laws for the ith subsystem are designed to have the form

$$v_{i,j} = \frac{1}{\eta_{i,j}(y_i)} v_{i,0},\ j=1,\cdots,\lambda_i, \tag{7}$$

where $v_{i,0}$ is to be determined later. Substituting (5) and (7) into (1), it becomes

$$\dot{x}_i = A_i x_i + \varphi_i(y_i)\theta_i + b_i G_i v_{i,0} + b_i \bar{u}_i^{\mathrm{T}}(t)\eta_i(y_i) + f_i, \tag{8}$$

where $G_i = \sum_{j=1}^{\lambda_i} g_{i,j},\ \bar{u}_i(t) = [\bar{u}_{i,1}(t),\cdots,\bar{u}_{i,\lambda_i}(t)]^{\mathrm{T}}$, and $\eta_i(y_i) = [\eta_{i,1}(y_i),\cdots,\eta_{i,\lambda_i}(y_i)]^{\mathrm{T}}$.

3.1 High-gain K-filters

In traditional output-feedback adaptive backstepping design, K-filters are widely adopted to estimate the unmeasured states. In this paper, we design a set of high-gain K-filters for each subsystem as follows:

$$\dot{\Lambda}_i = A_{\mu i}\Lambda_i + K_i y_i, \tag{9}$$

$$\dot{\Xi}_i = A_{\mu i}\Xi_i + \varphi_i(y_i), \tag{10}$$

$$\dot{\xi}_i = A_{\mu i}\xi_i + E_{n_i,n_i} v_{i,0}, \tag{11}$$

where $E_{n_i,j}$ denotes the jth coordinate vector in $\mathbb{R}^{n_i}$, $A_{\mu i} = A_i - K_i E_{n_i,1}^{\mathrm{T}}$, and $K_i = [\mu_i k_{i,1}, \mu_i^2 k_{i,2},\cdots,\mu_i^{n_i} k_{i,n_i}]^{\mathrm{T}}$ with $\mu_i \ge 1$ a design parameter and $k_{i,1},\cdots,k_{i,n_i}$ being chosen such that the polynomial $s^{n_i} + k_{i,1}s^{n_i-1} + \cdots + k_{i,n_i}$ is Hurwitz. Besides, define $\Psi_{i,j} = A_{\mu i}^j \xi_i,\ j = 0,\cdots,m_i$, whose derivatives by noting $A_{\mu i}^j E_{n_i,n_i} = E_{n_i,n_i-j}$ are explicitly available:

$$\dot{\Psi}_{i,j} = A_{\mu i}\Psi_{i,j} + E_{n_i,n_i-j} v_{i,0}. \tag{12}$$

With these signals, the state estimation of the ith subsystem, denoted by $\hat{x}_i$, is parametrized as

$$\hat{x}_i = \Lambda_i + \Xi_i\theta_i + \sum_{j=0}^{m_i} b_{i,j} G_i \Psi_{i,j}. \tag{13}$$

From (8)-(10), (12) and (13) and noting G_i is a constant during each time interval $[T_{i,q-1}^*, T_{i,q}^*)$, it can be verified that for all $[T_{i,q-1}^*, T_{i,q}^*)$, the state estimation error $\tilde{x}_i = x_i - \hat{x}_i$ satisfies

$$\dot{\tilde{x}}_i = A_{\mu i}\tilde{x}_i + b_i \bar{u}_i^{\mathrm{T}}(t)\eta_i(y_i) + f_i. \tag{14}$$

Further, applying the transformation

$$\varepsilon_i = W_i\tilde{x}_i,\ W_i = \mathrm{diag}\{1,\mu_i^{-1},\cdots,\mu_i^{1-n_i}\}, \tag{15}$$

we rewrite (14) as $\dot{\varepsilon}_i = \mu_i A_{oi}\varepsilon_i + W_i b_i \bar{u}_i^{\mathrm{T}}(t)\eta_i(y_i) + W_i f_i$, where

$$A_{oi} = \begin{bmatrix} -k_{i,1} & \\ \vdots & I_{n_i-1} \\ -k_{i,n_i} & 0\cdots 0 \end{bmatrix} \tag{16}$$

is Hurwitz. Let $P_i = P_i^{\mathrm{T}} > 0$ be the solution of $A_{oi}^{\mathrm{T}} P_i + P_i A_{oi} = -(3 + \frac{\rho_i}{2}) I_{n_i}$ and

$$V_{\varepsilon i} = \varepsilon_i^{\mathrm{T}} P_i \varepsilon_i. \tag{17}$$

Noting $\mu_i \ge 1$ and $\bar{u}_i(t)$ is bounded, we have

$$\begin{aligned} \dot{V}_{\varepsilon i} &= -\left(3+\frac{\rho_i}{2}\right)\mu_i\varepsilon_i^{\mathrm{T}}\varepsilon_i + 2\varepsilon_i^{\mathrm{T}}P_iW_i[b_i\bar{u}_i^{\mathrm{T}}(t)\eta_i(y_i)+f_i] \\ &\le -\left(1+\frac{\rho_i}{2}\right)\mu_i\varepsilon_i^{\mathrm{T}}\varepsilon_i + \varrho_{i,0}\eta_i^{\mathrm{T}}(y_i)\eta_i(y_i) + ||P_i||^2||f_i||^2, \end{aligned} \tag{18}$$

with $\varrho_{i,0} = ||P_i||^2 \sup_{t\ge 0} ||b_i\bar{u}_i^{\mathrm{T}}(t)||^2$. Write $\varepsilon_i = [\varepsilon_{i,1},\cdots,\varepsilon_{i,n_i}]^{\mathrm{T}}$, $\Lambda_i = [\Lambda_{i,1},\cdots,\Lambda_{i,n_i}]^{\mathrm{T}}$, $\xi_i = [\xi_{i,1},\cdots,\xi_{i,n_i}]^{\mathrm{T}}$, $\Psi_{i,j} = [\Psi_{i,j,1},\cdots,\Psi_{i,j,n_i}]^{\mathrm{T}}$, and $\Xi_i = [\Xi_{i,1},\cdots,\Xi_{i,n_i}]^{\mathrm{T}}$ with $\Xi_{i,q} \in \mathbb{R}^{p_i}$. Then, from (1), (13) and (15), the derivative of y_i yields

$$\begin{aligned} \dot{y}_i = x_{i,2} + \varphi_{i,1}^{\mathrm{T}}\theta_i + f_{i,1} = \Lambda_{i,2} + \theta_i^{\mathrm{T}}\left(\Xi_{i,2}+\varphi_{i,1}\right) \\ + \sum_{j=0}^{m_i} b_{i,j}G_i\Psi_{i,j,2} + \mu_i\varepsilon_{i,2} + f_{i,1}. \end{aligned} \tag{19}$$

Remark 3: The decentralized high-gain K-filters (9)-(11) are extended from our previous work [23] for centralized control systems without actuator failures. Compared with the traditional K-filters used in existing output-feedback adaptive compensation schemes [17]-[20], the high-gain K-filters provide an extra design parameter μ_i as well as an error transformation (15). After the error transformation, μ_i appears explicitly in the negative term of (18), which will be useful in our tracking performance analysis. If μ_i is fixed to 1, (9)-(11) reduce to the traditional K-filters and W_i in (15) becomes the identity matrix.

3.2 Backstepping design procedure

Based on the above high-gain K-filters, the adaptive backstepping design procedure for the ith $(i = 1, \cdots, N)$ subsystem consists of ρ_i steps, with the control signals being deduced at the last step. To simplify the expression, we first define

$$z_{i,1} = y_i - y_{ri},\ z_{i,q} = \Psi_{i,m_i,q} - y_{ri}^{(q-1)} - \alpha_{i,q-1}, \qquad (20)$$

where $q = 2, \cdots, \rho_i$ and $\alpha_{i,q-1}$ is a stabilizing function to be designed at the $(q-1)$th step. Let

$$\alpha_{i,\rho_i} := v_{i,0} + \Psi_{i,m_i,\rho_i+1} - y_{ri}^{(\rho_i)},\ z_{i,\rho_i+1} := 0. \qquad (21)$$

Besides, we will employ positive scalars $c_{i,q}$, $\delta_{i,q}$ $(q = 1, \cdots, \rho_i)$, γ_{di}, γ_{li}, $\sigma_{\Theta i}$, σ_{di}, σ_{li} and ϵ_i and symmetric positive definite matrices $\Gamma_i \in \mathbb{R}^{(p_i+1)\times(p_i+1)}$ as design parameters in the design procedure without restating.

Step $i, 1$: By (20) and (19), the derivative of the tracking error $z_{i,1}$ can be expressed as

$$\begin{aligned}\dot z_{i,1} = {} & b_{i,m_i} G_i (z_{i,2} + \alpha_{i,1}) + \theta_i^{\mathrm T} (\Xi_{i,2} + \varphi_{i,1}) + G_i \bar b_i^{\mathrm T} \omega_{i,1} \\ & + \Lambda_{i,2} - \dot y_{ri} + \mu_i \varepsilon_{i,2} + f_{i,1},\end{aligned} \qquad (22)$$

where $\bar b_i$ is given in (1) and $\omega_{i,1} = [\dot y_{ri}, \Psi_{i,m_i-1,2}, \cdots, \Psi_{i,0,2}]^{\mathrm T} \in \mathbb{R}^{m_i+1}$. From (3) and Assumptions 1 and 2, it can be checked that G_i is bounded and $G_i \ge \min_{1\le j\le \lambda_i}\{a_{i,j}\} > 0$. With respect to the uncertainties caused by actuator failures and interactions among subsystems, define

$$\vartheta_i = ||\bar b_i|| \sup_{t\ge 0} G_i(t),\ \varpi_i = |b_{i,m_i}| \inf_{t\ge 0} G_i(t),\ l_i = \frac{1}{\varpi_i}, \qquad (23)$$

$$d_i = \max_{1\le j\le N} \left\{ \varrho_{i,0}, \left(||P_j||^2 + \frac{\rho_j}{2}\right) \varrho_{j,i} \right\}. \qquad (24)$$

To counteract the effect of the interactions and the outputs of TLOE actuators, the following smooth function is introduced:

$$\bar\phi_i = \frac{2 z_{i,1}}{z_{i,1}^2 + \epsilon_i} \left[\eta_i^{\mathrm T}(y_i)\eta_i(y_i) + \sum_{j=1}^{N} \phi_{j,i}(y_i) \right]. \qquad (25)$$

Then, we consider the first Lyapunov function $V_{i,1} = \frac{1}{2} z_{i,1}^2 + \frac{1}{2}\tilde\Theta_i^{\mathrm T}\Gamma_i^{-1}\tilde\Theta_i + \frac{1}{2\gamma_{di}}\tilde d_i^2 + \frac{\varpi_i}{2\gamma_{li}}\tilde l_i^2 + V_{\varepsilon i}$, where $V_{\varepsilon i}$ is given by (17), $\tilde\Theta_i := \hat\Theta_i - \Theta_i$, $\tilde d_i := \hat d_i - d_i$, and $\tilde l_i := \hat l_i - l_i$, with $\hat\Theta_i$, $\hat d_i$ and $\hat l_i$ the estimations of $\Theta_i = [\theta_i^{\mathrm T}, \vartheta_i]^{\mathrm T} \in \mathbb{R}^{p_i+1}$, d_i and l_i, respectively. In view of (18) and (22), differentiating $V_{i,1}$ yields

$$\begin{aligned}\dot V_{i,1} \le {} & z_{i,1} b_{i,m_i} G_i z_{i,2} + z_{i,1} b_{i,m_i} G_i \alpha_{i,1} + z_{i,1}\theta_i^{\mathrm T} (\Xi_{i,2} + \varphi_{i,1}) \\ & + z_{i,1} G_i \bar b_i^{\mathrm T} \omega_{i,1} + z_{i,1}\Lambda_{i,2} - z_{i,1}\dot y_{ri} + \mu_i z_{i,1}\varepsilon_{i,2} + z_{i,1} f_{i,1} \\ & + \tilde\Theta_i^{\mathrm T}\Gamma_i^{-1}\dot{\hat\Theta}_i + \frac{1}{\gamma_{di}}\tilde d_i \dot{\hat d}_i + \frac{\varpi_i}{\gamma_{li}}\tilde l_i \dot{\hat l}_i - \left(1 + \frac{\rho_i}{2}\right)\mu_i \varepsilon_i^{\mathrm T}\varepsilon_i \\ & + \varrho_{i,0}\eta_i^{\mathrm T}(y_i)\eta_i(y_i) + ||P_i||^2||f_i||^2.\end{aligned} \qquad (26)$$

Using (23) and Lemma 1, it can be checked that

$$\begin{aligned} z_{i,1} G_i \bar b_i^{\mathrm T}\omega_{i,1} & \le |z_{i,1}|\vartheta_i||\omega_{i,1}|| \\ & \le \frac{\vartheta_i z_{i,1}^2 \omega_{i,1}^{\mathrm T}\omega_{i,1}}{\sqrt{z_{i,1}^2\omega_{i,1}^{\mathrm T}\omega_{i,1} + \delta_{i,1}^2}} + \vartheta_i\delta_{i,1}.\end{aligned} \qquad (27)$$

Besides, it follows from Young's inequality that

$$\mu_i z_{i,1}\varepsilon_{i,2} + z_{i,1} f_{i,1} \le \frac{\mu_i + 1}{2} z_{i,1}^2 + \frac{1}{2}\mu_i\varepsilon_i^{\mathrm T}\varepsilon_i + \frac{1}{2}||f_i||^2. \qquad (28)$$

Substituting (27) and (28) into (26) and noting $\Theta_i = [\theta_i^{\mathrm T}, \vartheta_i]^{\mathrm T}$, we have

$$\begin{aligned}\dot V_{i,1} \le {} & -c_{i,1} z_{i,1}^2 + z_{i,1} b_{i,m_i} G_i z_{i,2} + z_{i,1} b_{i,m_i} G_i \alpha_{i,1} + z_{i,1}\bar\alpha_{i,1} \\ & + \tilde\Theta_i^{\mathrm T}\Gamma_i^{-1}(\dot{\hat\Theta}_i - \Gamma_i\zeta_{i,1} z_{i,1}) + \frac{1}{\gamma_{di}}\tilde d_i(\dot{\hat d}_i - \gamma_{di} z_{i,1}\bar\phi_i) \\ & + \frac{\varpi_i}{\gamma_{li}}\tilde l_i \dot{\hat l}_i - \left(1 + \frac{\rho_i - 1}{2}\right)\mu_i\varepsilon_i^{\mathrm T}\varepsilon_i + \varrho_{i,0}\eta_i^{\mathrm T}(y_i)\eta_i(y_i) \\ & + \left(||P_i||^2 + \frac{1}{2}\right)||f_i||^2 - d_i z_{i,1}\bar\phi_i + \vartheta_i\delta_{i,1},\end{aligned} \qquad (29)$$

where $\zeta_{i,1} = [\Xi_{i,2}^{\mathrm T} + \varphi_{i,1}^{\mathrm T}, \frac{z_{i,1}\omega_{i,1}^{\mathrm T}\omega_{i,1}}{\sqrt{z_{i,1}^2\omega_{i,1}^{\mathrm T}\omega_{i,1} + \delta_{i,1}^2}}]^{\mathrm T}$ and

$$\bar\alpha_{i,1} = c_{i,1} z_{i,1} + \hat\Theta_i^{\mathrm T}\zeta_{i,1} + \Lambda_{i,2} - \dot y_{ri} + \frac{\mu_i + 1}{2} z_{i,1} + \hat d_i\bar\phi_i. \qquad (30)$$

Define the first tuning function for $\hat\Theta_i$, say, $\tau_{i,1}$, as

$$\tau_{i,1} = \Gamma_i\zeta_{i,1} z_{i,1} - \sigma_{\Theta i}\Gamma_i\hat\Theta_i, \qquad (31)$$

and let

$$\dot{\hat d}_i = \gamma_{di} z_{i,1}\bar\phi_i - \gamma_{di}\sigma_{di}\hat d_i. \qquad (32)$$

Choose the first stabilizing function

$$\alpha_{i,1} = -\frac{\mathrm{sign}(b_{i,m_i}) z_{i,1}\hat l_i^2\bar\alpha_{i,1}^2}{\sqrt{z_{i,1}^2\hat l_i^2\bar\alpha_{i,1}^2 + \delta_{i,1}^2}}, \qquad (33)$$

where $\mathrm{sign}(b_{i,m_i})$ is known by Assumption 4 and $\hat l_i$ is updated according to

$$\dot{\hat l}_i = \gamma_{li} z_{i,1}\bar\alpha_{i,1} - \gamma_{li}\sigma_{li}\hat l_i. \qquad (34)$$

With the aid of Lemma 1 and (23), (33) renders

$$z_{i,1}b_{i,m_i}G_i\alpha_{i,1} = -\frac{|b_{i,m_i}|G_i z_{i,1}^2\hat{l}_i^2\bar{\alpha}_{i,1}^2}{\sqrt{z_{i,1}^2\hat{l}_i^2\bar{\alpha}_{i,1}^2+\delta_{i,1}^2}}$$
$$\leq -\frac{\varpi_i z_{i,1}^2\hat{l}_i^2\bar{\alpha}_{i,1}^2}{\sqrt{z_{i,1}^2\hat{l}_i^2\bar{\alpha}_{i,1}^2+\delta_{i,1}^2}} \leq \varpi_i\delta_{i,1} - z_{i,1}\varpi_i\hat{l}_i\bar{\alpha}_{i,1}. \quad (35)$$

Substituting (31)-(35) and the equality $\varpi_i\tilde{l}_i - \varpi_i\hat{l}_i = -\varpi_i l_i = -1$ into (29), it follows that

$$\dot{V}_{i,1} \leq -c_{i,1}z_{i,1}^2 + z_{i,1}b_{i,m_i}G_i z_{i,2} + \tilde{\Theta}_i^{\mathrm{T}}\Gamma_i^{-1}(\dot{\hat{\Theta}}_i - \tau_{i,1})$$
$$-\frac{\rho_i-1}{2}\left(\mu_i\varepsilon_i^{\mathrm{T}}\varepsilon_i + ||f_i||^2\right) + \vartheta_i\delta_{i,1} + L_i, \quad (36)$$

where

$$L_i = \varpi_i\delta_{i,1} - \sigma_{\Theta i}\tilde{\Theta}_i^{\mathrm{T}}\hat{\Theta}_i - \sigma_{di}\tilde{d}_i\hat{d}_i - \varpi_i\sigma_{li}\tilde{l}_i\hat{l}_i - \mu_i\varepsilon_i^{\mathrm{T}}\varepsilon_i$$
$$+\varrho_{i,0}\eta_i^{\mathrm{T}}(y_i)\eta_i(y_i) + \left(||P_i||^2 + \frac{\rho_i}{2}\right)||f_i||^2 - d_i z_{i,1}\bar{\phi}_i. (37)$$

Step $i,2$: Noting $\alpha_{i,1}$ is a smooth function of y_i, $\hat{\Theta}_i$ and $\chi_{i,1} = [y_{ri}, \dot{y}_{ri}, \Lambda_i^{\mathrm{T}}, \Xi_{i,1}^{\mathrm{T}}, \cdots, \Xi_{i,n_i}^{\mathrm{T}}, \xi_{i,1}, \cdots, \xi_{i,m_i+1}, \hat{d}_i, \hat{l}_i]^{\mathrm{T}}$, the derivative of $z_{i,2} = \Psi_{i,m_i,2} - \dot{y}_{ri} - \alpha_{i,1}$ can be expressed as

$$\dot{z}_{i,2} = z_{i,3} + \alpha_{i,2} + \beta_{i,2} - \frac{\partial\alpha_{i,1}}{\partial y_i}\theta_i^{\mathrm{T}}(\Xi_{i,2} + \varphi_{i,1}) - G_i\bar{b}_i^{\mathrm{T}}\omega_{i,2}$$
$$-b_{i,m_i}G_i z_{i,1} - \mu_i\frac{\partial\alpha_{i,1}}{\partial y_i}\varepsilon_{i,2} - \frac{\partial\alpha_{i,1}}{\partial y_i}f_{i,1} - \frac{\partial\alpha_{i,1}}{\partial\hat{\Theta}_i}\dot{\hat{\Theta}}_i, (38)$$

where $\beta_{i,2} = -\mu_i^2 k_{i,2}\Psi_{i,m_i,1} - \frac{\partial\alpha_{i,1}}{\partial y_i}\Lambda_{i,2} - \frac{\partial\alpha_{i,1}}{\partial\chi_{i,1}}\dot{\chi}_{i,1}$ and $\omega_{i,2} = [\frac{\partial\alpha_{i,1}}{\partial y_i}\Psi_{i,m_i,2} - z_{i,1}, \frac{\partial\alpha_{i,1}}{\partial y_i}\Psi_{i,m_i-1,2}, \cdots, \frac{\partial\alpha_{i,1}}{\partial y_i}\Psi_{i,0,2}]^{\mathrm{T}} \in \mathbb{R}^{m_i+1}$. Choose the second Lyapunov function $V_{i,2} = V_{i,1} + \frac{1}{2}z_{i,2}^2$. Using (36), (38) and the same technique of obtaining (27)-(29), it can be checked that

$$\dot{V}_{i,2} \leq -c_{i,1}z_{i,1}^2 + z_{i,2}\Big[z_{i,3} + \alpha_{i,2} + \beta_{i,2} + \hat{\Theta}_i\zeta_{i,2}$$
$$+\frac{\mu_i+1}{2}\left(\frac{\partial\alpha_{i,1}}{\partial y_i}\right)^2 z_{i,2} - \frac{\partial\alpha_{i,1}}{\partial\hat{\Theta}_i}\dot{\hat{\Theta}}_i\Big] + \tilde{\Theta}_i^{\mathrm{T}}\Gamma_i^{-1}(\dot{\hat{\Theta}}_i$$
$$-\tau_{i,1} - \Gamma_i\zeta_{i,2}z_{i,2}) - \frac{\rho_i-2}{2}\left(\mu_i\varepsilon_i^{\mathrm{T}}\varepsilon_i + ||f_i||^2\right)$$
$$+\vartheta_i(\delta_{i,1} + \delta_{i,2}) + L_i, \quad (39)$$

where $\zeta_{i,2} = [-\frac{\partial\alpha_{i,1}}{\partial y_i}\left(\Xi_{i,2}^{\mathrm{T}} + \varphi_{i,1}^{\mathrm{T}}\right), \frac{z_{i,2}\omega_{i,2}^{\mathrm{T}}\omega_{i,2}}{\sqrt{z_{i,2}^2\omega_{i,2}^{\mathrm{T}}\omega_{i,2}+\delta_{i,2}^2}}]^{\mathrm{T}}$. Let $\tau_{i,2} = \tau_{i,1} + \Gamma_i\zeta_{i,2}z_{i,2}$ and $\alpha_{i,2} = -c_{i,2}z_{i,2} - \beta_{i,2} - \hat{\Theta}_i\zeta_{i,2} - \frac{\mu_i+1}{2}(\frac{\partial\alpha_{i,1}}{\partial y_i})^2 z_{i,2} + \frac{\partial\alpha_{i,1}}{\partial\hat{\Theta}_i}\tau_{i,2}$. Then, (39) becomes

$$\dot{V}_{i,2} \leq -c_{i,1}z_{i,1}^2 - c_{i,2}z_{i,2}^2 + z_{i,2}z_{i,3} + z_{i,2}\frac{\partial\alpha_{i,1}}{\partial\hat{\Theta}_i}(\tau_{i,2} - \dot{\hat{\Theta}}_i)$$
$$+\tilde{\Theta}_i^{\mathrm{T}}\Gamma_i^{-1}(\dot{\hat{\Theta}}_i - \tau_{i,2}) - \frac{\rho_i-2}{2}\left(\mu_i\varepsilon_i^{\mathrm{T}}\varepsilon_i + ||f_i||^2\right)$$
$$+\vartheta_i(\delta_{i,1} + \delta_{i,2}) + L_i. \quad (40)$$

Step i,q ($3 \leq q \leq \rho_i$): Noting $\alpha_{i,q-1}$ is a smooth function of y_i, $\hat{\Theta}_i$ and $\chi_{i,q-1} = [y_{ri}, \cdots, y_{ri}^{(q-1)}, \Lambda_i^{\mathrm{T}}, \Xi_{i,1}^{\mathrm{T}}, \cdots, \Xi_{i,n_i}^{\mathrm{T}}, \xi_{i,1}, \cdots, \xi_{i,m_i+q-1}, \hat{d}_i, \hat{l}_i]^{\mathrm{T}}$, the derivative of $z_{i,q} = \Psi_{i,m_i,q} - y_{ri}^{(q-1)} - \alpha_{i,q-1}$ can be expressed as

$$\dot{z}_{i,q} = z_{i,q+1} + \alpha_{i,q} + \beta_{i,q} - \frac{\partial\alpha_{i,q-1}}{\partial y_i}\theta_i^{\mathrm{T}}(\Xi_{i,2} + \varphi_{i,1})$$
$$-G_i\bar{b}_i^{\mathrm{T}}\omega_{i,q} - \mu_i\frac{\partial\alpha_{i,q-1}}{\partial y_i}\varepsilon_{i,2} - \frac{\partial\alpha_{i,q-1}}{\partial y_i}f_{i,1} - \frac{\partial\alpha_{i,q-1}}{\partial\hat{\Theta}_i}\dot{\hat{\Theta}}_i, (41)$$

where $\beta_{i,q} = -\mu_i^q k_{i,q}\Psi_{i,m_i,1} - \frac{\partial\alpha_{i,q-1}}{\partial y_i}\Lambda_{i,2} - \frac{\partial\alpha_{i,q-1}}{\partial\chi_{i,q-1}}\dot{\chi}_{i,q-1}$ and $\omega_{i,q} = \frac{\partial\alpha_{i,q-1}}{\partial y_i}[\Psi_{i,m_i,2}, \Psi_{i,m_i-1,2}, \cdots, \Psi_{i,0,2}]^{\mathrm{T}} \in \mathbb{R}^{m_i+1}$. Choose the qth Lyapunov function $V_{i,q} = V_{i,q-1} + \frac{1}{2}z_{i,q}^2$, where the derivative of $V_{i,q-1}$ satisfies

$$\dot{V}_{i,q-1} \leq -\sum_{j=1}^{q-1}c_{i,j}z_{i,j}^2 + z_{i,q-1}z_{i,q} + \sum_{j=2}^{q-1}z_{i,j}\frac{\partial\alpha_{i,j-1}}{\partial\hat{\Theta}_i}$$
$$\times(\tau_{i,q-1} - \dot{\hat{\Theta}}_i) + \tilde{\Theta}_i^{\mathrm{T}}\Gamma_i^{-1}(\dot{\hat{\Theta}}_i - \tau_{i,q-1})$$
$$-\frac{\rho_i-q+1}{2}\left(\mu_i\varepsilon_i^{\mathrm{T}}\varepsilon_i + ||f_i||^2\right) + \vartheta_i\sum_{j=1}^{q-1}\delta_{i,j} + L_i. (42)$$

Letting $\tau_{i,q} = \tau_{i,q-1} + \Gamma_i\zeta_{i,q}z_{i,q}$ and $\alpha_{i,q} = -c_{i,q}z_{i,q} - z_{i,q-1} - \beta_{i,q} - \hat{\Theta}_i\zeta_{i,q} - \frac{\mu_i+1}{2}(\frac{\partial\alpha_{i,q-1}}{\partial y_i})^2 z_{i,q} + \frac{\partial\alpha_{i,q-1}}{\partial\hat{\Theta}_i}\tau_{i,q} + \sum_{j=2}^{q-1}z_{i,j}\frac{\partial\alpha_{i,j-1}}{\partial\hat{\Theta}_i}\Gamma_i\zeta_{i,q}$ with $\zeta_{i,q} = [-\frac{\partial\alpha_{i,q-1}}{\partial y_i}\left(\Xi_{i,2}^{\mathrm{T}} + \varphi_{i,1}^{\mathrm{T}}\right), \frac{z_{i,q}\omega_{i,q}^{\mathrm{T}}\omega_{i,q}}{\sqrt{z_{i,q}^2\omega_{i,q}^{\mathrm{T}}\omega_{i,q}+\delta_{i,q}^2}}]^{\mathrm{T}}$, it can be proved that

$$\dot{V}_{i,q} \leq -\sum_{j=1}^{q}c_{i,j}z_{i,j}^2 + z_{i,q}z_{i,q+1} + \sum_{j=2}^{q}z_{i,j}\frac{\partial\alpha_{i,j-1}}{\partial\hat{\Theta}_i}(\tau_{i,q} - \dot{\hat{\Theta}}_i)$$
$$+\tilde{\Theta}_i^{\mathrm{T}}\Gamma_i^{-1}(\dot{\hat{\Theta}}_i - \tau_{i,q}) - \frac{\rho_i-q}{2}\left(\mu_i\varepsilon_i^{\mathrm{T}}\varepsilon_i + ||f_i||^2\right)$$
$$+\vartheta_i\sum_{j=1}^{q}\delta_{i,j} + L_i. \quad (43)$$

After obtaining τ_{i,ρ_i} and α_{i,ρ_i} at Step i,ρ_i, let

$$\dot{\hat{\Theta}}_i = \tau_{i,\rho_i}. \quad (44)$$

In view of (21) and (7), $v_{i,0}$ is given by $v_{i,0} = \alpha_{i,\rho_i} - \Psi_{i,m_i,\rho_i+1} + y_{ri}^{(\rho_i)}$, and the control signals are chosen as

$$v_{i,j} = \frac{1}{\eta_{i,j}(y_i)}(\alpha_{i,\rho_i} - \Psi_{i,m_i,\rho_i+1} + y_{ri}^{(\rho_i)}),\ j = 1, \cdots, \lambda_i. (45)$$

Since $z_{i,\rho_i+1}=0$, in (43) setting $q=\rho_i$ gives

$$\dot{V}_{i,\rho_i} \leq -\sum_{j=1}^{\rho_i} c_{i,j} z_{i,j}^2 + \vartheta_i \sum_{j=1}^{\rho_i} \delta_{i,j} + L_i. \quad (46)$$

Remark 4: The above scheme is totally decentralized because for each subsystem, only local signals are used to construct the controller. Compared with existing adaptive actuator failure compensation schemes [10]-[22], it has the following features:

- Instead of directly estimating failure uncertainties as in [10]-[21], here we estimate the bounds of the failure uncertainties (i.e., the uncertainties related to $g_{i,j,h}$ and $\bar{u}_{i,j,h}$). This approach results in a nice feature that, though the failure uncertainties vary with failure pattern, the Lyapunov function constructed at each design step is independent of failure pattern and has no jump when failures occur. As a result, we no long need to assume that the total number of failures is finite as in [10]-[20] or the bounds of all uncertainties are known as in [21]. However, as a tradeoff, our control signals may be relatively higher since the estimated bounds are used in the design. Also, it is noticed that the schemes in [10]-[21] cannot be directly applied to decentralized control of interconnected systems.
- Compared with our previous work [22], this paper extends adaptive actuator failure compensation from decentralized output regulation via state-feedback to decentralized output tracking via output-feedback. Unlike the state-feedback case in [22], where no effort is needed for state estimation and failure uncertainties appear only at the last step of the backstepping design, in this paper failure uncertainties appear at each design step, including the design procedure of the high-gain K-filters for state estimation. Hence, more efforts are needed here to tackle the failure uncertainties.
- As explained in Remark 2, this paper also relaxes the assumption on interactions in [22]. To counteract the effect of the interactions among subsystems as well as the outputs of TLOE actuators, the smooth function $\bar{\phi}_i$ is introduced in (25) and the term $\hat{d}_i\bar{\phi}_i$ is added in (30), with the adaptive law for $\hat{d}_i$ being designed in (32). Such efforts, as will be shown in the next section, can force the corresponding residue terms to have upper bounds. By comparison, in [22], since a more conservative assumption is imposed on interactions and only output regulation is considered, the disposal of interactions is much easier and $\bar{\phi}_i$ in (25) is not needed.

Remark 5: The bound estimation approach also brings challenge to ensure the differentiability of stabilizing functions, which is a basic requirement of backstepping design. To this end, the smooth function $\frac{z^2}{\sqrt{z^2+\delta^2}}$ is introduced in Lemma 1. With the aid of this function and Lemma 1, we can replace the failure uncertainties by their bounds and then cancel the bound terms via differentiable stabilizing functions (see, e.g., (27) and (35)).

4 Stability and tracking performance analysis

We first establish the following theorem.

Theorem 1: *Consider the overall closed-loop system consisting of the interconnected system (1), the high-gain K-filters (9)-(11), the adaptive laws (32), (34) and (44) and the control laws (45) with possible unknown actuator failures (2). Suppose that Assumptions 1-5 hold. Then, all signals of the overall closed-loop system are globally uniformly bounded, and the tracking errors converge to a residue set that can be made arbitrarily small by adjusting the design parameters.*

Proof: Define a Lyapunov function of the overall closed-loop system as $V=\sum_{i=1}^{N} V_{i,\rho_i}$, whose derivative, by summing (46) for $i=1,\cdots,N$ and taking (37) into consideration, yields

$$\begin{aligned}\dot{V} \leq& \sum_{i=1}^{N}\Bigg[-\sum_{j=1}^{\rho_i} c_{i,j} z_{i,j}^2 - \sigma_{\Theta i}\tilde{\Theta}_i^{\mathrm{T}}\hat{\Theta}_i - \sigma_{di}\tilde{d}_i\hat{d}_i \\ & -\varpi_i\sigma_{li}\tilde{l}_i\hat{l}_i - \mu_i\varepsilon_i^{\mathrm{T}}\varepsilon_i + \bar{\delta}_i\Bigg] + \sum_{i=1}^{N}\varrho_{i,0}\eta_i^{\mathrm{T}}(y_i)\eta_i(y_i) \\ & + \sum_{i=1}^{N}\left(||P_i||^2+\frac{\rho_i}{2}\right)||f_i||^2 - \sum_{i=1}^{N} d_i z_{i,1}\bar{\phi}_i, \end{aligned} \quad (47)$$

where $\bar{\delta}_i=\varpi_i\delta_{i,1}+\vartheta_i\sum_{j=1}^{\rho_i}\delta_{i,j}$, and the last three terms are residue terms related to the interactions among subsystems and the outputs of TLOE actuators. Recalling (6) and (24) we have

$$\begin{aligned}\sum_{i=1}^{N}\left(||P_i||^2+\frac{\rho_i}{2}\right)||f_i||^2 \leq& \sum_{i=1}^{N}\sum_{j=1}^{N}\left(||P_i||^2+\frac{\rho_i}{2}\right)\varrho_{i,j}\phi_{i,j}(y_j) \\ =& \sum_{i=1}^{N}\sum_{j=1}^{N}\left(||P_j||^2+\frac{\rho_j}{2}\right)\varrho_{j,i}\phi_{j,i}(y_i) \\ \leq& \sum_{i=1}^{N}\sum_{j=1}^{N} d_i\phi_{j,i}(y_i), \end{aligned} \quad (48)$$

which, together with (25) and the fact $\varrho_{i,0}\leq d_i$, implies $\sum_{i=1}^{N}\varrho_{i,0}\eta_i^{\mathrm{T}}(y_i)\eta_i(y_i)+\sum_{i=1}^{N}\left(||P_i||^2+\frac{\rho_i}{2}\right)||f_i||^2-\sum_{i=1}^{N} d_i z_{i,1}\bar{\phi}_i \leq \sum_{i=1}^{N}\Delta_i$, where $\Delta_i = d_i\frac{\epsilon_i - z_{i,1}^2}{z_{i,1}^2+\epsilon_i}[\eta_i^{\mathrm{T}}(y_i)\eta_i(y_i)$ $+\sum_{j=1}^{N}\phi_{j,i}(y_i)]$. It can be checked that, for each $i=1,\cdots,N$, if $|z_{i,1}|>\sqrt{\epsilon_i}$, $\Delta_i<0$, and on the other hand, if $|z_{i,1}|\leq\sqrt{\epsilon_i}$, y_i is bounded by (20) and thus Δ_i has an upper bound $\bar{\Delta}_i\geq 0$, which is independent of all the design parameters except ϵ_i. With this relationship in mind and using the facts $-2\tilde{\Theta}_i^{\mathrm{T}}\hat{\Theta}_i \leq -\tilde{\Theta}_i^{\mathrm{T}}\tilde{\Theta}_i + \Theta_i^{\mathrm{T}}\Theta_i$, $-2\tilde{d}_i\hat{d}_i \leq -\tilde{d}_i^2 + d_i^2$, $-2\tilde{l}_i\hat{l}_i \leq -\tilde{l}_i^2 + l_i^2$ and $\varpi_i l_i = 1$, the inequality (47) can be rewritten as

$$\begin{aligned}\dot{V} \leq & \sum_{i=1}^{N}\Bigg(-\sum_{j=1}^{\rho_i} c_{i,j}z_{i,j}^2 - \frac{\sigma_{\Theta i}}{2}\tilde{\Theta}_i^{\mathrm{T}}\tilde{\Theta}_i - \frac{\sigma_{di}}{2}\tilde{d}_i^2 \\ & - \frac{\varpi_i\sigma_{li}}{2}\tilde{l}_i^2 - \mu_i\varepsilon_i^{\mathrm{T}}\varepsilon_i\Bigg) + \Omega \\ \leq & -2\kappa V + \Omega, \end{aligned} \tag{49}$$

where $\Omega = \sum_{i=1}^{N}(\frac{\sigma_{\Theta i}}{2}\Theta_i^{\mathrm{T}}\Theta_i + \frac{\sigma_{di}d_i^2}{2} + \frac{\sigma_{li}l_i}{2} + \bar{\delta}_i + \bar{\Delta}_i)$ and

$$\begin{aligned}\kappa = \min_{1\leq i\leq N, 1\leq j\leq \rho_i} & \Bigg\{c_{i,j}, \frac{\sigma_{\Theta i}\lambda_{\min}(\Gamma_i)}{2}, \frac{\gamma_{di}\sigma_{di}}{2}, \\ & \frac{\gamma_{li}\sigma_{li}}{2}, \frac{\mu_i}{2\lambda_{\max}(P_i)}\Bigg\}. \end{aligned} \tag{50}$$

It follows from (49) that

$$0 \leq V(t) \leq \frac{\Omega}{2\kappa} + \left[V(0) - \frac{\Omega}{2\kappa}\right]e^{-2\kappa t}. \tag{51}$$

As a result, the signals $z_{i,j}, \hat{\Theta}_i, \hat{d}_i, \hat{l}_i$ and ε_i, for $i = 1, \cdots, N$ and $j = 1, \cdots, \rho_i$, are bounded. Then, following similar analysis to [1, Chapter 8], it can be shown that all signals of the overall closed-loop system are globally uniformly bounded. Furthermore, (51) gives $\lim_{t\to+\infty} V(t) \leq \frac{\Omega}{2\kappa}$. Note that we can first fix $\sigma_{\Theta i}$, σ_{di}, σ_{li} and then increase $c_{i,j}, \lambda_{\min}(\Gamma_i), \gamma_{di}, \gamma_{li}, \mu_i$ to increase κ. In this way, Ω is independent of κ. Thus, the tracking errors $z_{i,1}$ can converge to an arbitrarily small residual set by increasing κ. This completes the proof.□

In the following theorem, we will further show that, by introducing an initialization technique, the $\mathcal{L}_\infty$ performance of the tracking errors can be guaranteed.

***Theorem 2**: Consider the overall closed-loop system described by Theorem 1 under Assumptions 1-5. Set the initial conditions of (9)-(11), (32), (34) and (44) to zeros except $\Lambda_{i,1}(0) = y_i(0)$. Let $y_{ri}(0) = y_i(0)$ and $y_{ri}^{(q)}(0) = -\alpha_{i,q}(0)$, $q = 1, \cdots, \rho_i - 1$. Then the $\mathcal{L}_\infty$ norms of the tracking errors are bounded by $\|z_{i,1}\|_\infty = \sup_{t\geq 0}|z_{i,1}(t)| \leq \sqrt{\frac{\Omega_0}{\kappa}}$ with $\Omega_0 = \Omega + \sum_{i=1}^{N}\sum_{q=2}^{n_i} x_{i,q}^2(0)$, which can be made arbitrarily small with a sufficiently large κ.*

Proof: In view of (13), setting the initial conditions of (9)-(11), (32), (34) and (44) to zeros except $\Lambda_{i,1}(0) = y_i(0)$ gives $\tilde{\Theta}_i(0) = -\Theta_i$, $\tilde{d}_i(0) = -d_i$, $\tilde{l}_i(0) = -l_i$, and

$$\hat{x}_i(0) = [y_i(0), 0, \cdots, 0]^{\mathrm{T}}. \tag{52}$$

From (15) and (52), we have

$$\varepsilon_i(0) = [0, \mu_i^{-1}x_{i,2}(0), \cdots, \mu_i^{1-n_i}x_{i,n_i}(0)]^{\mathrm{T}}. \tag{53}$$

Letting $y_{ri}(0) = y_i(0)$ results in $z_{i,1}(0) = 0$. Besides, it can be checked that $\frac{\partial\alpha_{i,q}}{\partial y_{ri}^{(q)}} = -\mathrm{sign}(b_{i,m_i})z_{i,1}\hat{l}_i^2 \times\partial(\frac{\bar{\alpha}_{i,1}^2}{\sqrt{z_{i,1}^2\hat{l}_i^2\bar{\alpha}_{i,1}^2 + \delta_{i,1}^2}})/\partial\dot{y}_{ri}$, $q = 1, \cdots, \rho_i - 1$. Since $z_{i,1}(0) = 0$ and $\hat{l}_i(0) = 0$, it is true that $\alpha_{i,q}(0)$ is independent of $y_{ri}^{(q)}(0)$. Hence, noting $\Psi_{i,m_i}(0) = 0$, we can choose $y_{ri}^{(q)}(0) = -\alpha_{i,q}(0)$ to obtain $z_{i,q+1}(0) = 0$, $q = 1, \cdots, \rho_i - 1$. Then, $V(0)$ satisfies

$$\begin{aligned}V(0) = & \sum_{i=1}^{N}\left[\frac{1}{2}\Theta_i^{\mathrm{T}}\Gamma_i^{-1}\Theta_i + \frac{d_i^2}{2\gamma_{di}} + \frac{\varpi_i l_i^2}{2\gamma_{li}} + \varepsilon_i^{\mathrm{T}}(0)P_i\varepsilon_i(0)\right] \\ \leq & \frac{\Omega}{2\kappa} + \frac{1}{2\kappa}\sum_{i=1}^{N}\sum_{q=2}^{n_i} x_{i,q}^2(0). \end{aligned} \tag{54}$$

Substituting (54) into (51), we arrive at $0 \leq V(t) \leq \frac{\Omega}{2\kappa} + \frac{1}{2\kappa}\sum_{i=1}^{N}\sum_{q=2}^{n_i} x_{i,q}^2(0)e^{-2\kappa t} \leq \frac{\Omega_0}{2\kappa}$, $\forall t \geq 0$, where $\Omega_0 = \Omega + \sum_{i=1}^{N}\sum_{q=2}^{n_i} x_{i,q}^2(0)$. As a result, $\|z_{i,1}\|_\infty \leq \sqrt{2\|V\|_\infty} \leq \sqrt{\frac{\Omega_0}{\kappa}}$, $i = 1, \cdots, N$, which implies the $\mathcal{L}_\infty$ norms of the tracking errors can be made arbitrarily small with a sufficiently large κ. □

Remark 6: To the best of our knowledge, Theorem 2 can be regarded as the first extension of the results concerning initialization and $\mathcal{L}_\infty$ tracking performance in standard backstepping design [1] to adaptive systems with unknown actuator failures. Such extension is not available in existing adaptive compensation schemes [10]-[22]. Thanks to the bound estimation approach, the initialization technique in Theorem 2 is failure-independent and does not need to be performed repeatedly at the time instants when failures occur.

Remark 7: As discussed in [1, p. 120], in many cases, the desired trajectories y_{ri} are generated by some reference models and one can set $y_{ri}(0), \cdots, y_{ri}^{(\rho_i-1)}(0)$ by initializing the reference models. If, on the other hand, y_{ri} are precomputed functions of time, then they can be initialized through the addition of exponentially decaying terms which define the reference transients.

Remark 8: It is worth pointing out that the adjustable parameters μ_i introduced by the high-gain K-filters play an important role in the tracking performance analysis, which allow us to arbitrarily increase κ in (50) and consequently, arbitrarily decrease the tracking errors.

5 Simulation results

To illustrate the effectiveness of the proposed scheme, we consider an interconnected system composed of two inverted pendulums connected by a spring [24], as shown in Fig. 1. The ith ($i = 1, 2$) pendulum is controlled by two torques $u_{i,1}$ and $u_{i,2}$ from two servomotors. The dynamics of the pendulums can be described as

$$\begin{aligned} J_1\ddot{y}_1 &= u_{1,1} + u_{1,2} + M_1\bar{g}R\sin y_1 \\ &\quad -0.5k(X - X_0)R\cos(y_1 - y_0), \\ J_2\ddot{y}_2 &= u_{2,1} + u_{2,2} + M_2\bar{g}R\sin y_2 \\ &\quad +0.5k(X - X_0)R\cos(y_2 - y_0), \end{aligned} \tag{55}$$

where y_1 and y_2 are the pendulum angles, $J_1 = M_1R^2$ and $J_2 = M_2R^2$ are the moment of inertia, M_1 and M_2 are the pendulum end masses, R is the pendulum length, $\bar{g}$ is the gravitational acceleration, k is the spring constant, X_0 is the natural length of the spring, X is the distance between the points O_1 and O_2, given by $X = \sqrt{D^2 + DR(\sin y_1 - \sin y_2) + 0.5R^2[1 - \cos(y_2 - y_1)]}$ with D denoting the distance between O_3 and O_4, and $y_0 = \arctan\left(\frac{0.5R(\cos y_2 - \cos y_1)}{D+0.5R(\sin y_1 - \sin y_2)}\right)$. It is assumed that the angle velocities of the pendulums are unmeasured and the system parameters are unknown, while in the simulation we set $M_1 = 0.8$ kg, $M_2 = 0.7$ kg, $R = 0.3$ m, $\bar{g} = 9.81$ m/s^2, $k = 45$ N/m, $X_0 = 0.35$ m and $D = 0.4$ m. The unknown failures of the servomotors are set as

$$\begin{aligned} u_{1,1}(t) &= \begin{cases} v_{1,1}(t), & \text{if } t \in [2hT, (2h+1)T), \\ 0.6v_{1,1}(t), & \text{if } t \in [(2h+1)T, (2h+2)T), \end{cases} \\ u_{1,2}(t) &= \begin{cases} v_{1,2}(t), & \text{if } t \in [0, t_{f1}), \\ 0.5v_{1,2}(t), & \text{if } t \in [t_{f1}, +\infty), \end{cases} \\ u_{2,1}(t) &= \begin{cases} v_{2,1}(t), & \text{if } t \in [2hT, (2h+1)T), \\ 0.85v_{2,1}(t), & \text{if } t \in [(2h+1)T, (2h+2)T), \end{cases} \\ u_{2,2}(t) &= \begin{cases} v_{2,2}(t), & \text{if } t \in [0, t_{f2}), \\ v_{2,2}(t_{f2})e^{(t_{f2}-t)}, & \text{if } t \in [t_{f2}, +\infty), \end{cases} \end{aligned} \tag{56}$$

where $h = 0, 1, 2, \cdots, T = 4$ s, $t_{f1} = 5$ s and $t_{f2} = 7$ s. Our objective is to apply the proposed scheme to (55) such that the pendulum angles y_1 and y_2 track desired trajectories y_{r1} and y_{r2} generated by

$$\ddot{y}_{ri} + 2\dot{y}_{ri} + y_{ri} = 1.4\sin(0.25t), \; i = 1, 2. \tag{57}$$

Letting $x_i = [y_i, \dot{y}_i]^{\mathrm{T}}$, $\varphi_i(y_i) = [0, \sin y_i]^{\mathrm{T}}$, $\theta_i = \frac{M_i\bar{g}R}{J_i}$, $b_{i,0} = \frac{1}{J_i}, \eta_{i,j}(y_i) = 1, f_1 = [0, -\frac{0.5k(X-X_0)R\cos(y_1-y_0)}{J_1}]^{\mathrm{T}}$ and $f_2 = [0, \frac{0.5k(X-X_0)R\cos(y_2-y_0)}{J_2}]^{\mathrm{T}}$, we can express (55) in the general form of (1), where Assumptions 1-5 hold with $\varrho_{i,1} = \varrho_{i,2} = \frac{k^2R^2\max\{(D+R-X_0)^2, X_0^2\}}{8J_i^2}$ and $\phi_{i,1} = \phi_{i,2} = 1$. The high-gain K-filters, adaptive laws and control laws follow directly the design in Section 3.

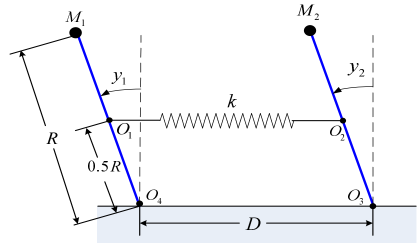

Fig. 1. Two inverted pendulums connected by a spring.

In the simulation, we choose $k_{i,1} = 2$ and $k_{i,2} = 1$ for the high-gain K-filters. To start with, the case without using the initialization technique in Theorem 2 is considered. In this case, all initial conditions are zeros except $y_1(0) = -0.5$ and $y_2(0) = 0.5$, and the design parameters are chosen as $\mu_i = 4.5$, $c_{i,1} = c_{i,2} = 10$, $\delta_{i,1} = \delta_{i,2} = 0.5$, $\epsilon_i = 1$, $\Gamma_i = 3I_2$, $\gamma_{di} = 3$, $\gamma_{li} = 3$, and $\sigma_{\Theta i} = \sigma_{di} = \sigma_{li} = 0.075$. The simulation results are shown in Fig. 2, which indicate that the pendulum angles track the desired trajectories with the tracking errors converging to a small residue set, despite the existence of an infinite number of unknown actuator failures and unknown interactions between the pendulums.

Further, to illustrate Theorem 2, we initialize the reference models (57) and the high-gain K-filters such that $y_{r1}(0) = -0.5$, $y_{r2}(0) = 0.5$, $\Lambda_{1,1}(0) = -0.5$ and $\Lambda_{2,1}(0) = 0.5$, while the other initial conditions are the same as before. We then consider two cases. In the first case, all the design parameters are the same as before, and the simulation results are shown in Fig. 3. Comparing Fig. 3 with Fig. 2, one can see that both the control effort and the tracking performance are improved by using the initialization technique. In the second case, we increase κ in (50) by choosing $\mu_i = 15$, $c_{i,1} = c_{i,2} = 80$, $\delta_{i,1} = \delta_{i,2} = 0.5$, $\epsilon_i = 1$, $\Gamma_i = 15I_2$, $\gamma_{di} = 15$, $\gamma_{li} = 15$, and $\sigma_{\Theta i} = \sigma_{di} = \sigma_{li} = 0.075$, and the simulation results are shown in Fig. 4, which indicate that the $\mathcal{L}_\infty$ performance of the tracking errors is significantly improved.

6 Conclusion

In this paper, a decentralized output-feedback adaptive compensation control scheme has been proposed for a class of interconnected nonlinear systems with unknown actuator failures. A kind of high-gain K-filters has been introduced to estimate the unmeasured states. With the aid of a bound estimation approach, the Lyapunov function constructed for the overall closed-loop system has no jump when failures occur and the total number of failures is allowed to be infinite. The proposed scheme is capable of guaranteeing the $\mathcal{L}_\infty$ performance of tracking errors and global stability of the overall closed-loop system. Simulation results have been presented to illustrate the effectiveness of the proposed scheme.

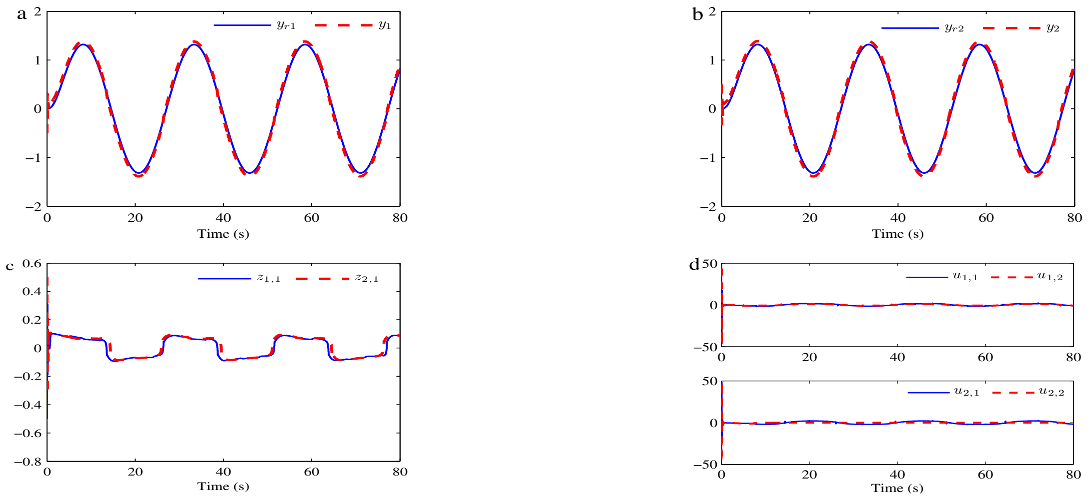

Fig. 2. Simulation results without using the initialization technique: (a) y_{r1} and y_1 (rad); (b) y_{r2} and y_2 (rad); (c) tracking errors (rad); (d) outputs of servomotors (N·m).

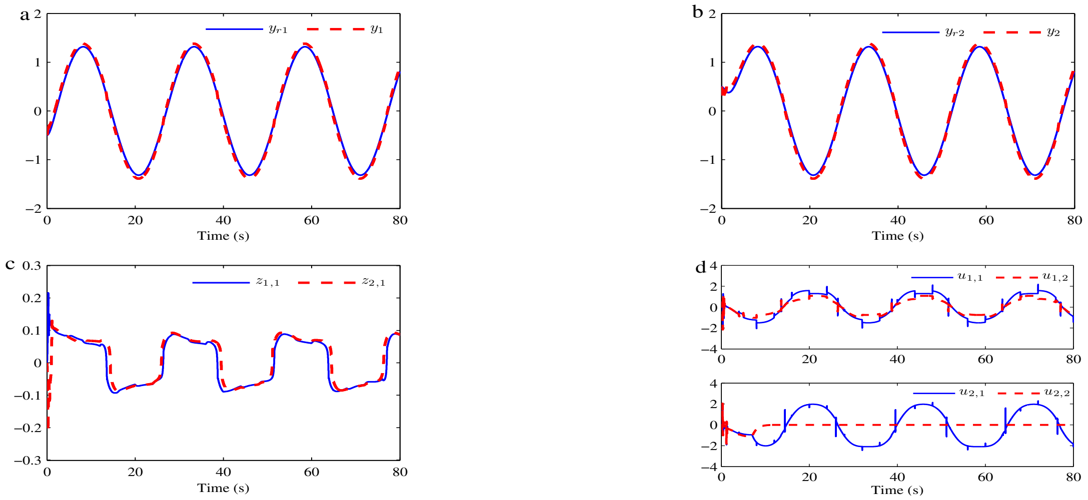

Fig. 3. Simulation results using the initialization technique: (a) y_{r1} and y_1 (rad); (b) y_{r2} and y_2 (rad); (c) tracking errors (rad); (d) outputs of servomotors (N·m).

References

[1] Krstic, M., Kanellakopoulos, I., and Kokotovic, P. V. (1995). *Nonlinear and adaptive control design*. New York: Wiley.

[2] Wen, C. (1994). Decentralized adaptive regulation. *IEEE Transactions on Automatic Control*, 39(10), 2163-2166.

[3] Jiang, Z. P. (2000). Decentralized and adaptive nonlinear tracking of large-scale systems via output feedback. *IEEE Transactions on Automatic Control*, 45(11), 2122-2128.

[4] Wang, C., and Lin, Y. (2015). Decentralized adaptive tracking control for a class of interconnected nonlinear time-varying systems. *Automatica*, 54, 16-24.

[5] Zhao, Q., and Jiang, J. (1998). Reliable state feedback control system design against actuator failures. *Automatica*, 34(10), 1267-1272.

[6] Yang, G. H., Wang, J. L., and Soh, Y. C. (2001). Reliable H_∞ controller design for linear systems. *Automatica*, 37(5), 717-725.

[7] Boskovic, J. D., and Mehra, R. K. (2002). Multiple-model adaptive flight control scheme for accommodation of actuator failures. *Journal of Guidance, Control, and Dynamics*, 25(4), 712-724.

[8] Jiang, J. (1994). Design of reconfigurable control systems using eigenstructure assignment. *International Journal of Control*, 59(2), 395-410.

[9] Corradini, M. L., and Orlando, G. (2007). Actuator failure identification and compensation through sliding modes. *IEEE Transactions on Control Systems Technology*, 15(1), 184-190.

[10] Tao, G., Joshi, S. M., and Ma, X. L. (2001). Adaptive state feedback control and tracking control of systems with actuator failures. *IEEE Transactions on Automatic Control*, 46(1), 78-95.

[11] Tao, G., Chen, S. H., and Joshi, S. M. (2002). An adaptive control scheme for systems with unknown actuator failures. *Automatica*, 38(6), 1027-1034.

[12] Tao, G., Chen, S. H., and Joshi, S. M. (2002). An adaptive

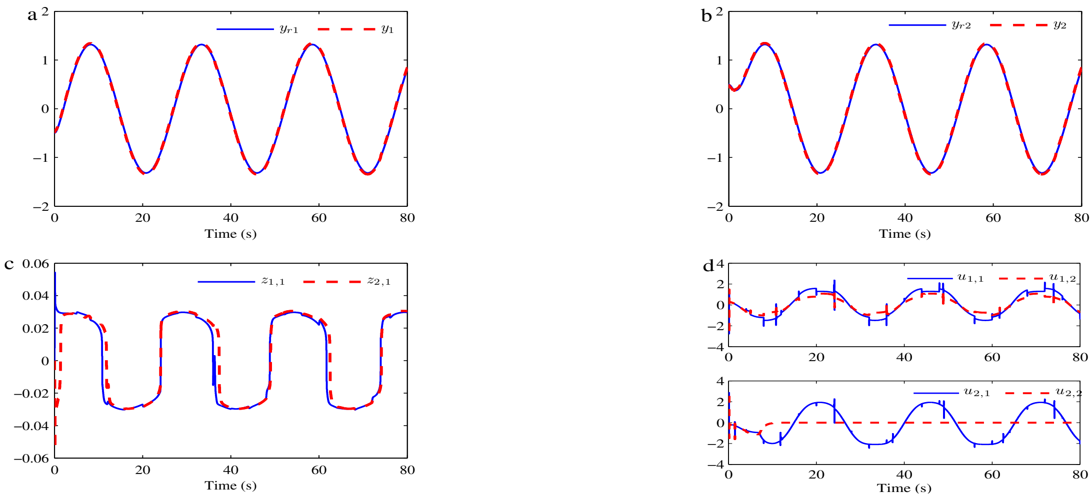

Fig. 4. Simulation results using the initialization technique and a larger κ: (a) y_{r1} and y_1 (rad); (b) y_{r2} and y_2 (rad); (c) tracking errors (rad); (d) outputs of servomotors (N·m).

failure compensation controller using output feedback. *IEEE Transactions on Automatic Control*, 47(3), 506-511.

[13] Tang, X. D., Tao, G., and Joshi, S. M. (2003). Adaptive actuator failure compensation for parametric strict feedback systems and an aircraft application. *Automatica*, 39(11), 1975-1982.

[14] Tang, X. D., Tao, G., and Joshi, S. M. (2007). Adaptive actuator failure compensation for nonlinear MIMO systems with an aircraft control application. *Automatica*, 43(11), 1869-1883.

[15] Wang, W., and Wen, C. (2010). Adaptive actuator failure compensation control of uncertain nonlinear systems with guaranteed transient performance. *Automatica*, 46(12), 2082-2091.

[16] Cai, J., Wen, C., Su, H., and Liu, Z. (2013). Robust adaptive failure compensation of hysteretic actuators for a class of uncertain nonlinear systems. *IEEE Transactions on Automatic Control*, 58(9), 2388-2394.

[17] Tang, X. D., Tao, G., and Joshi, S. M. (2005). Adaptive output feedback actuator failure compensation for a class of non-linear systems. *International Journal of Adaptive Control and Signal Processing*, 19(6), 419-444.

[18] Tang, X. D., and Tao, G. (2009). An adaptive nonlinear output feedback controller using dynamic bounding with an aircraft control application. *International Journal of Adaptive Control and Signal Processing*, 23(7), 609-639.

[19] Zhang, Z., and Chen, W. (2009). Adaptive output feedback control of nonlinear systems with actuator failures. *Information Sciences*, 179(24), 4249-4260.

[20] Zhang, Z., Xu, S., Guo, Y., and Chu, Y. (2010). Robust adaptive output-feedback control for a class of nonlinear systems with time-varying actuator faults. *International Journal of Adaptive Control and Signal Processing*, 24(9), 743-759.

[21] Wang, W., and Wen, C. (2011). Adaptive compensation for infinite number of actuator failures or faults. *Automatica*, 47(10), 2197-2210.

[22] Wang, C., Wen, C., and Lin, Y. (2015). Decentralized adaptive backstepping control for a class of interconnected nonlinear systems with unknown actuator failures. *Journal of the Franklin Institute*, 352(3), 835-850.

[23] Wang, C., and Lin, Y. (2013). Output-feedback robust adaptive backstepping control for a class of multivariable nonlinear systems with guaranteed L_∞ tracking performance. *International Journal of Robust and Nonlinear Control*, 23(18), 2082-2096.

[24] Bechlioulis, C. P., and Rovithakis, G. A. (2011). Robust partial-state feedback prescribed performance control of cascade systems with unknown nonlinearities. *IEEE Transactions on Automatic Control*, 56(9), 2224-2230.