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Effects of residual suction and residual water content on the estimation of permeability function

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Abstract

Soil-water characteristic curve (SWCC), which defines the relationship between water amount in the soil and matric suction, contains key information for the application of unsaturated soil mechanics principles to engineering practice. Best fit equations are commonly used for representation of the SWCC for unsaturated soils. Normally, these best fit equations are mathematically continuous and governed by a few fitting parameters. Either volumetric water content, θ_w or normalized volumetric water content, Θ, is adopted in the different best fit equations. If θ_w is used to establish SWCC, the parameter related to residual suction, C_r needs to be defined prior to fitting process of SWCC data. On the other hand, if Θ is used to develop SWCC, the residual volumetric water content, θ_r, needs to be defined prior to fitting process of SWCC data. Results of analyses in this study indicate that the performance of the best fit equation is not affected by the value of C_r, but it is significantly affected by the value of θ_r. As a result, the performance on the estimation of the permeability function is also affected by the value of θ_r. Different type of soils are used to investigate the effect of C_r and θ_r on the performance of the best fit equation and the estimation of the permeability function.

Key words: unsaturated soil, soil-water characteristic curve, best fit equation, fitting parameters, SWCC variables, permeability function

1. Introduction

Soil-water characteristic curve (SWCC) defines the relationship between the amount of water in soil and soil suction. The SWCC is normally determined from experimental data and the best fit equation is adopted to provide a continuous model. The best fit equations are normally governed by a few fitting parameters, and these fitting parameters are typically determined using a curve fitting technique. As the fitting parameters are determined from a regression procedure, these fitting parameters are mathematical solutions rather than physical soil properties.

Two forms of water content, either volumetric water content, θ_w, (e.g., Fredlund and Xing 1994) or normalized volumetric water content, Θ, (e.g., van Genuchten 1980), are adopted in different best fit equations, where, $\Theta=(\theta_w-\theta_r)/(\theta_s-\theta_r)$, θ_w= volumetric water content, θ_s = saturated volumetric water content, and θ_r = residual volumetric water content. Parameter C_r in Fredlund and Xing's (1994) equation which is related to the residual suction and parameter θ_r in van Genuchten's (1980) which is related to the residual volumetric water content need to be defined before regression analysis is carried out. The effect of the value of C_r and θ_r on the performance of Fredlund and Xing's (1994) equation and van Genuchten's (1980) equations is investigated and discussed in this paper. Four sets of soil, including two sets of coarse-grained soils and two sets of fine-grained soils, are used to investigate the effect of the value of C_r and θ_r on the performance of best fit SWCC equations and the estimation of the permeability function.

2. Literature review

There are various best fit equations that have been proposed by different researchers (Brooks and Corey 1964; Gardner 1958; Farrel and Larson 1972; van Genuchten 1980; Williams et al. 1983; Fredlund and Xing 1994; Kosugi 1994 and Satyanaga et al.2013). Studies by Leong and Rahardjo (1997) and Zapata (1999) concluded that Fredlund and Xing's (1994) equation, as illustrated in Equation (1), performed best among the available equations for best fitting the SWCC data. Zapata (1999) also suggested that van Genuchten's (1980) equation, as illustrated in Equation (2), performed well for best fitting fine-grained soils.

$$\theta = C(\psi)\frac{\theta_s}{\left\{\ln\left[e+\left(\frac{\psi}{a}\right)^n\right]\right\}^m} \qquad \text{----- (1)}$$

where:

$C(\psi)$ = correction factor $C(\psi) = 1 - \frac{\ln\left(1+\frac{\psi}{C_r}\right)}{\ln\left(1+\frac{10^6}{C_r}\right)}$,

C_r = input value, can be a roughly estimated value for residual suction, C_r = 1500 kPa for most cases as suggested by Fredlund and Xing (1994) and Zhai and Rahardjo (2012a, 2012b)

θ = volumetric water content,

ψ = matric suction,

e = Euler’s number and

a, n and m = fitting parameters.

$$\Theta = \left[\frac{1}{1 + (ah)^b} \right]^c \qquad \text{----- (2)}$$

where:

$$\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r},$$

θ = volumetric water content,

θ_s = saturated volumetric water content,

θ_r = residual volumetric water content,

h = pressure head and

a, b and c = fitting parameters.

Vanapalli et al. (1996) presented that water phase might not be continuous in the residual zone as illustrated in Figure 1. It is observed from the shrinkage curve, that insignificant soil volume changes occurs after air-entry value. The statistical method for estimation of the permeability function is based on two major assumptions such as Poiseuille’s law for liquid flow and constant pore-size distribution function. From the shrinkage curve, a significant soil volume change is observed at suctions before the air-entry value and it becomes insignificant beyond the air-entry value. On the other hand, the water flow at suctions beyond the residual suction was more in a vapor form than in a liquid form. Therefore, SWCC variables are important parameters for the estimation of the permeability function. Zhai and Rahardjo (2012a) derived equations for determination of SWCC variables (i.e. air-entry value, residual suction and residual water content) from the SWCC fitting parameters. Inflection point is required for determining the air-entry value, residual suction and residual water content. Zhang and Fredlund (2015) and Zhai and Rahardjo (2012a, 2013) derived equations for determination of the inflection point on arithmetic scale. In this paper, a new mathematical equation was developed for the determination of the inflection point on log-scale.

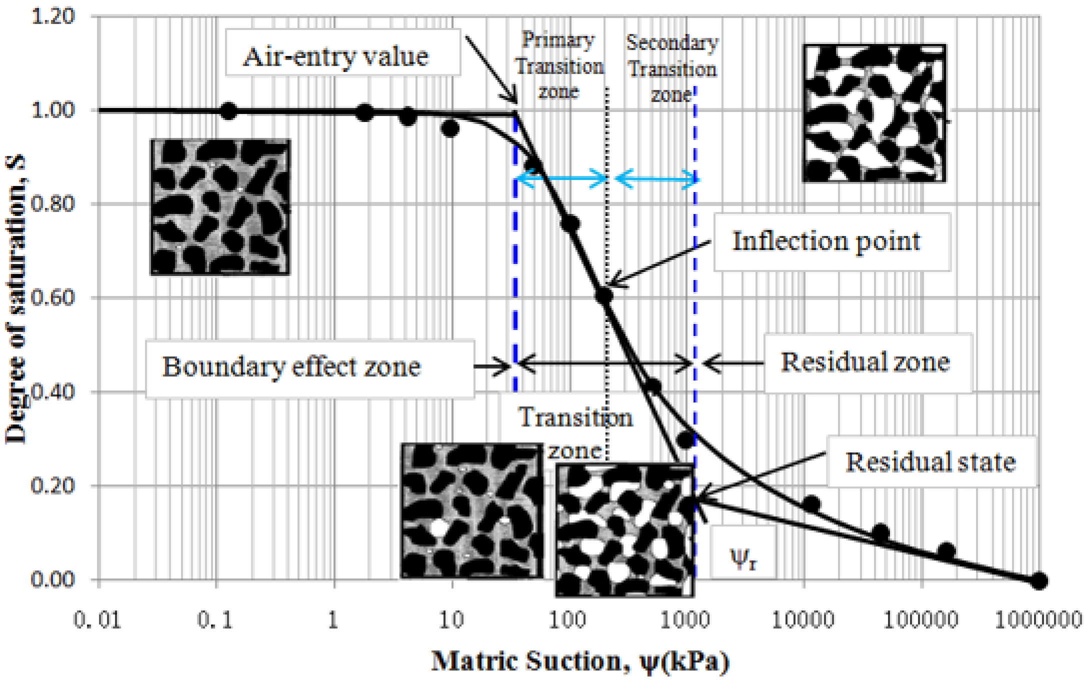


Figure 1. Illustration of the condition of water phase at different zones (Vanapalli et al. 1996)

Based on theory from Child and Collis George (1950) and simplification method from Kunze et al. (1968), Fredlund et al. (1994) proposed a simple equation with the integration form for the estimation of the permeability function from SWCC. To simplify the problem (i.e., make all the individual group of pores to have the same pore-size density), Kunze et al. (1968) proposed to evenly divide the volumetric water content into finite numbers of interval. Kunze et al. (1968) divide the entire SWCC into finite numbers of segment and each segment has the same range of volumetric water content but different ranges of matric suction. As explained by Rahimi et al. (2015), inverse function $\psi=f^{-1}(\theta_w)$ needs to be adopted to calculate suction, ψ, from the volumetric water content, θ_w using Kunze et al.'s (1968) method. Sometimes, it is not easy to calculate the suction from the volumetric water content due to the complex form of the best fit equation. In addition, iteration may be needed to solve the inverse function. To avoid the calculation of suction from the volumetric water content, Zhai and Rahardjo (2015) proposed to evenly divide the suction instead of volumetric water content into finite numbers of interval. Zhai and Rahardjo's (2015) method divided the SWCC into finite numbers of segment and each segment has the same range of matric suction and different ranges of degree of saturation. The degree of saturation of each segment represents the pore-size density in the pore-size distribution function. Bharat and Sharma (2012) presented that Fredlund-Xing-Kunze model required the first derivative of volumetric water content and commented the small value of C_r would affect the SWCC and the prediction result on the permeability function. On the other hand, Zhai and Rahardjo's (2015) method does not require the first derivative of volumetric water content and gives good prediction on the permeability function.

By varying the value of C_r and fixing values of a, n and m in Fredlund and Xing's (1994) equation, Bharat and Sharma (2012) found that small values of C_r influenced the SWCC near saturation. Consequently, Bharat and Sharama (2012) and Bharat (2014) concluded that Fredlund-Xing-Kunze model had limitation in prediction of the permeability function. Bharat and Sharma (2012) also indicated the value of C_r could affect the shape of SWCC near saturation but they did not comment on the effect of C_r on the performance of Fredlund and Xing's (1994) equation in best fitting SWCC data. Bharat and Sharma (2012) only carried out parametric study (i.e., varying value of C_r and fixing value of a, n and m), but they did not perform best fitting study (i.e., varying a, n and m to fit experimental data). In this study, regression procedure was carried out to investigate the effect of C_r on the performance of Fredlund and Xing's (1994) equation in best fitting SWCC data. The recent work from Rahimi et al. (2015) suggested that Fredlund-Xing-Kunze model could give reasonable prediction on the permeability function. Zhai and Rahardjo (2013) pointed out that fitting parameters in Fredlund and Xing's (1994) equation for the SWCC in the form of degree of saturation could be different from that for the SWCC in the form of gravimetric water content or volumetric water content if soil volume change is considered. In addition, Zhai and Rahardjo (2015) also pointed out that prediction on the permeability function from SWCC can be significantly improved if the SWCC in the form of degree of saturation rather than other forms is adopted. Zhang and Fredlund (2015) also suggested that the permeability function to be predicted from the SWCC in the form of degree of saturation

3. Theory

Mathematical equations for determination of the inflection point from Fredlund and Xing's (1994) equation and van Genutchen's (1980) equation are derived in this study. The statistical method for the estimation of the permeability function from the SWCC is also explained in this section. SWCC can be presented in the form either in gravimetric water content, w, volumetric water content, θ_w, and degree of saturation, S. Soil volume change is considered in volumetric water content and degree of saturation. Fredlund (2006) suggested that only SWCC in the form of degree of saturation yielded the correct air-entry value and Zhai and Rahardjo (2015) also suggested that prediction results on the permeability function could be significantly improved if the SWCC in the form of degree of saturation was adopted. Therefore, all the mathematical equations were derived based on the SWCC in the form of degree of saturation, S. If the SWCC in the form of degree of saturation is converted from other forms such as volumetric water content or gravimetric water content, then the soil volume change needs to be considered and the fitting parameters will also be different.

3.1 Determination of the inflection point

Fredlund and Xing (1994) introduced the physical definition of air-entry value as the matric suction where air starts to enter the largest pores in soil and residual water content is the water content where a large suction change is required to remove additional water from soil. The matric suction corresponding to the residual water content is commonly named as residual suction. Although the physical definitions of the air-entry value, residual suction and residual water content are clear, it is difficult to measure them directly. Vanapalli et al. (1998) proposed the graphical method for determination of these SWCC variables. Fredlund (2006) suggested that SWCC variables should be determined from the SWCC in the form of degree of saturation. Zhai and Rahardjo (2012a) demonstrated the geometrical relationship between these SWCC variables as illustrated in Figure 2.

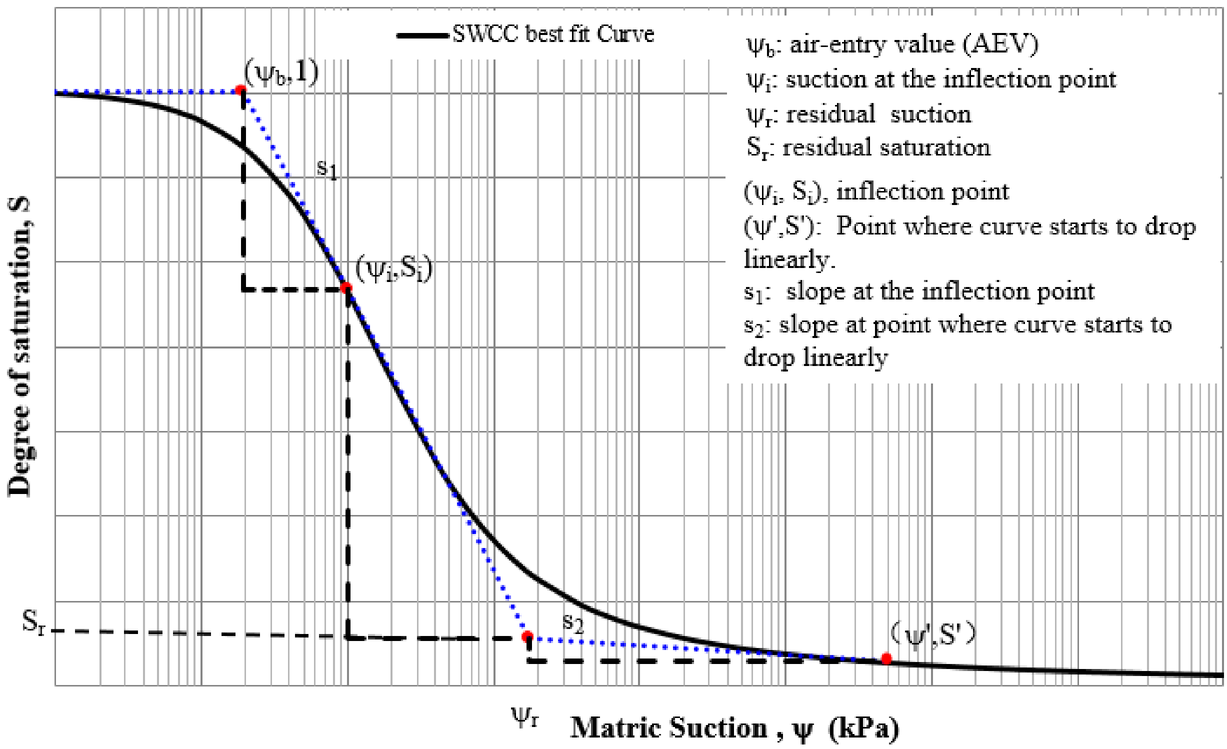


Figure 2. Definitions of SWCC variables (i.e., air-entry value, residual suction and residual saturation) after Zhai and Rahardjo (2012)

Zhai and Rahardjo (2012a, 2013) and Zhang and Fredlund (2015) derived equations for determination of the suction corresponding to the maximum slope on the arithmetic scale. In fact, the SWCC is commonly plotted in a logarithmic scale rather than in an arithmetic scale. The slope of the plot in the logarithmic scale is different from the slope of the plot in the arithmetic scale. The relationship between the two slopes using the two different scales can be obtained from Equation (3).

$$s = s'\ln(10)\psi \qquad \text{----- (3)}$$

where:

s = slope plotted in logarithmic scale,

s’= slope plotted in arithmetic scale

ψ = matric suction.

The inflection point is the point where the curve changes from being concave (concave downward) to convex (concave upward), or vice versa. If x is an inflection point for the function f(x) then the second derivative, f″(x), should be equal to zero if the inflection point exists. Therefore, the inflection point on the SWCC can be obtained by solving the differential equation given in Equation (4):

$$\frac{d^2S}{d(\log(\psi))^2} = \ln(10)\psi\left[\frac{d^2S}{d\psi^2}\ln(10)\psi + \frac{dS}{d\psi}\ln(10)\right] = 0 \qquad \text{----- (4)}$$

Equation (4) can be simplified as Equation (5) as follows:

$$\frac{d^2S}{d\psi^2}\psi + \frac{dS}{d\psi} = 0 \qquad \text{----- (5)}$$

Equation (5) provides a convenient method to obtain the inflection point using an electronic spreadsheet.

Substituting Equation (1) into Equation (5), the equation for determination of the inflection point using Fredlund and Xing’s (1994) equation can be obtained as follows:

$$\left[1 - \frac{\ln\left(1+\frac{\psi}{C_r}\right)}{\ln\left(1+\frac{10^6}{C_r}\right)}\right]\frac{mn}{a}\frac{1}{\left[\ln\left(e+\left(\frac{\psi}{a}\right)^n\right)\right]}\frac{1}{e+\left(\frac{\psi}{a}\right)^n}\left(\frac{\psi}{a}\right)^{n-1}$$

$$\left\{\frac{(m+1)n}{\left[\ln\left(e+\left(\frac{\psi}{a}\right)^n\right)\right]}\frac{1}{e+\left(\frac{\psi}{a}\right)^n}\left(\frac{\psi}{a}\right)^n + \frac{n}{e+\left(\frac{\psi}{a}\right)^n}\left(\frac{\psi}{a}\right)^n - n\right\} + \frac{\psi}{\ln\left(1+\frac{10^6}{C_r}\right)}\left(\frac{1}{C_r+\psi}\right)^2$$

$$2\frac{m\,n}{\ln\left(1+\frac{10^6}{C_r}\right)}\frac{1}{C_r+\psi}\frac{1}{\left[\ln\left(e+\left(\frac{\psi}{a}\right)^n\right)\right]}\frac{1}{e+\left(\frac{\psi}{a}\right)^n}\left(\frac{\psi}{a}\right)^n - \frac{1}{\ln\left(1+\frac{10^6}{C_r}\right)}\frac{1}{C_r+\psi} = 0$$

----- (6)

If the correction factor C(ψ)=1 (without correction factor), as suggested by Leong and

Rahardjo (1997), Equation (6) can be simplified as Equation (7).

$$\left\{ \frac{(m+1)n}{\left[\ln\left(e+\left(\frac{\psi}{a}\right)^n\right)\right]} \frac{1}{e+\left(\frac{\psi}{a}\right)^n}\left(\frac{\psi}{a}\right)^n + \frac{n}{e+\left(\frac{\psi}{a}\right)^n}\left(\frac{\psi}{a}\right)^n - n \right\} = 0 \qquad \text{----- (7)}$$

Equations (6) and (7) are the general equations for determination of the inflection point using Fredlund and Xing's (1994) equation with and without correction factor, respectively.

Substituting Equation (2) into Equation (5) generates equation for determination of the inflection point using van Genuchten's (1980) equation as follows:

$$c(ah)^b - 1 = 0 \qquad \text{----- (8)}$$

Equation (8) is the general equation for locating the inflection point using van Genuchten's (1980) equation.

3.2 Determination of air-entry value, residual suction and residual saturation

Zhai and Rahardjo (2012a) showed that the air-entry value (AEV), residual suction (ψ_r) and residual saturation (S_r) could be obtained from Equations (9) to (11) as follows:

$$AEV = \psi_i 0.1^{\frac{1-S_i}{s_1}} \qquad \text{----- (9)}$$

$$\psi_r = 10^{\frac{S_i - S' + s_1 \log(\psi_i) - s_2 \log(\psi')}{s_1 - s_2}} \qquad \text{----- (10)}$$

$$S_r = S_i - s_1 \log\left(\psi_r / \psi_i\right) \qquad \text{----- (11)}$$

where:

AEV = air-entry value,

ψ_i = suction corresponding to the inflect ion point,

S_i = degree of saturation corresponding to the inflection point,

s_1 = slope at the inflection point,

ψ' = suction where the curve starts to drop linearly (ψ=3000 kPa is suggested for sandy and silty soil),

S' = degree of saturation corresponding to suction ψ' and

s_2 = slope at the point (ψ', S')

By substituting the inflection point computed from Equation (6), (7) or (8) into Equations (9), (10), (11), the air-entry value, residual suction and residual saturation can be computed for each best fit equation from the respective fitting parameters. Compared with Zhai and Rahardjo's (2012a) method, Equations (6), (7) and (8) provide convenient method to obtain the inflection point for determination of SWCC variables.

3.3 Estimation of permeability function from SWCC fitting parameters

Zhai and Rahardjo (2015) explained the concept of the soil model which considers the pores in soil as a series of capillary tubes with different radii. Fredlund and Rahardjo (1993), Fredlund et al. (2012), Zhai and Rahardjo (2015) and Zhai et al. (2016) presented that SWCC was related to the permeability function. Zhai and Rahardjo (2015) pointed out that only the SWCC in the form of degree of saturation could be treated as analogous to the pore-size distribution function. Using the concept of random connection, Poiseuille's law and the SWCC in the form of degree of saturation as a probability function, Zhai and Rahardjo (2015) derived an equation to estimate permeability function from the SWCC as follows:

$$\frac{k(\psi_{m+i})}{k(\psi_m)} = \frac{ {n_{m+i}}^2 \left\{ \begin{array}{l} \left(S(\psi_{m+i}) - S(\psi_{m+i+1})\right)^2 \dfrac{1}{\psi_{m+i}^2} + \\ \displaystyle\sum_{j=m+i+1}^{N} \left[\left(S(\psi_{m+i}) - S(\psi_j)\right)^2 - \left(S(\psi_{m+i}) - S(\psi_{j-1})\right)^2\right] \dfrac{1}{\psi_j^2} \end{array} \right\} }{ {n_m}^2 \left\{ \begin{array}{l} \left(S(\psi_m) - S(\psi_{m+1})\right)^2 \dfrac{1}{\psi_m^2} + \\ \displaystyle\sum_{i=m+1}^{N} \left[\left(S(\psi_m) - S(\psi_i)\right)^2 - \left(S(\psi_m) - S(\psi_{i-1})\right)^2\right] \dfrac{1}{\psi_i^2} \end{array} \right\} }$$

----- (12)

where:

$k(\psi_{m+i})$ = hydraulic conductivity corresponding to suction of ψ_{m+i}, which is to be calculated,

$k(\psi_m)$ = hydraulic conductivity corresponding to suction of ψ_m, which is known as the reference point,

$S(\psi_m)$ = degree of saturation corresponding to suction of ψ_m,

$S(\psi_{m+i})$ = degree of saturation corresponding to suction of ψ_{m+i},

$S(\psi_j)$ = degree of saturation corresponding to suction of ψ_j,

n_m = porosity of soil corresponding to the suction of ψ_m and

n_{m+i} = porosity of soil corresponding to the suction of ψ_{m+i}.

Fredlund and Rahardjo (1993), Fredlund et al. (2012) and Rahimi et al. (2015) introduced the procedure for the estimation of the permeability function using the statistical method. In the conventional statistical method, volumetric water content was equally divided into certain intervals. Rahimi et al. (2015) demonstrated that if $\theta_w=f(\psi)$ expresses the volumetric water content as a function of the suction, then inverse of $f(\psi)$ ($\psi=f^{-1}(\theta_w)$ which expresses the suction as a function of volumetric water content) need to be calculated using the conventional statistical method. It is difficult to obtain the suction from the volumetric water content if a complex form of the best fit equation was adopted (e.g. Fredlund and Xing's 1994 equation or Kosugi's 1994 equation). Zhai and Rahardjo (2015) proposed to divide the suction into certain intervals and all the corresponding volumetric water contents (or degree of saturations) can be directly computed from the suction using the best fit equation regardless the complexity of the best fit equation. Zhai and Rahardjo's (2015) method provides convenience for using the statistical method to estimate the permeability function because inverse function, $\psi=f^{-1}(\theta_w)$, is not adopted.

By substituting Fredlund and Xing's (1994) equation into Equation (12), the permeability function can be computed using the fitting parameters a, n and m, as shown in Equation (13). On the other hand, by substituting van Genuchten's (1980) equation into Equation (12), the permeability function can be computed using the fitting parameters a, b and c, as illustrated in Equation (14).

$$\frac{k(\psi_{m+i})}{k(\psi_m)}=\frac{1}{A_{FX}}\left\{\begin{array}{l}\left(\dfrac{C(\psi_{m+i})}{\left\{\ln\left[e+\left(\dfrac{\psi_{m+i}}{a}\right)^{n}\right]\right\}^{m}}-\dfrac{C(\psi_{m+i+1})}{\left\{\ln\left[e+\left(\dfrac{\psi_{m+i+1}}{a}\right)^{n}\right]\right\}^{m}}\right)^{2}\dfrac{1}{\psi_{m+i}^{2}}+\\ \displaystyle\sum_{j=m+i+1}^{N}\left\{\begin{array}{l}\left\{\dfrac{C(\psi_{m+i})}{\left\{\ln\left[e+\left(\dfrac{\psi_{m+i}}{a}\right)^{n}\right]\right\}^{m}}-\dfrac{C(\psi_{j})}{\left\{\ln\left[e+\left(\dfrac{\psi_{j}}{a}\right)^{n}\right]\right\}^{m}}\right\}^{2}-\\ \left\{\dfrac{C(\psi_{m+i})}{\left\{\ln\left[e+\left(\dfrac{\psi_{m+i}}{a}\right)^{n}\right]\right\}^{m}}-\dfrac{C(\psi_{j-1})}{\left\{\ln\left[e+\left(\dfrac{\psi_{j-1}}{a}\right)^{n}\right]\right\}^{m}}\right\}^{2}\end{array}\right\}\dfrac{1}{\psi_{j}^{2}}\end{array}\right\}$$

----- (13)

where,

$$A_{FX} = \left\{ \begin{array}{l} \left(\dfrac{C(\psi_m)}{\left\{ \ln\left[e + \left(\dfrac{\psi_m}{a} \right)^n \right] \right\}^m} - \dfrac{C(\psi_{m+1})}{\left\{ \ln\left[e + \left(\dfrac{\psi_{m+1}}{a} \right)^n \right] \right\}^m} \right)^2 \dfrac{1}{\psi_m^2} + \\ \displaystyle\sum_{j=m+1}^{N} \left\{ \begin{array}{l} \left\{ \dfrac{C(\psi_m)}{\left\{ \ln\left[e + \left(\dfrac{\psi_m}{a} \right)^n \right] \right\}^m} - \dfrac{C(\psi_i)}{\left\{ \ln\left[e + \left(\dfrac{\psi_i}{a} \right)^n \right] \right\}^m} \right\}^2 - \\ \left\{ \dfrac{C(\psi_m)}{\left\{ \ln\left[e + \left(\dfrac{\psi_m}{a} \right)^n \right] \right\}^m} - \dfrac{C(\psi_{i-1})}{\left\{ \ln\left[e + \left(\dfrac{\psi_{i-1}}{a} \right)^n \right] \right\}^m} \right\}^2 \end{array} \right\} \dfrac{1}{\psi_i^2} \end{array} \right\}$$

$$\frac{k(\psi_{m+i})}{k(\psi_m)} = \frac{1}{A_{VG}} \left\{ \begin{array}{l} \left(\dfrac{1}{\left[1 + (a\psi_{m+i})^b\right]^c} - \dfrac{1}{\left[1 + (a\psi_{m+i+1})^b\right]^c} \right)^2 \dfrac{1}{\psi_{m+i}^2} + \\ \displaystyle\sum_{j=m+i+1}^{N} \left\{ \begin{array}{l} \left\{ \dfrac{1}{\left[1 + (a\psi_{m+i})^b\right]^c} - \dfrac{1}{\left[1 + (a\psi_j)^b\right]^c} \right\}^2 - \\ \left\{ \dfrac{1}{\left[1 + (a\psi_{m+i})^b\right]^c} - \dfrac{1}{\left[1 + (a\psi_{j-1})^b\right]^c} \right\}^2 \end{array} \right\} \dfrac{1}{\psi_j^2} \end{array} \right\} \quad \text{----- (14)}$$

where,

$$A_{VG} = \left\{ \begin{array}{l} \left(\dfrac{1}{\left[1 + (a\psi_m)^b\right]^c} - \dfrac{1}{\left[1 + (a\psi_{m+1})^b\right]^c} \right)^2 \dfrac{1}{\psi_m^2} + \\ \displaystyle\sum_{j=m+1}^{N} \left\{ \begin{array}{l} \left\{ \dfrac{1}{\left[1 + (a\psi_m)^b\right]^c} - \dfrac{1}{\left[1 + (a\psi_i)^b\right]^c} \right\}^2 - \\ \left\{ \dfrac{1}{\left[1 + (a\psi_m)^b\right]^c} - \dfrac{1}{\left[1 + (a\psi_{i-1})^b\right]^c} \right\}^2 \end{array} \right\} \dfrac{1}{\psi_i^2} \end{array} \right\}$$

4. Effects of C_r and θ_r on the estimation of permeability function

The value of C_r in Fredlund and Xing (1994) and θ_r in van Genuchten (1980) equations can be obtained from either regression analysis or predefined before the regression analysis. Therefore, the value of C_r or θ_r may affect the performance of best fit equation and prediction result of other unsaturated properties (e.g. the permeability function). To evaluate the performance of Fredlund and Xing (1994) and van Genuchten (1980) equations with respect to the value of θ_r and C_r, three case studies were considered in the analyses. In case 1, C_r or θ_r was considered as a fitting parameter (determined from the regression process). In case 2, the value of θ_r and C_r which corresponds to the oven-dry condition (i.e., $C_r=10^6$ kPa, $\theta_r= 0$) was assigned. In case 3, the value of θ_r and C_r which corresponds to the air-dry condition (i.e., $C_r=1500$kPa, $\theta_r=5\%$) was assigned.

Equation (12) does not limit the usage of the best fit SWCC equation. Any best fit SWCC equation can be substituted into Equation (12). Substituting van Genuchten's (1980) equation into Equation (12) generates the estimation of the unsaturated hydraulic conductivity from the fitting parameters of van Genuchten's (1980) equation. In this paper, it is named as ZR-VG model. On the other hand, substituting Fredlund and Xing's (1994) equation into Equation (12) provides the estimation of the unsaturated hydraulic conductivity from the fitting parameters of Fredlund and Xing's (1994) equation. In this paper, it is named as ZR-FX model.

Two sets of coarse-grained soil data including Volcanic sand and Fine sand (from Brooks and Corey, 1964) and two sets of fine-grained soil data including clayey sand with gravel (SC) and sandy silt (ML) (from Li et al., 2009) were selected in this paper for the evaluation of the performance of ZR-VG and ZR-FX models. The experimental data on SWCC and unsaturated hydraulic conductivity for four types of soils are illustrated in Figures 3 and 4. The best fitted parameters for the three case studies are summarized in Table 1 and the computed SWCC variables are summarized in Table 2.

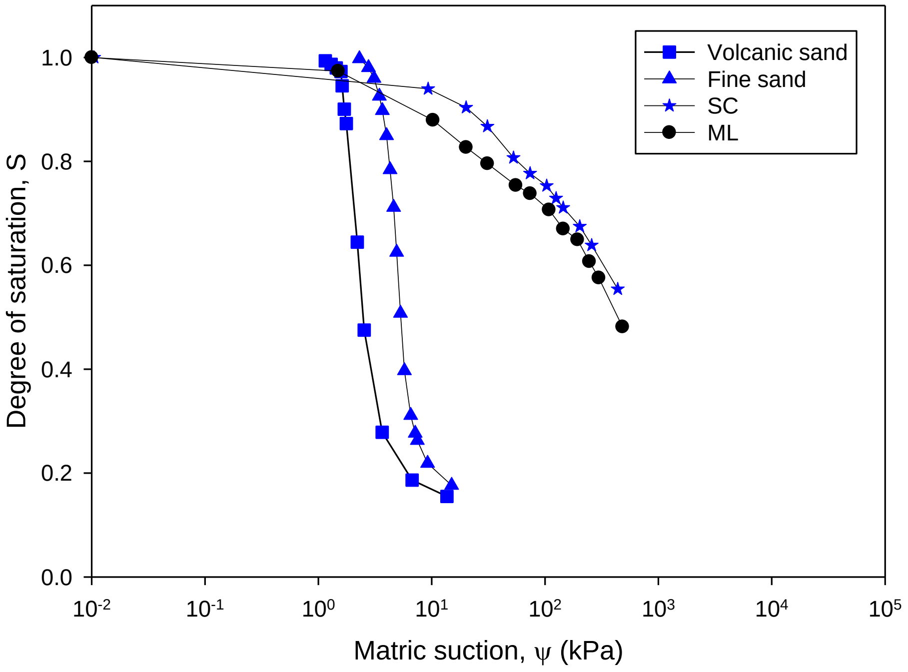
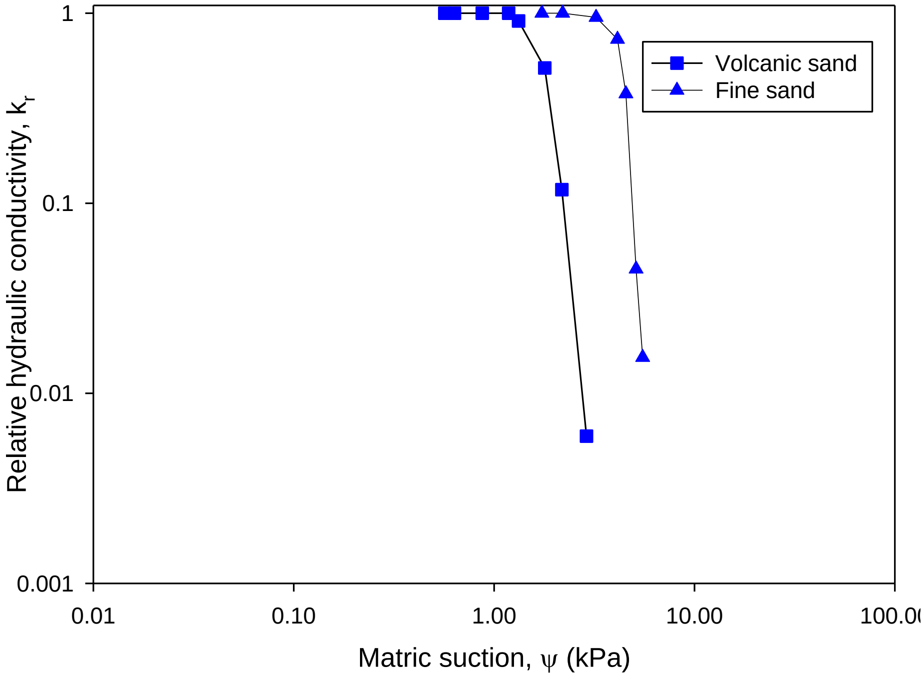
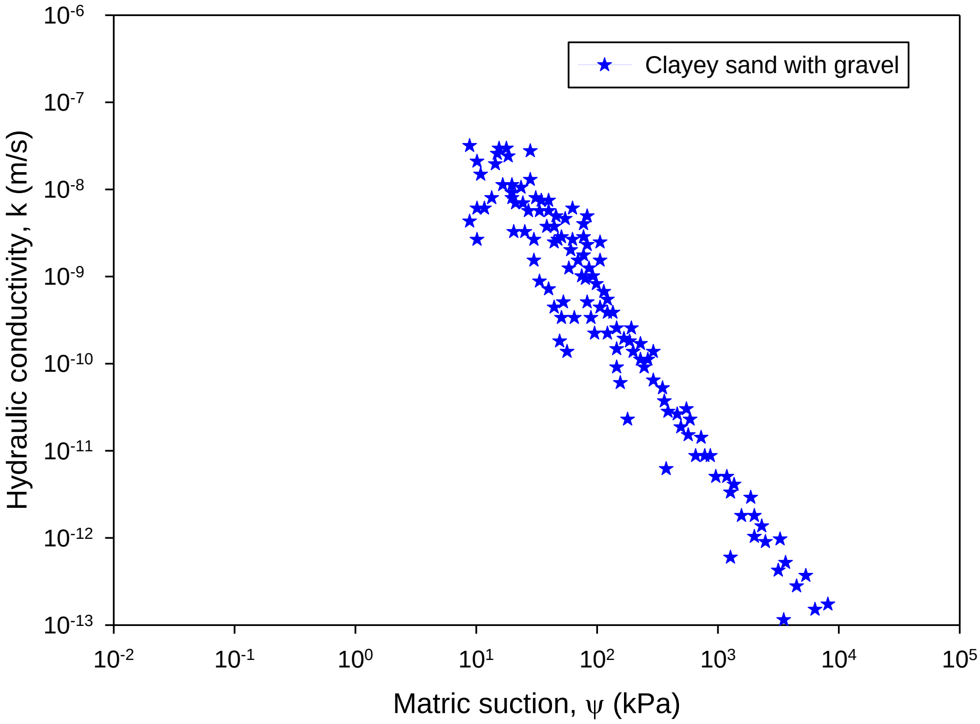


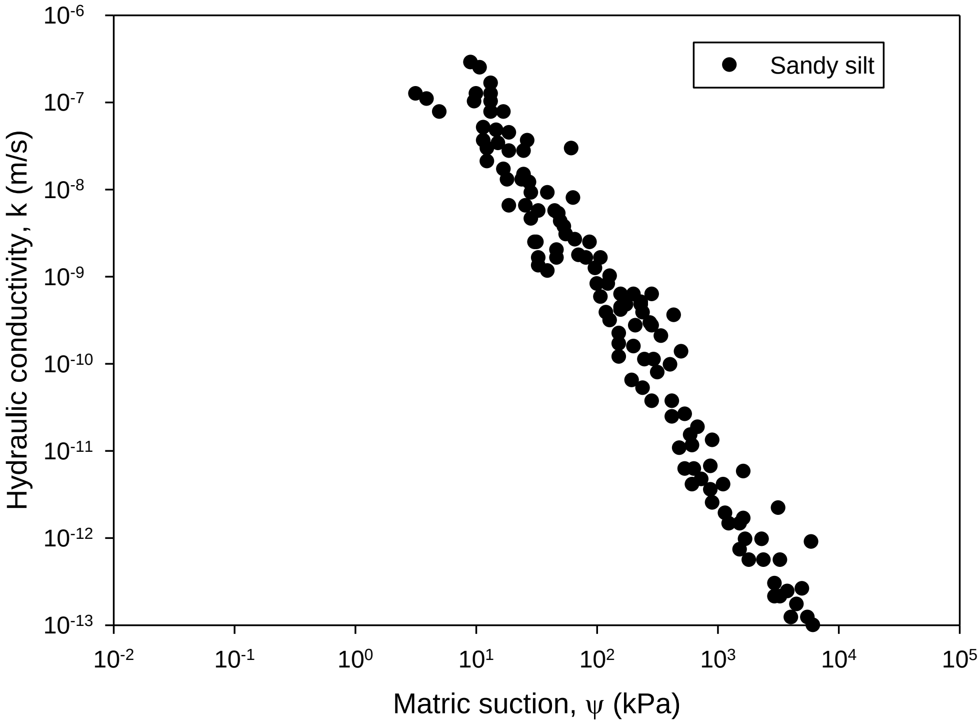
Figure 3. SWCC of four types of soil



(a) Measured relative hydraulic conductivity, k_r, for Volcanic sand and Fine sand



(b) Measured hydraulic conductivity, k, for clayey sand with gravel (SC)



(c) Measured hydraulic conductivity, k, for sandy silt (ML)

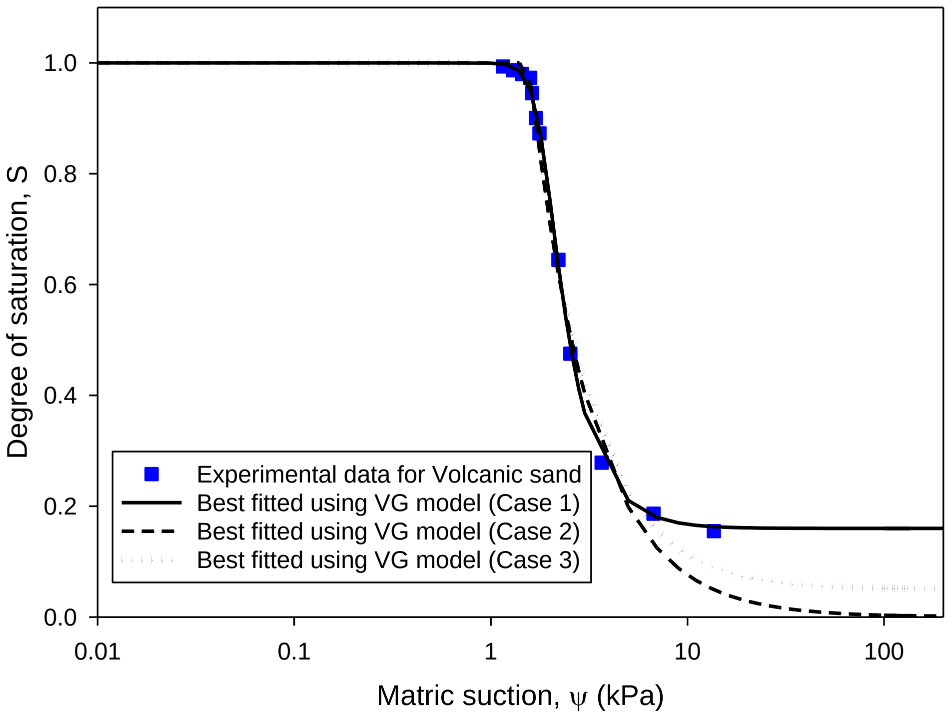
Figure 4. Measured hydraulic conductivity for four types of soil

Table 1: Fitting parameters of SWCC for four types of soil.

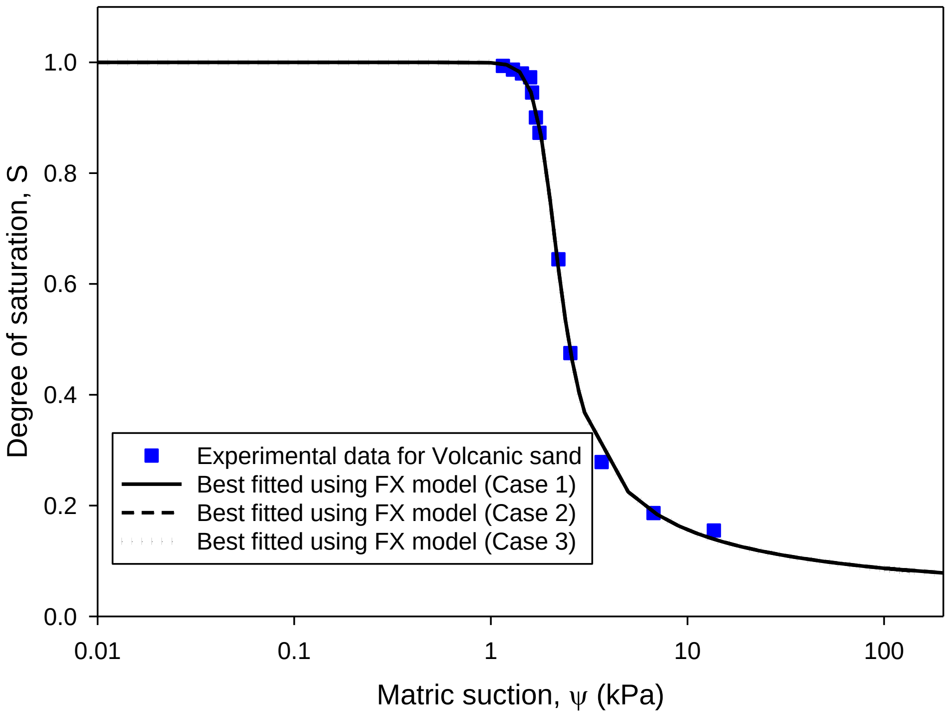
Soil	Best fit equation	van Genuchten's (1980) equation											
Volcanic Sand from Brooks and Corey (1964)	case 1	a=	0.55	n=	10.30	m=	0.27	θ_r=	0.16	R^2=	99.97%	SSE=	1.20e-3
	case 2	a=	0.64	n=	46.29	m=	0.03	θ_r=	0	R^2=	99.54%	SSE=	1.73e-2
	case 3	a=	0.63	n=	24.69	m=	0.06	θ_r=	0.05	R^2=	99.74%	SSE=	9.96e-3
	Best fit equation	Fredlund and Xing's (1994) equation											
	case 1	a=	1.87	n=	9.13	m=	0.68	C_r=	8.29e+6	R^2=	99.96%	SSE=	1.6e-3
	case 2	a=	1.87	n=	9.13	m=	0.68	C_r=	1e+6	R^2=	99.96%	SSE=	1.6e-3
	case 3	a=	1.87	n=	9.14	m=	0.68	C_r=	1500	R^2=	99.96%	SSE=	1.6e-3
Fine sand from Brooks and Corey (1964)	**Best fit equation**	van Genuchten's (1980) equation											
	case 1	a=	0.21	n=	6.82	m=	0.87	θ_r=	0.19	R^2=	99.94%	SSE=	1.96e-3
	case 2	a=	0.26	n=	10.42	m=	0.19	θ_r=	0	R^2=	99.33%	SSE=	2.16e-2
	case 3	a=	0.25	n=	9.43	m=	0.25	θ_r=	0.05	R^2=	99.64%	SSE=	1.48e-2
	Best fit equation	Fredlund and Xing's (1994) equation											
	case 1	a=	4.30	n=	9.88	m=	0.74	C_r=	7.24	R^2=	99.87%	SSE=	4.30e-3
	case 2	a=	4.23	n=	8.23	m=	0.85	C_r=	1e+6	R^2=	99.82%	SSE=	5.60e-3
	case 3	a=	4.23	n=	8.25	m=	0.85	C_r=	1500	R^2=	99.82%	SSE=	5.60e-3
SC from Li et al. (2009)	**Best fit equation**	van Genuchten's (1980) equation											
	case 1	a=	0.007	n=	0.71	m=	0.51	θ_r=	0	R^2=	99.98%	SSE=	5.40e-4
	case 2	a=	0.007	n=	0.71	m=	0.51	θ_r=	0	R^2=	99.98%	SSE=	5.40e-4
	case 3	a=	0.006	n=	0.70	m=	0.57	θ_r=	0.05	R^2=	99.98%	SSE=	5.50e-4
	Best fit equation	Fredlund and Xing's (1994) equation											
	case 1	a=	100.12	n=	0.70	m=	1	C_r=	1499.56	R^2=	99.98%	SSE=	5.00e-4
	case 2	a=	63.47	n=	0.79	m=	0.81	C_r=	1e+6	R^2=	99.98%	SSE=	7.40e-4
	case 3	a=	62.99	n=	0.75	m=	0.79	C_r=	1500	R^2=	99.98%	SSE=	6.00e-4
ML from Li et al. (2009)	**Best fit equation**	van Genuchten's (1980) equation											
	case 1	a=	0.001	n=	0.50	m=	1.28	θ_r=	0	R^2=	99.91%	SSE=	2.70e-3
	case 2	a=	0.001	n=	0.50	m=	1.28	θ_r=	0	R^2=	99.91%	SSE=	2.70e-3
	case 3	a=	0.001	n=	0.51	m=	1.38	θ_r=	0.05	R^2=	99.91%	SSE=	2.80e-3
	Best fit equation	Fredlund and Xing's (1994) equation											
	case 1	a=	12167	n=	0.44	m=	7.71	C_r=	1092.19	R^2=	99.92%	SSE=	2.40e-3
	case 2	a=	792009	n=	0.44	m=	50	C_r=	1e+6	R^2=	99.92%	SSE=	2.40e-3
	case 3	a=	288203	n=	0.43	m=	27.94	C_r=	1500	R^2=	99.92%	SSE=	2.40e-3

Table 2. SWCC variables obtained from fitting parameters for four types of soil.

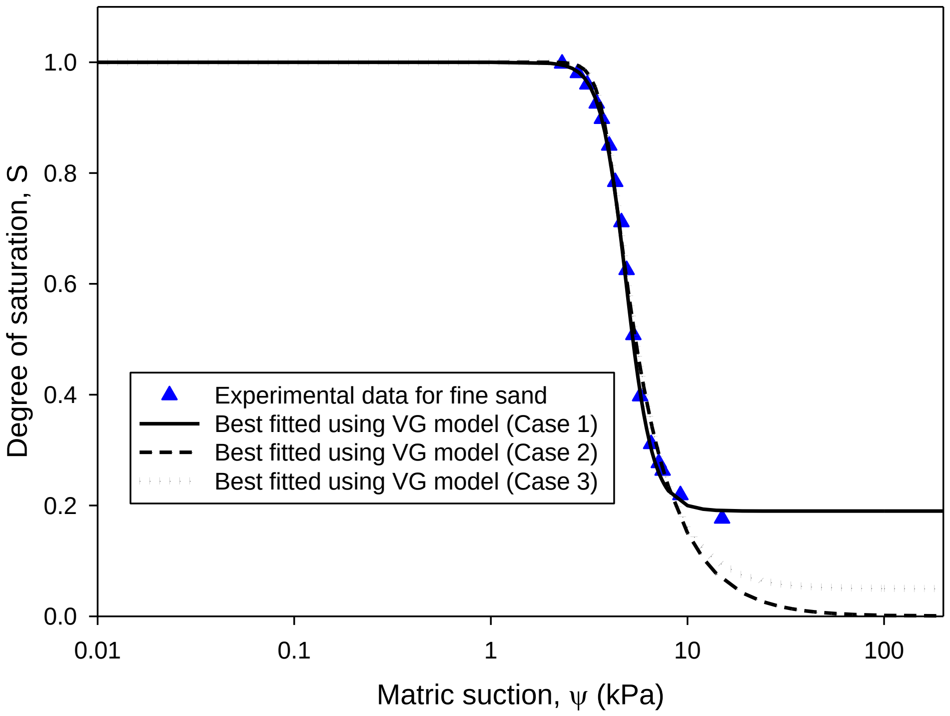
Soil	Best fit equation	Scenarios	Air-entry value AEV (kPa)	Residual suction ψ_r (kPa)	Residual saturation, S_r
Volcanic sand	van Genuchten's (1980) equation	Case 1	1.62	3.29	0.16
		Case 2	1.55	3.55	0
		Case 3	1.55	3.66	0.05
	Fredlund and Xing's (1994) equation	Case 1	1.64	3.40	0.09
		Case 2	1.64	3.40	0.09
		Case 3	1.64	3.34	0.12
Fine sand	van Genuchten's (1980) equation	Case 1	3.58	6.71	0.19
		Case 2	3.49	8.35	0
		Case 3	3.54	7.98	0.05
	Fredlund and Xing's (1994) equation	Case 1	3.54	8.38	0.06
		Case 2	3.51	8.20	0.06
		Case 3	3.51	8.11	0.07
SC	van Genuchten's (1980) equation	Case 1	16.89	1391.9	0.39
		Case 2	16.89	1391.9	0.39
		Case 3	17.18	1398.54	0.40
	Fredlund and Xing's (1994) equation	Case 1	17.99	1477	0.40
		Case 2	13.57	975.9	0.47
		Case 3	14.67	1270	0.43
ML	van Genuchten's (1980) equation	Case 1	12.40	1764.70	0.34
		Case 2	12.40	1764.70	0.34
		Case 3	11.98	1673.49	0.34
	Fredlund and Xing's (1994) equation	Case 1	21.66	2077	0.27
		Case 2	20.69	2182	0.26
		Case 3	24.48	2178	0.26



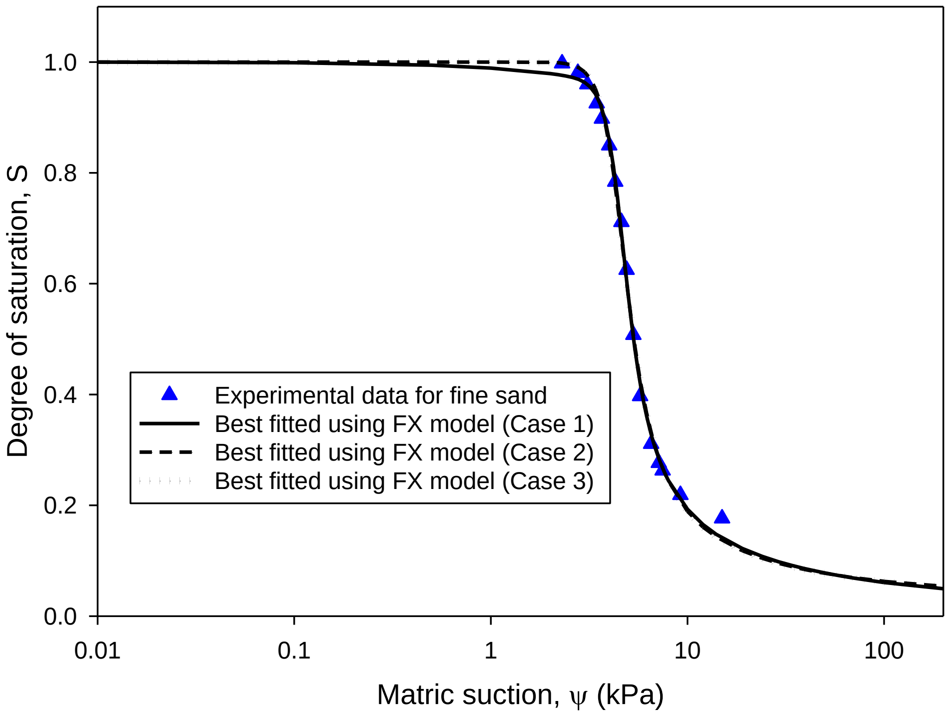
(a)


1.0
0.8
0.6
0.4
0.2
0.0
Degree of saturation, S
0.01
0.1
1
10
100
Matric suction, ψ (kPa)
Experimental data for Volcanic sand
Best fitted using FX model (Case 1)
Best fitted using FX model (Case 2)
Best fitted using FX model (Case 3)

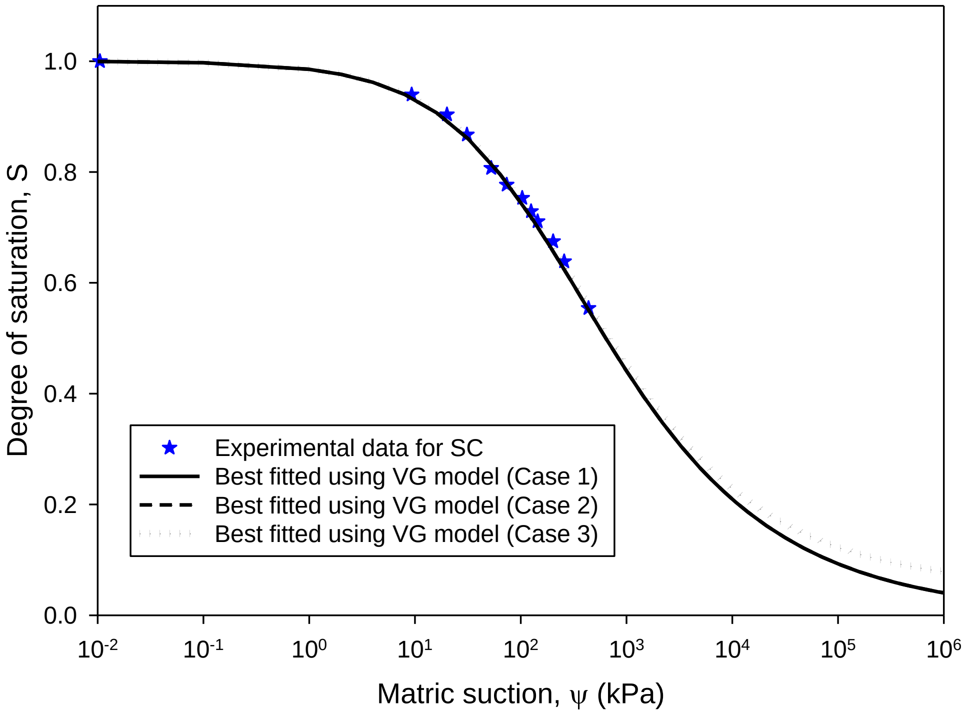
(b)


1.0
0.8
0.6
0.4
0.2
0.0
Degree of saturation, S
0.01
0.1
1
10
100
Matric suction, ψ (kPa)
Experimental data for fine sand
Best fitted using VG model (Case 1)
Best fitted using VG model (Case 2)
Best fitted using VG model (Case 3)

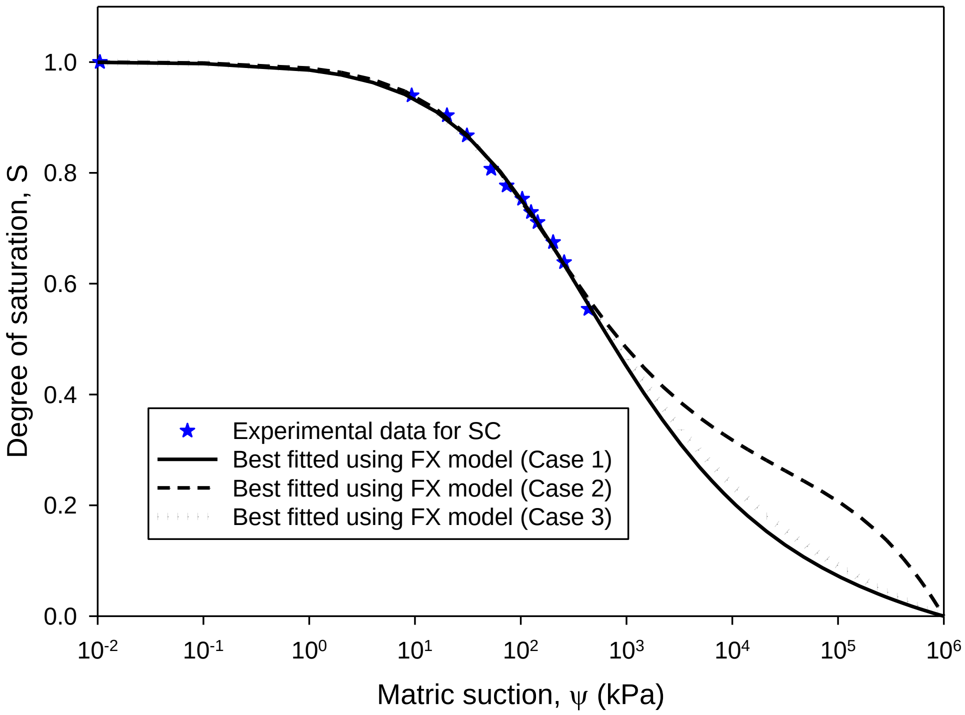
(c)


1.0
0.8
0.6
0.4
0.2
0.0
Degree of saturation, S
0.01
0.1
1
10
100
Matric suction, ψ (kPa)
Experimental data for fine sand
Best fitted using FX model (Case 1)
Best fitted using FX model (Case 2)
Best fitted using FX model (Case 3)

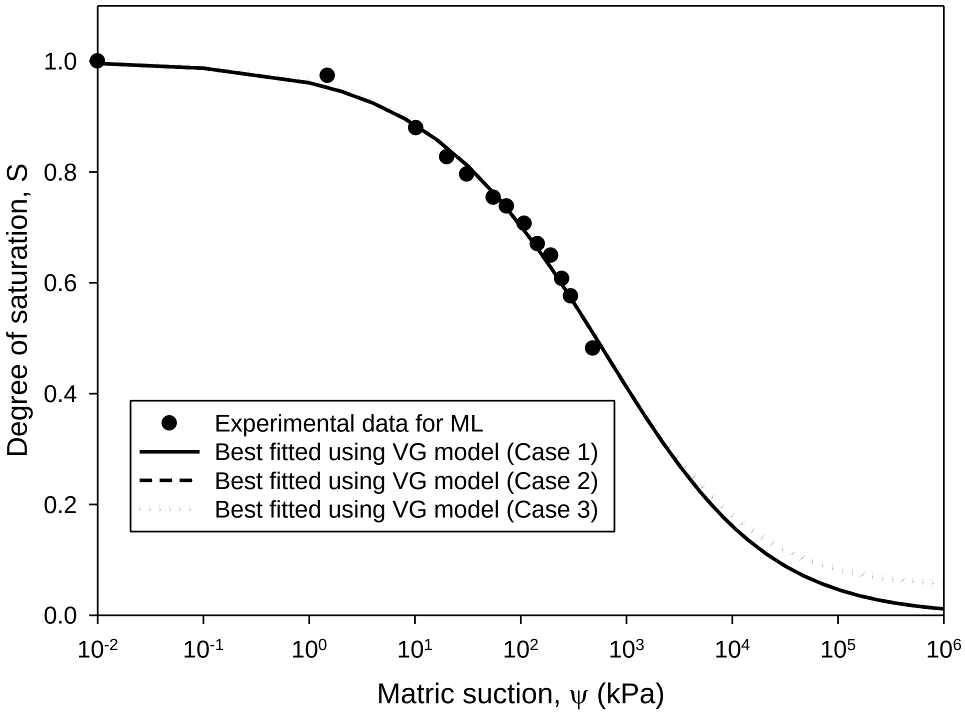
(d)


1.0
0.8
0.6
0.4
0.2
0.0
Degree of saturation, S
10-2
10-1
100
101
102
103
104
105
106
Matric suction, ψ (kPa)
Experimental data for SC
Best fitted using VG model (Case 1)
Best fitted using VG model (Case 2)
Best fitted using VG model (Case 3)

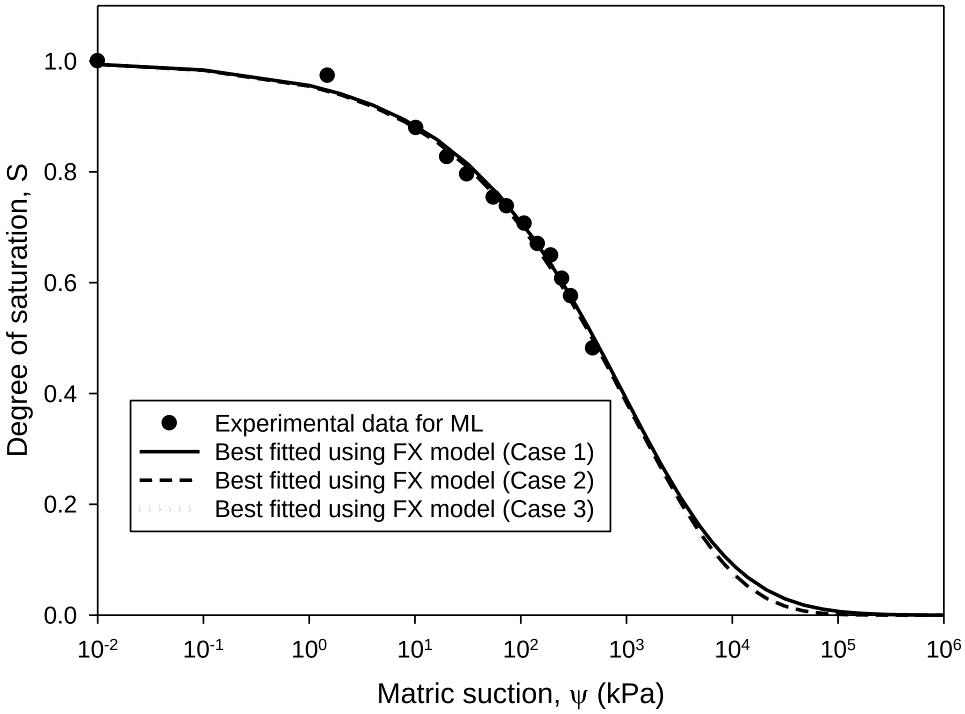
(e)


Degree of saturation, S
1.0
0.8
0.6
0.4
0.2
0.0
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
Matric suction, ψ (kPa)
Experimental data for SC
Best fitted using FX model (Case 1)
Best fitted using FX model (Case 2)
Best fitted using FX model (Case 3)

(f)

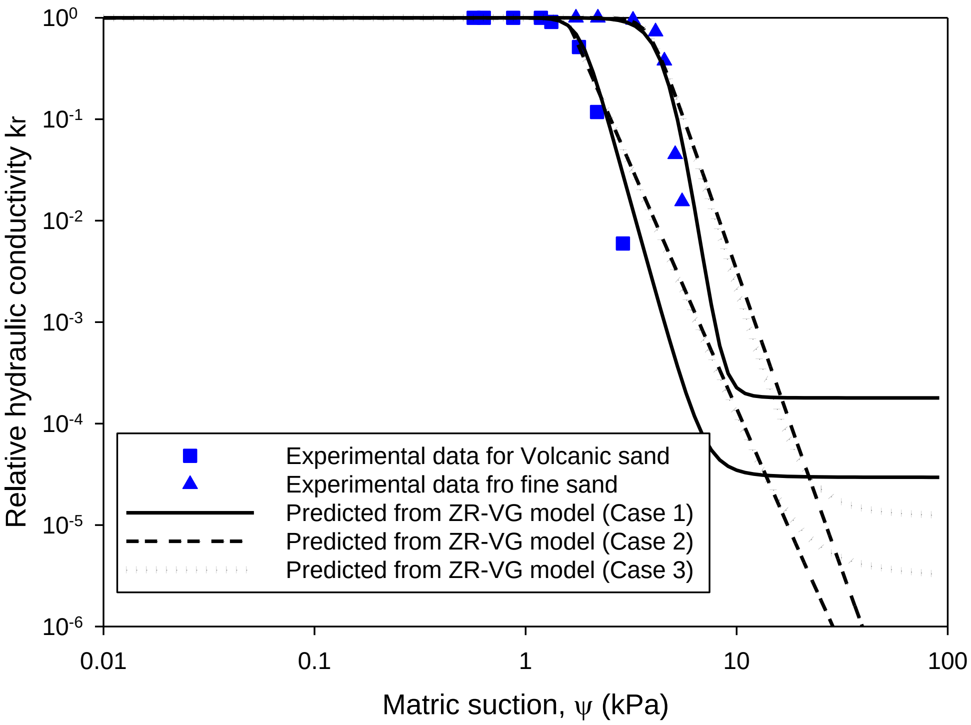

Degree of saturation, S
1.0
0.8
0.6
0.4
0.2
0.0
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
Matric suction, ψ (kPa)
Experimental data for ML
Best fitted using VG model (Case 1)
Best fitted using VG model (Case 2)
Best fitted using VG model (Case 3)

(g)

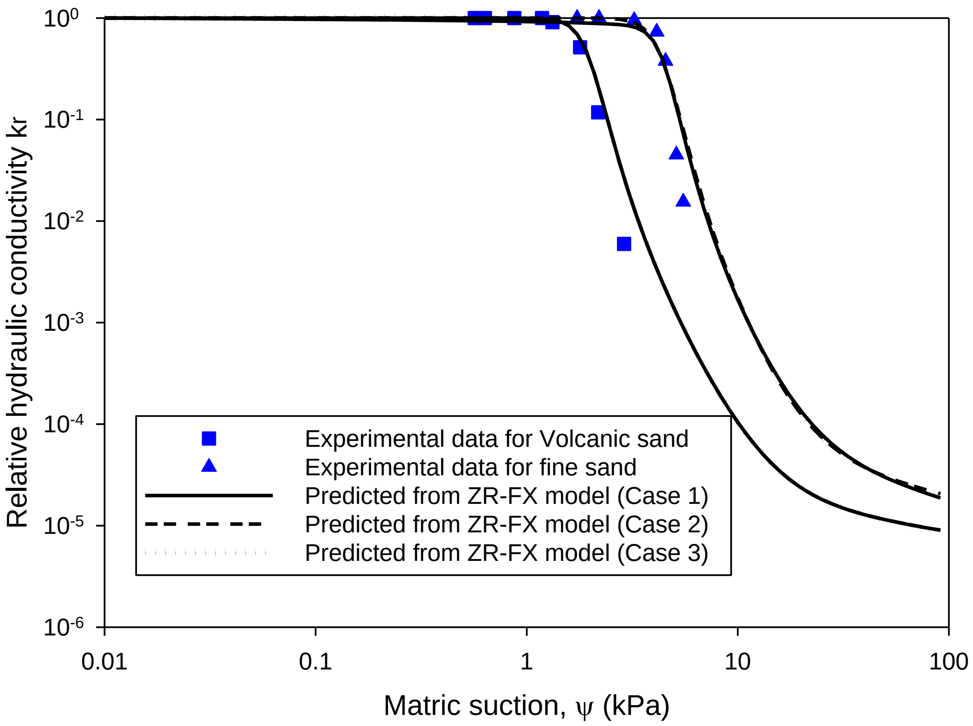


(h)

Figure 5. Best fitted SWCC for four types of soil.

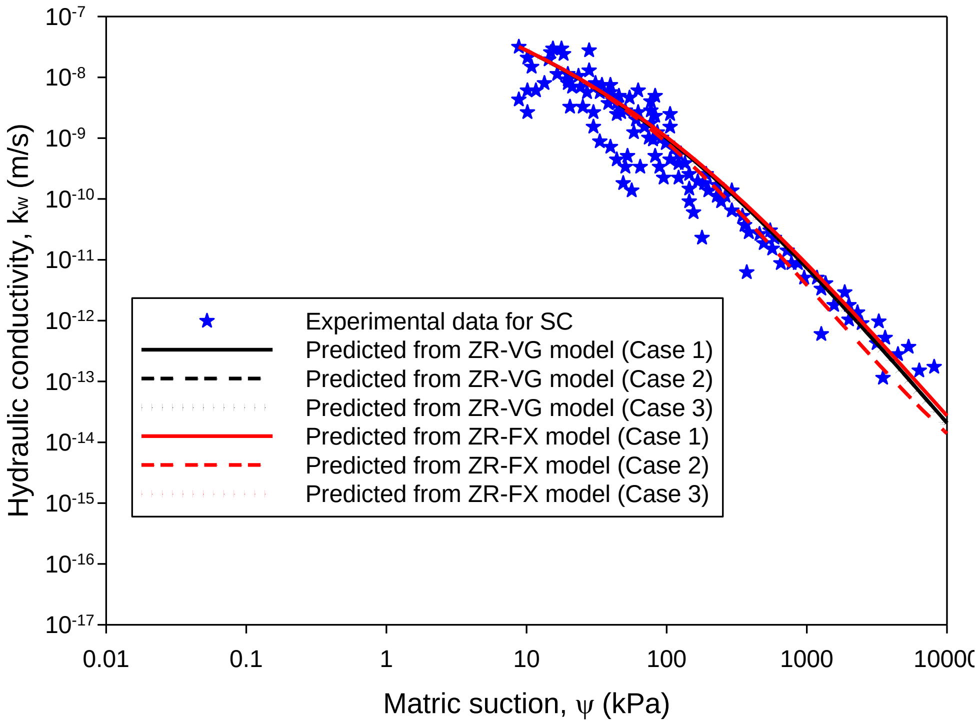


(a)



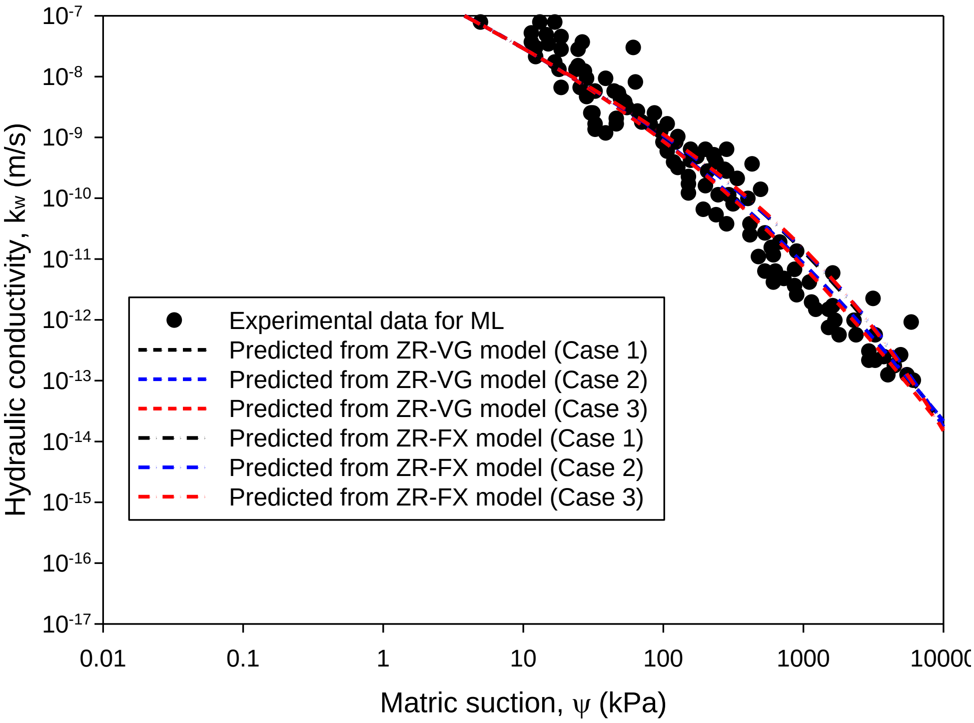
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
Relative hydraulic conductivity kr
0.01
0.1
1
10
100
Matric suction, ψ (kPa)
Experimental data for Volcanic sand
Experimental data for fine sand
Predicted from ZR-FX model (Case 1)
Predicted from ZR-FX model (Case 2)
Predicted from ZR-FX model (Case 3)

(b)



10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{-17}
Hydraulic conductivity, k_w (m/s)
0.01
0.1
1
10
100
1000
10000
Matric suction, ψ (kPa)
Experimental data for SC
Predicted from ZR-VG model (Case 1)
Predicted from ZR-VG model (Case 2)
Predicted from ZR-VG model (Case 3)
Predicted from ZR-FX model (Case 1)
Predicted from ZR-FX model (Case 2)
Predicted from ZR-FX model (Case 3)

(c)



(d)

Figure 6. Predicted permeability functions for four types of soil

As illustrated in Figure 5, there are insignificant differences in the SWCCs obtained from Fredlund and Xing's (1994) equation with different values of C_r for the soils investigated in this study. As a result, the value of C_r has insignificant effect on the estimated permeability function from ZR-FX model as illustrated in Figure 6. Therefore, parameter C_r = 1500 kPa could be used in Fredlund and Xing's (1994) equation as recommended by Fredlund and Xing (1994) and Zhai and Rahardjo (2012a, 2012b). On the other hand, there are significant differences in the SWCC obtained from van Genuchten's (1980) equation with different values of θ_r for coarse-grained soils while insignificant differences are observed for fine-grained soils. As a result, the value of θ_r has significant effects on the estimated permeability function from ZR-VG model. Therefore, parameter θ_r should be treated as fitting parameter in van Genuchten's (1980) equation to ensure good estimation on the permeability function.

5. Conclusions

A new mathematical equation was proposed to determine the inflection point which was used for determination of the SWCC variables such as the air-entry value, the residual suction and residual water content. Results of analyses show that value of C_r has insignificant effects on the performance of Fredlund and Xing's (1994) equation and the estimation of the unsaturated hydraulic conductivity for soils investigated in this study. Significant differences in the best fitted SWCCs using van Genuchten's (1980) equation for coarse-grained soils were observed if different values of θ_r were adopted for the best fit procedure. Consequently, the value of θ_r has significant effects on the estimation of the permeability function. It is concluded that C_r= 1500 kPa can be used in Fredlund and Xing's (1994) equation whereas θ_r should be treated as a fitting parameter in van Genuchten's (1980) equation which can be obtained from the regression analyses.

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