<table>
<thead>
<tr>
<th><strong>Title</strong></th>
<th>Quantum entanglement in nanocavity arrays</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Author(s)</strong></td>
<td>Liew, Timothy Chi Hin; Savona, V.</td>
</tr>
<tr>
<td><strong>Citation</strong></td>
<td>Liew, T. C. H., &amp; Savona, V. (2012). Quantum entanglement in nanocavity arrays. Physical Review A, 85(5), 050301-.</td>
</tr>
<tr>
<td><strong>Date</strong></td>
<td>2012</td>
</tr>
<tr>
<td><strong>URL</strong></td>
<td><a href="http://hdl.handle.net/10220/43888">http://hdl.handle.net/10220/43888</a></td>
</tr>
<tr>
<td><strong>Rights</strong></td>
<td>© 2012 American Physical Society. This paper was published in Physical Review A and is made available as an electronic reprint (preprint) with permission of American Physical Society. The published version is available at: <a href="http://dx.doi.org/10.1103/PhysRevA.85.050301">http://dx.doi.org/10.1103/PhysRevA.85.050301</a>. One print or electronic copy may be made for personal use only. Systematic or multiple reproduction, distribution to multiple locations via electronic or other means, duplication of any material in this paper for a fee or for commercial purposes, or modification of the content of the paper is prohibited and is subject to penalties under law.</td>
</tr>
</tbody>
</table>
Quantum entanglement in nanocavity arrays

T. C. H. Liew and V. Savona
Institute of Theoretical Physics, Ecole Polytechnique Fédérale de Lausanne EPFL, CH-1015 Lausanne, Switzerland
(Received 28 November 2011; published 7 May 2012)

We show theoretically how quantum interference between linearly coupled modes with weak local nonlinearity allows the generation of continuous variable entanglement. By solving the quantum master equation for the density matrix, we show how the entanglement survives realistic levels of pure dephasing. The generation mechanism forms a paradigm for entanglement generation in arrays of coupled quantum modes.

DOI: 10.1103/PhysRevA.85.050301 PACS number(s): 03.67.Bg, 42.50.Ex, 42.50.Pq, 71.36.+c
in the case of a semiconductor structure. Equation (2) can be solved numerically for the steady state density matrix using a truncated number state basis [14] (see Supplemental Material for details [27]).

Our aim is to evidence continuous variable entanglement [30] between the modes in the first and third quantum boxes. In analogy to Bell’s result for discrete variable entanglement [30] between the modes in the first and third quantum boxes. In contrast, the modes $a_1$ and $a_2$ (or symmetrically $a_2$ and $a_3$) are not entangled, as evidenced by the dashed curve showing the value of $S_{13}$, evaluated from Eq. (3) by replacing $a_1$ with $a_2$. While the quantity $S_{13}$ is capable of witnessing entanglement and useful given its experimental accessibility, it is important to note that it does not fulfill the requirements of a direct measure of the amount of entanglement [36]. In fact, there is no unique, universally accepted, measure of the entanglement for our system.

For increasing dephasing rate, the amount of violation decreases and the entanglement is lost at high dephasing rate. Dephasing rates in semiconductor microcavities have been calculated [37] and measured [38] in the range of tenths of $\mu$eV. Even for a hypothetical dephasing rate an order of magnitude stronger, we still find that the predicted violation is sufficient for experimental detection.

Figure 1(c) shows the corresponding average populations of the modes in the signal quantum boxes (solid curves) and central box (dashed curves). For small pump amplitudes, corresponding to the linear regime, the populations grow according to a power law as expected. Since $J$ is large, the largest occupations are those of modes $a_1$ and $a_3$, even though only mode $a_2$ is driven. This trend is best understood by expressing the $a_n$ operators in terms of eigenmodes of the coupling $J$. Then, similarly to the two-mode system [19], these eigenmodes are driven by the pump in a way that results in destructive interference for the occupation of mode $a_2$. Figure 1(d) shows the variation of $S_{13}$ as a function of a finite detuning between the mode energies $E_1$ and $E_3$. The strong resonance at zero detuning is an indication of the underlying quantum interference mechanism. The level of control, required to fabricate a device with such a range of detuning to minimize the entanglement parameter $S_{13}$, is achievable in state-of-the-art arrays of semiconductor micropillars [21].

We stress that the reported results are also of significance in several other systems. Since the Jaynes-Cummings model can be linked to an effective Kerr nonlinearity [39], Eq. (1) is also applicable to quantum dots embedded in nanocavities and circuit QED systems [40], where the value of $U$ is related to the cooperativity parameter. In addition, the value of $U$ has been recently evaluated in passive nanocavities [41], which represent a particularly promising system given the low decay and dephasing rates. Values of $J$ and $\Gamma_p$ suitable for the present proposal have also been measured for photonic crystal nanostructures. As an example, the coupling of nanocavities has been recently studied in Ref. [42] and an upper bound to dephasing rates in quantum dots of $1 \mu$eV has been experimentally established [43].
In order to better understand the origin and the nature of the observed entanglement, we carry out an approximate analysis by expanding the quantum state on a truncated set of photon number states and solving the time-dependent Schrödinger equation for this state. This approach does not include the effect of (Lindblad type) dissipation and pure dephasing, and is expected to give an upper bound to the violation of inequality (3). The expansion reads

\[ |ψ⟩ = \sum_{n_1, n_2, n_3} C_{n_1, n_2, n_3} |n_1 n_2 n_3⟩, \]  

where the basis vectors

\[ |n_1 n_2 n_3⟩ = \hat{a}_1^{n_1} \hat{a}_2^{n_2} \hat{a}_3^{n_3} |000⟩/\sqrt{n_1!n_2!n_3!} \]  

represent states with \( n_1 \), \( n_2 \), and \( n_3 \) particles in modes 1, 2, and 3, respectively. For the analysis, expansion (4) has to be truncated to a maximum occupation, \( N = \sum n_i \). The first ten states, used in expansion (4), are depicted schematically in Fig. 2, together with their couplings caused by the pump and tunneling terms in the Hamiltonian.

The states containing two quanta in the same mode experience slight energy shifts by an amount \( 2U \) above the bare energy levels (shown in gray) due to the local nonlinear interactions.

The Schrödinger equation, \( i\hbar d|ψ⟩/dt = \hat{H}|ψ⟩ \), can be solved iteratively under the assumption of small occupations (see the supplemental material for more details) for the steady state (including the effect of particle loss). The coefficients \( C_{n_1, n_2, n_3} \) are then calculated and shown in Fig. 3 for the cases with \( U \neq 0 \) with green/light gray bars) and without \( U = 0 \) with red/dark gray bars) nonlinearity.

In accordance with Fig. 1(c), we observe that the quantum state is in general characterized by very low occupancy of mode 2. Each photon that is initially injected in this mode, tunnels to modes 1 and 3. This behavior can be easily understood in the linear case \( (U = 0) \), for which the Hamiltonian can be diagonalized exactly. In this case, the Schrödinger equation shows that only the mode generated by the operator \((\hat{a}_1 + \hat{a}_3)\) is effectively driven by the pump, thus giving rise to a fully separable quantum state, expressed as a linear combination of states \((\hat{a}_1 + \hat{a}_3)^N|000⟩\) at varying occupancy \( N \). Consequently, the relative weights of the coefficients \( C_{n_1, n_2, n_3} \), for each given value of the total occupancy \( N \), are exactly given by binomial coefficients, as shown by the dashed lines in Fig. 3. In the nonlinear regime, the system changes to a state characterized by the green (light gray) bars in Fig. 3, where it is clear that states containing particles in both modes 1 and 3 (e.g., \(|101⟩\)) are suppressed with respect to the linear case, while those with all particles in the same mode are enhanced (e.g., \(|200⟩\)). This result is a consequence of the nonlinear shift when photons occupy the same box, which has an effect on the quantum interference of possible time evolution paths in the Fock basis. As an example, within the manifold of states with \( N = 2 \) occupancy, the couplings of the states \(|110⟩\) and \(|011⟩\) to the states \(|200⟩\) and \(|002⟩\), respectively, change the phase of any time-evolution path passing through those states. If we consider the system initially in the state \(|011⟩\) for example, then to reach the state \(|101⟩\) two possible options are clear from Fig. 2: the direct path \(|011⟩ \leftrightarrow |101⟩\) or the path \(|011⟩ \leftrightarrow |020⟩ \leftrightarrow |110⟩ \leftrightarrow |101⟩\). The relative quantum phase of the two paths is affected by coupling to the state \(|200⟩\), which in the presence of the small
nonlinear shift of this state induces a destructive interference of the two paths and suppression of the state \[101\]. We are left with a situation where the detection of at least one photon in either signal mode, 1 or 3, grants that no photon will be detected in the other mode. This result solely depends on the nonlinearity in modes 1 and 3. We have verified that the parameter \(3_{13}\) experiences negligible change when the nonlinearity in mode 2 is removed.

In summary, arrays of coupled photonic modes are able to display striking quantum correlations despite their modest nonlinearity in the low occupation limit. This allows continuous variable entanglement to be generated between degenerate spatially separated modes that are coupled via quantum tunneling, in a way that is robust to typical decoherence rates in these systems. The set of three coupled modes here described serves as a building block that can be repeated on an array of modes with appropriate topology, which could be further controlled using electric or magnetic fields \[44\]. This sets a viable paradigm for the generation of multiparty entanglement in arrays of quantum boxes on a single device.

Our work was supported by NCCR Quantum Photonics (NCCR QP), a research instrument of the Swiss National Science Foundation (SNSF).

\[27\] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevA.85.050301 details on the numerical solution of the master equation and an approximate analytic treatment, which is useful for parameter optimization.
\[35\] E. Wertz et al., Nat. Phys. 6, 860 (2010).