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Synchronized oscillations and acoustic fluidization in confined granular materials

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According to the acoustic fluidization hypothesis, elastic waves at a characteristic frequency form inside seismic faults even in the absence of an external perturbation. These waves are able to generate a normal stress which contrasts the confining pressure and promotes failure. Here, we study the mechanisms responsible for this wave activation via numerical simulations of a granular fault model. We observe the particles belonging to the percolating backbone, which sustains the stress, to perform synchronized oscillations over ellipticlike trajectories in the fault plane. These oscillations occur at the characteristic frequency of acoustic fluidization. As the applied shear stress increases, these oscillations become perpendicular to the fault plane just before the system fails, opposing the confining pressure, consistently with the acoustic fluidization scenario. The same change of orientation can be induced by external perturbations at the acoustic fluidization frequency.

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Confined granular materials under shear display the typical stick-slip dynamics observed in real fault systems. In the last years this dynamics has been deeply investigated in several experimental settings as well as by means of molecular dynamics simulations [1–14]. These studies mostly focus on two central questions: (i) Why is the stress responsible for seismic failure usually orders of magnitude smaller than the value expected on the basis of rock fracture mechanics? (ii) Why are seismic faults very susceptible to even small amplitude transient seismic waves? Indeed, the resistance to shear stress of seismic faults is typically much larger than the one obtained in experiments measuring the friction coefficient of sliding rocks [15]. Furthermore, remote triggering of earthquakes [16–20] at distances of thousand kilometers from the main shock epicenter indicates a high susceptibility of seismic faults to the passage of seismic waves. The hypothesis of acoustic fluidization (AF), formulated by Melosh [21,22], provides an answer to both questions. According to AF, the elastic waves produced by seismic fracture, at a characteristic frequency ωAF, diffuse and scatter inside the fault and then generate a normal stress which can contrast the confining pressure. In this way, seismic failure is promoted. To investigate this scenario, experimental studies [1,2,6,7] have demonstrated that acoustic perturbations modify granular rheology and lead to autoacoustic compaction [7]. Recently, the AF scenario has been explored in three-dimensional (3D) molecular dynamics simulations [23] which have shown that weak external perturbations, at a frequency ωAF, even if increasing the confining pressure or reducing the applied shear, induce slip instabilities. Interestingly, simulations have also shown that oscillations at a frequency ωAF are activated immediately before each slip, even in the absence of an external perturbation. Nevertheless, the mechanisms responsible for this activation, as well as the nonlinear response to external perturbations, are not fully understood.

In this Rapid Communication, we shed light on these mechanisms by means of a detailed investigation of grain trajectories during the stick phase in 3D molecular dynamics simulations. Differently from previous studies, which mainly focus on the confining plate dynamics, we follow the evolution of each grain. This analysis shows that oscillations at ωAF are always present during the stick phase. Indeed, grains exhibit vibrational modes describing quasielliptic trajectories which are oriented, most of the time, parallel to the drive direction. In proximity of slip instabilities, conversely, the orientation of the ellipses changes and oscillations perpendicular to the drive direction emerge. These oscillations reduce the confining pressure and promote failure. The same mechanism is observed when the system undergoes an external perturbation at a frequency ωAF.

The model. The system is composed of N spheres enclosed between two rigid plates of dimension $L_x \times L_y = 20d \times 5d$. Each plate is made of $L_x/L_y/d^2$ spheres of diameter $d$ placed in random positions in the fault plane, namely, i.e., the $x$-$y$ plane. Spheres are shifted by a random $\delta z \in [0,d/2]$ in the $z$ direction. In order to make the plates rigid, the particles keep their relative positions. This preparation protocol ensures the roughness of both confining plates. The fault width is roughly of size $L_z \approx 10d$. While the bottom plate is kept fixed, the top one is subject to a constant pressure $p$ and attached to a spring of elastic constant $k_n$, which is pulled at constant velocity $V$ along the $x$ direction. We employ a contact force model that captures the major features of granular interactions, known as the spring-dashpot model, which also takes into account the presence of static friction [24–26]. The normal interaction between two contacting spheres is characterized by a spring constant $k_n = 2 \times 10^3 k_m$ and a damping coefficient $\gamma_n = 50 \sqrt{k_m/m}$. Model parameters are chosen according to Refs. [25,27] in order to have long stick phases interrupted by rapid plate displacements, i.e., the slips. The duration

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of the stick phase is inversely proportional to the driving velocity $V$. We measure the mass in units of $m$, the length in units of $d$, and time in units of $\sqrt{m/k_m}$, where the typical duration of a slip instability is of the order of one time unit. The confining pressure is $p = k_m/d$ and the driving velocity $V = 0.01 d/\sqrt{m/k_m}$.

In Fig. 1 (upper panels) we plot the top-plate position in four time intervals of different durations which present six slips. We evaluate the autocorrelation function of particle velocities

$$\hat{C}(t,t') = \sum_{i=1}^{N} \frac{\hat{v}_i(t) \cdot \hat{v}_i(t')}{\sum_{j=1}^{N} \hat{v}_j(t) \cdot \hat{v}_j(t)} \sum_{j=1}^{N} \hat{v}_j(t) \cdot \hat{v}_j(t),$$

where $\hat{v}_i$ is the velocity of the $i$th particle. More precisely, at each time $t$, we create a replica of the system decoupled from the external drive ($V = 0$) and measure the particle velocity $\hat{v}_i$ at subsequent times $t' \geq t$. We study the temporal evolution of the power spectral density $\hat{C}(t,\omega)$ defined as the Fourier transform, with respect to $t'$, of $C(t,t')$. We find that for all values of $\omega$, the spectral density takes very small values so that $\hat{C}(t,\omega) < 0.01$, whereas much larger values are observed in a narrow range, $\omega \in (1.3, 1.5)\pi$ (red squares in Fig. 1, central panels). In particular, Fig. 1 (central panels) shows that $\hat{C}(t,\omega^*)$, with $\omega^* = 1.4\pi$, presents nonmonotonic behavior as a function of $t$. More precisely, $\hat{C}(t,\omega^*)$ rapidly increases as $t$ approaches $t_s$, decreasing after $t_s$. Since the dissipation is relevant only on time scales much larger than $1/\omega^*$, the integral of $\hat{C}(t,\omega)$ over the entire frequency range is always close to 1. Hence, the value of $\hat{C}(t,\omega^*)$ can be interpreted as the percentage of energy of the modes in the frequency range $(\omega^* - d\omega/2, \omega^* + d\omega/2)$. The frequency $\omega^*$ can be related to the characteristic frequency $\omega_{AF}$ predicted by the AF scenario. Indeed, according to AF, the resonant frequency $\omega_{AF}$ characterizes the typical acoustic waves bouncing back and forth within the medium [23]. These waves propagate with velocity $v_A = \sqrt{M/\rho}$, where $M$ is the $P$-wave modulus and $\rho$ is the system density. The evaluation of $M$ in the confined granular medium is very complicated and, indeed, experimental and numerical studies [28–30] indicate that it increases when the confining pressure is increased. In first approximation we can use the result $M \approx k_m/(6d)$ obtained for a single grain under normal compressional stresses [31]. Then, using $\rho \approx mN/(L_xL_yL_z)$ and taking into account that a time $T_a = 2L_z/v_A$ is necessary to reach the bottom plate and return to the top, the typical AF resonant frequency is

$$\omega_{AF} = 2\pi/T_a = (\pi/L_z)/\sqrt{k_m/(6d\rho)}.$$
of particles are always in contact with their neighbors, forming an almost rigid structure, i.e., the backbone. Conversely, a small fraction (less than 10%) of particles, the rattlers, are located inside the cages formed by the particles in the backbone [32,33] and most of the time do not interact with other particles [34]. These cages form soon after each slip and keep their configuration substantially unaltered during the whole stick phase, as shown in Fig. 1 in the Supplemental Material [35]. Rattlers present a kinetic energy order of magnitude larger than the backbone energy [34]. They are then identified with the shear transforming zone [36,37] according to a local strain measure as $D_{\text{min}}$, introduced in Ref. [38]. Nevertheless, rattlers are not directly responsible for slip instabilities caused by the collapse of the force chains made by backbone particles. In the left panels of Fig. 2 we plot the $x$ position of four neighboring particles during a short time window: Three particles $(x_1, x_2, x_3)$ exhibit regular oscillations along the $x$ direction. Differently, particle 4 is a rattler and moves along a straight line up to an abrupt change in the direction caused by a collision. In the following, we restrict the study to backbone particles. As shown in the left panel of Fig. 2, the particle motion in the $x$ direction corresponds to vibrational modes at a characteristic frequency $\omega = 1.4 \tau \simeq \omega_{AF}$. More precisely, when we superimpose the centers of each trajectory in a common point, as in Fig. 2 (upper right panel), trajectories are roughly confined in a plane and exhibit an elliptic-like shape. The same oscillating behavior (not shown) is observed for the other backbone particles and for all time intervals during the stick phase. This pattern is recovered for all particles as confirmed by the power spectrum of the particle position $\hat{C}_i(\omega)$ obtained from the Fourier transform of the $i$th particle position $x_i(t)$. Figure 3 indeed shows that all particles present characteristic oscillations at a frequency $\omega \simeq \omega_{AF}$. Hence, this study shows that vibrational modes at a characteristic frequency $\omega_{AF}$ do not form at the onset of slips but are already present inside the system at all times. The energy responsible for these oscillations originates from the energy stored, during the stick phase, through the spring which couples the system to the external drive. Most of this energy is released very rapidly during the slip, but a significant fraction contributes to the activation of harmonic oscillations. Because of the vertical confinement, only the mode at the frequency $\omega_{AF}$ [Eq. (2)] survives. Even if these modes have been explained in terms of compressional waves propagating along the $z$ direction, because of the heterogeneous structure of the granular packing, these waves induce also displacements along the $x$ and $y$ directions. Far from the slip, the confinement along the $z$ direction and periodic boundary conditions along $x$ and $y$ lead to $x$ and $y$ displacements larger than $z$ displacements.

We wish to stress that $\hat{C}_i(\omega)$ is the Fourier transform of a one-time quantity $x(t)$, whereas $\hat{C}(t, \omega)$ is obtained from the correlation function $C(t, t')$, which is a two-time quantity. Hence the existence of a temporal interval with $\hat{C}(t, \omega_{AF}) \simeq 0$ before $t_s$ indicates a decorrelation of grain velocities. To rationalize the origin of this decorrelation we investigate the orientation of the quasielliptic trajectories during the evolution. We find that the orientations of these trajectories are preferentially oriented along the $x$ direction and that, just before and after the occurrence of a slip, the orientation of the quasielliptic trajectories during the evolution is large temporal distances from $t_s$, $P(\theta)$ is sharply
peaked at $\theta \approx 90^\circ$, corresponding to an oscillatory motion in the $x$-$y$ plane. This distribution does not change significantly during the evolution and only in proximity of the slip time does it spread towards smaller values of $\theta$ (red squares and blue triangles). Therefore, when $t$ approaches $t_s$, we find the presence of oscillations also in the direction parallel to the $z$ axis ($\theta \approx 0$). This is confirmed by the behavior of the $\theta$ coordinates as functions of time and as functions of the $x$ coordinate (Fig. 2). Far from the slip (upper panels of Fig. 2), the displacement in the $z$ direction presents oscillation at the frequency $\omega_{AF}$. As already observed, $z$ displacements are small compared to the $x$ displacements and the trajectory is mostly confined in the $x$-$y$ plane ($\theta = 90^\circ$) (upper left panels of Fig. 2). At the onset of slip instability (lower panels of Fig. 2) the angle $\theta$ is no longer stable and $z$ displacements, of a size comparable to the $x$ displacements, are indeed observed. This behavior is also clearly enlightened by animations presented in the Supplemental Material [35]. The above findings support the hypothesis of weakening by AF. Indeed, when oscillations are confined in the $x$-$y$ plane ($\theta \approx 90^\circ$) they do not affect the confining pressure. Conversely, when $\theta \approx 0^\circ$, oscillations can reduce the confining pressure, promoting failure. In order to support this interpretation in Fig. 1 (lower panels) we plot the standard deviation of the $\theta$ angle,

$$\sigma(t) = \frac{1}{N} \sum_{i=1}^{N} \theta_i(t)^2 - \left(\frac{1}{N} \sum_{i=1}^{N} \theta_i(t)\right)^2,$$

where $\theta_i(t)$ is the angle formed by the velocity of the $i$th particle at a time $t$ with the $z$ axis. We find small values of $\sigma(t)$ when most of the trajectories are aligned in the $x$-$y$ plane ($\theta \approx 90^\circ$) whereas larger values are found when oscillations along the $z$ direction ($\theta \approx 0^\circ$) are observed. Figure 1 (lower panels) shows that $\sigma(t)$ typically increases before slip instabilities where $z$ oscillations increase the slip probability. A decay of $\sigma(t)$ is conversely observed at times $t > t_s$. Interestingly, we observe that the onset of the increase of $\sigma(t)$ slightly anticipates the onset of the increase of $C(t, \omega_{AF})$. This indicates that the configuration with all ellipses oriented along the $x$-$y$ direction is no longer stable since oscillations along other directions start to appear in the system. Finally, the increase of $C(t, \omega_{AF})$ immediately before $t_s$ indicates that a coherent behavior of grain trajectories is recovered at the onset of slip instability with $z$ oscillations promoting failure. These oscillations along the $z$ direction are probably activated by collisions of rattlers with backbone particles, sufficiently energetic to destabilize oscillations originally confined in the $x$-$y$ plane. The occurrence time of these collisions appear to be unpredictable.

The overall picture is confirmed by the evolution of the quasielliptic trajectories when a compressive periodic perturbation is applied at a resonant frequency $\omega_{AF}$, which increases the confining pressure of $5\%$. The response to the external perturbation clearly depends on the temporal distance from the slip instability. When the system is far from the slip, the external perturbation promotes a change in the ellipse orientation which tends to align along the $z$ direction. This is confirmed by an animation (see Supplemental Material [35]) and by the distribution of $\theta$ (Fig. 4 right panels), which, in the presence of a perturbation, moves towards smaller values of $\theta$. The same behavior is observed at all times during the stick phase. Conversely, only close to instabilities are the trajectories weakly affected by the perturbation, and indeed only small differences are found in the angle distribution $P(\theta)$ with or without the perturbation.

Summarizing, our results provide support for the AF scenario, indicating that vibrational modes at the characteristic frequency $\omega_{AF}$ are present during the entire stick phase. These vibrations affect the confining pressure only at the onset of slip instabilities, or in the presence of external perturbations, when oscillations along the $z$ direction are observed.

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[31] In our model, a single grain under normal compression is deformed as a cube. A compressional stress $\sigma_{ii}$ applied in the $i$th direction, on the two faces perpendicular to the $i$th direction, produces a deformation $2\delta x_i$, along the $i$th direction, with $k_i\delta x_i = \sigma_{ii} d^2$. As a consequence, $\delta V/V \approx -2\sum_{i=1}^{3} \delta x_i/d = -2/(d k_i) \sum_{i=1}^{3} \sigma_{ii} = -6/(d k_i) P$, where $P$ is the applied pressure.