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# Stochastic Channel Allocation for Nonlinear Systems with Markovian Packet Dropout*

**LONG Yushen · LIU Shuai · XIE Lihua · CHEN Jie**

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**Abstract** This paper addresses a channel scheduling problem for group of dynamically decoupled nonlinear subsystems with actuators connected through digital communication channels and controlled by a centralized controller. Due to the limited communication capacity, only one channel can be activated and hence there is only one pair of sensor and actuator can communicate with the controller at each time instant. In addition, the communication channels are not reliable so Markovian packed dropout is introduced. A predictive control framework is adopted for controller/scheduler co-design to alleviate the performance loss caused by the limited communication capacity. Instead of sending a single control value, the controller sends a sequence of predicted control values to a selected actuator so that there are control input candidates which can be fed to the subsystem when the actuator does not communicate with the controller. A stochastic algorithm is proposed to schedule the usage of the communication medium and sufficient conditions on stochastic stability are given under some mild assumptions.

**Keywords** Markovian packet dropout, model predictive control, networked control systems, nonlinear systems.

LONG Yushen
*School of Electrical and Electronic Engineering, Nanyang Technological University,* 639798*, Singapore.*
Email: yslong@ntu.edu.sg.
LIU Shuai
*School of Control Science and Engineering, Shandong University, Jinan* 250061*, China.*
Email: liushuai@sdu.edu.cn.
XIE Lihua
*School of Electrical and Electronic Engineering, Nanyang Technological University,* 639798*, Singapore.*
Email: elhxie@ntu.edu.sg.
CHEN Jie
*School of Automation, Beijing Institute of Technology, Beijing* 100081*, China.* Email: chenjie@bit.edu.cn.
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## 1 Introduction

Digital communication network has numerous advantages such as low cost, high reliability, interoperability of devices, and simple installation and maintenance. Therefore, it has been widely used in control systems as a communication medium between controllers and actuators/sensors. A lot of new research topics such as channel resource allocation, quantization and packet dropout have arisen in the resulted networked control systems (NCS).

For example, feedback design problem with a static quantizer is studied in [1] and the classic sector bound approach is proved to be non-conservative. In [2] the authors designed a predictive controller for a linear discrete-time plant with external disturbance where signal transmission is over a network with bit-rate constraint and random packet dropout. The stochastic stability is guaranteed provided some conditions on the packet dropout rate, prediction horizon and the spectral radius of the system matrix are satisfied. In [3], several necessary and/or sufficient conditions on the network capacity are proved for mean square stabilizability of linear discrete-time systems. In [4], the authors showed that the channel allocation problem can be solved if and only if the channel capacity is larger than the topological entropy of the system. More comprehensive literature review on control and estimation over communication networks can be found in [5] and [6].

There are two types of communication networks[7]. One is the control network which transmits a large number of small packets to a large set of nodes for time-critical requirement. The other one is the data network which transmits large data packets less frequently and it does not have hard real-time constraints. In data networks, the size of the data packet frame can be more than one thousand bytes so the quantization issue is relatively minor. On the other hand, the effects of time-delay, packet dropout and so on could be alleviated by using a predictive control framework, see, e.g., [2, 8–10]. Instead of sending a single control value, a predictive controller sends a packet which contains a sequence of predicted control values to the actuator. Such a control sequence provides a control input candidate in case when the communication is affected by packet dropouts and/or time-delay.

Robust stability with respect to packet dropout has been widely studied in the literature. For example, in [10, 11], the authors obtained stability conditions when the maximum number of consecutive packets dropouts is bounded. In recent years, stochastic packet dropout has also been analyzed for the stability of NCS. In [2], the packet dropout is modeled as i.i.d. process and sufficient conditions for mean square stability are derived for LTI systems with quantized control. In [9], $s$-th moment stability conditions are established for nonlinear systems with Markovian packet dropout.

In this paper we consider an NCS structure where a centralized controller communicates with sensors and actuators of a group of dynamically decoupled discrete-time nonlinear subsystems through a data network, which is shown in Figure 1. Such a control system structure can be found in a lot of application areas including space exploration, factory automation, tele-robots and tele-operation, and so on, see, e.g., [12, 13]. Actuators of a group of dynamically decoupled subsystems communicate with the controller through digital communication channels. In such a control system, only one channel can be activated due to limited communication resource. Therefore, at each time instant, the controller can only communicate with the sensor and actuator associated with one subsystem so scheduling the usage of the communication channel is a key issue to ensure stability. The scheduling issue in controller-actuator links under the predictive control framework for general nonlinear systems has been studied in [10, 14]. In these two works, the authors studied a single nonlinear system and each actuator corresponds to an element of the partition of the control input vector. One key assumption is that there exist a few 'good' actuators by using any one of which the system can be stabilized. However, in the dynamically decoupled case, this assumption does not hold any more if all subsystems are open-loop unstable. Furthermore, the authors also assume that the maximum number of consecutive packet dropouts is bounded. In this paper, we propose a stochastic channel scheduling strategy and adopt the predictive control idea to alleviate the negative effects of the limited communication capacity. The packet dropout is modeled as a Markovian process so the maximum number of consecutive packet dropouts could be unbounded. In [15], the authors used a similar stochastic scheduling approach for an estimation problem. However, in this work, since the systems under consideration are nonlinear, the approach based on Riccati equation used in [15] cannot be applied here. In our framework, the controller receives the measurement from a sensor associated with a subsystem and sends a sequence of predicted control values to an actuator associated with the same subsystem and the actuator has a buffer to store and process the control sequence. In this way, when the actuator does not communicate with the controller, the predicted control values can be fed to the subsystem to improve its performance. Under some mild assumptions commonly used in standard model predictive control[16], we show that stochastic stability is ensured if some design parameters are properly chosen.

The rest of this paper is organized as follows. In Section 2 we introduce the model of the NCS, including subsystem models and a buffer scheme. In Section 3 we design a stochastic selection scheme for actuators and solve a constrained optimization problem for predictive control. In Section 4 we analyze the stochastic stability of the system. In Section 5 a numerical example is given to illustrate our results. In Section 6 we draw some conclusions.

Some remarks on notations are introduced as follows. We use $\mathbb{R}$, $\mathbb{Z}$ and $\mathbb{R}^n$ to denote the set of real number, positive integers and $n$-dimensional Euclidean space, respectively. $\mathbb{R}^{m\times n}$ denotes the $m\times n$-dimensional Euclidean space. $I_n$ and $0_n$ denote the $n\times n$ identity matrix and zero matrix, respectively. For a vector $x$, $\|x\|$ denotes its Euclidean norm and $x^{\mathrm{T}}$ stands for its transpose. We use $P(\omega)$ to denote the probability of event $\omega$. For a given random variable $\xi$, $E(\xi)$ and $E(\xi|\sigma)$ denote its expectation and conditional expectation given $\sigma$, respectively. For simplicity, we define $\sum_{i=l_1}^{l_2} a_i = 0$ for any sequence $a_i,\ i=0,1,\cdots$, if $l_2 < l_1$. For a matrix $A$, $A(i,j)$ denotes its $i,j$-th component. Finally, we use $y(k+l|k),\ l=0,1,\cdots$ to indicate that $y$ is calculated based on the information available at time instant $k$.

# 2 Problem Formulation

## 2.1 Network and System Model

In this paper, we consider an NCS whose schematic diagram is shown in Figure 1.

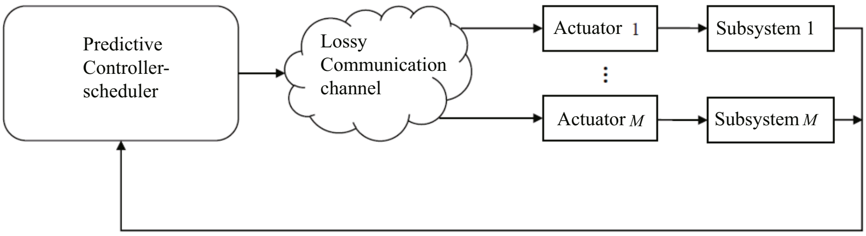


**Figure 1** System structure

The dynamic model of the $i$-th subsystem is given by

$$x_i(k+1) = f_i(x_i(k), u_i(k), w_i(k)), \quad i \in \mathcal{N}, \tag{1}$$

where $\mathcal{N} = \{1, 2, \cdots, M\}$, $x_i(k) \in \mathbb{R}^{n_i}$, $u_i(k) \in \mathbb{U}_i \subset \mathbb{R}^{m_i}$ are the state and control input respectively, $w_i(k) \in \mathbb{W}_i \subset \mathbb{R}^{n_i}$ is the disturbance. The following assumption is made for system dynamics.

**Assumption 2.1** There exist positive constants $L_{f_i}$, $L_{w_i}$ and $s$ such that

$$\|f_i(a, b, c) - f_i(a', b, c')\|^s \le L_{f_i}\|a - a'\|^s + L_{w_i}\|c - c'\|^s.$$

Furthermore, $\mathbb{U}_i$ and $\mathbb{W}_i$ are compact and contain the origin as interior point. $\mathbb{W}_i$ also satisfies that $\|w_i\|^s \le d_i^s$, $\forall w_i \in \mathbb{W}_i$.

**Remark 2.1** Assumption 2.1 is a more general type of Lipschitz condition. Indeed, any function satisfying Assumption 2.1 for $s = 1$ and some positive constants $L_{f_i}$ and $L_{w_i}$ satisfies Assumption 2.1 for any positive integer $s$ with the following inequality:

$$\|f_i(a, b, c) - f_i(a', b, c')\|^s \le 2^s(L_{f_i}\|a - a'\|^s + L_{w_i}\|c - c'\|^s).$$

In the sequel we assume that for each subsystem the sampling time instants are synchronized. The reliability of each channel is modeled as a time-homogeneous binary Markov process $\delta_i$:

$$\delta_i(k) = \begin{cases} 1, & \text{the } i\text{-th channel is unreliable at time instant } k, \\ 0, & \text{the } i\text{-th channel is reliable at time instant } k. \end{cases}$$

When a channel is called reliable, it means that a packet could be transmitted successfully through this channel. The transition probability is given by

$$\begin{aligned} &P(\delta_i(k+1) = 0|\delta_i(k) = 0) = q_i, \\ &P(\delta_i(k+1) = 1|\delta_i(k) = 0) = 1 - q_i, \\ &P(\delta_i(k+1) = 1|\delta_i(k) = 1) = p_i, \\ &P(\delta_i(k+1) = 0|\delta_i(k) = 1) = 1 - p_i. \end{aligned}$$

### 2.2 Actuator and Buffer

Since at each time instant only the sensor and actuator of one subsystem can communicate with the controller, other subsystems will work in an open-loop manner. To improve system performance during the open-loop period, we adopt the predictive control framework and assume that there is a buffer in each actuator which can be used to store a sequence of predicted control inputs. In this way, when the subsystems are running in an open-loop manner, the predicted control inputs can still be used to improve system performance. To this end, at each time instant, the controller sends a packet $\pi_{in} = \{a, U_a(k)\}$ through the communication network to the $a$-th actuator, where $a \in \mathcal{N}$ is the index of a subsystem whose control input needs to be updated, $U_a(k) = (u_a^{\rm T}(k|k), u_a^{\rm T}(k+1|k), \cdots, u_a^{\rm T}(k+N_a-1|k))^{\rm T}$ is the predicted control sequence and $N_a$ is the prediction horizon. The buffer state of the $i$-th actuator is updated as follows:

$$b_i(k+1) = \begin{cases} U_i(k+1), & \text{if } i = a, \\ S_i b_i(k), & \forall i \neq a, \end{cases} \tag{2}$$

where the shift matrix $S_i$ is defined as

$$S_i = \begin{pmatrix} 0_{m_i} & I_{m_i} & 0_{m_i} & \cdots & 0_{m_i} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0_{m_i} & \cdots & 0_{m_i} & I_{m_i} & 0_{m_i} \\ 0_{m_i} & \cdots & \cdots & 0_{m_i} & I_{m_i} \\ 0_{m_i} & \cdots & \cdots & \cdots & 0_{m_i} \end{pmatrix} \in \mathbb{R}^{m_i N_i \times m_i N_i}.$$

The outputs of actuators are given by

$$u_i(k) = (I_{m_i} 0_{m_i} \cdots 0_{m_i}) b_i(k), \quad \forall i \in \mathcal{N}. \tag{3}$$

## 3 Control and Scheduling Design

As mentioned in the previous section, the predictive controller sends a packet $\pi_{in}$ to the communication network at each time instant. In this section, we will illustrate how to determine the index $a$ and control sequence $U_a$.

We propose a randomized approach to assign the usage of the communication channel. At each time instant $k$, the $i$-th subsystem is chosen with probability $\theta_i$. Once the $i$-th subsystem is chosen, the controller receives the current state $x_i(k)$ from the corresponding sensor and solves the following optimization problem to obtain $U_i(k)$:

**Problem 3.1**

$$\min J_i(x_i(k), \widehat{U}_i(k))$$

subject to

$$\widehat{x}_i(k+l|k) = f_i(\widehat{x}_i(k+l-1|k), \widehat{u}_i(k+l-1|k), 0),$$

$$\widehat{x}_i(k|k) = x_i(k),$$
$$\widehat{u}_i(k+l-1|k) \in \mathbb{U}_i, \quad l = 1, 2, \cdots, N_i,$$

where $J_i(x_i(k), \widehat{U}_i(k)) = \sum_{j=k}^{k+N_i-1} l_i(\widehat{x}_i(j|k), \widehat{u}_i(j|k)) + V_i(\widehat{x}_i(k+N_i|k))$, $l_i$ is the stage cost and $V_i$ is the terminal cost.

Denote the optimal solution of the above problem as $U_i^*(k) = (u_i^{*\mathrm{T}}(k|k), u_i^{*\mathrm{T}}(k+1|k), \cdots, u_i^{*\mathrm{T}}(k+N_i-1|k))^{\mathrm{T}}$, the corresponding state trajectory as $X_i^*(k) = (x_i^{*\mathrm{T}}(k|k), x_i^{*\mathrm{T}}(k+1|k), \cdots, x_i^{*\mathrm{T}}(k+N_i|k))^{\mathrm{T}}$ and the optimal value $J_i^*(x_i(k)) = J_i(x_i(k), U_i^*(k))$. Then the control sequence $U_i(k)$ is set to be equal to the minimizer of the above optimization problem and transmitted to the $i$-th actuator.

## 4 Analysis

The following assumptions are made to facilitate the analysis of stability.

**Assumption 4.1** There exist positive constants $\alpha_{l_i}$, $\alpha_{V_i}$, $L_{l_i}$ and $L_{V_i}$, $\forall i \in \mathcal{N}$ such that $\forall x \in \mathbb{R}^{n_i},\ u \in \mathbb{U}_i$,

$$|l_i(x,u) - l_i(x',u)| \le L_{l_i}\|x - x'\|^s,$$
$$|V_i(x) - V_i(x')| \le L_{V_i}\|x - x'\|^s,$$
$$l_i(x,u) \ge \alpha_{l_i}\|x\|^s,$$
$$V_i(x) \ge \alpha_{V_i}\|x\|^s,$$

where $s$ is the constant defined in Assumption 2.1.

**Assumption 4.2** There exists a constrained control law $\kappa_i : \mathbb{R}^{n_i} \to \mathbb{U}_i$ such that $V_i(f_i(x_i, \kappa_i(x_i), 0)) - V_i(x_i) \le -l_i(x_i, \kappa_i(x_i))$, $\forall i \in \mathcal{N},\ x_i \in \mathbb{R}^{n_i}$.

**Assumption 4.3** There exist positive constants $\gamma_i$ and $c_i$ such that $V_i(x_i^+) \le \gamma_i V_i(x_i) + c_i d_i^s$, where $x_i^+ = f_i(x_i, 0, w_i),\ \forall x_i \in \mathbb{R}^{n_i},\ w_i \in \mathbb{W}_i$.

**Remark 4.1** Note that the constrained controller $\kappa_i$ in Assumption 4.2 will only be used to construct a feasible control sequence for stability analysis as in [16] and will not be implemented to the system.

Finally, we introduce the following lemma from [9].

**Lemma 4.2** *Suppose that Assumption* 4.2 *holds. Then*

$$l_i(x_i(k), u_i^*(k|k)) \le J_i^*(x_i(k)) \le V_i(x_i(k)), \quad \forall\ x_i(k) \in \mathbb{R}^{n_i}.$$

To derive conditions under which the system is stable, we use $E(J_i^*(x_i(k)))$ as a Lyapunov functional candidate for the $i$-th subsystem and study how it evolves. Considering the Markovian property of $E(J_i^*(x_i(k_i^j))|x_i(k_i^{j-1}))$, where $k_i^j$ denotes the $j$-th time instant when the $i$-th subsystem is chosen, without loss of generality, we can assume that $k = 0$ and $k = \Delta_i$ are the first two time instants when the actuator of the $i$-th subsystem communicates with the controller and check the relationship between $E(J_i^*(x_i(\Delta_i))|x_i(0))$ and $J_i^*(x_i(0))$.

It is obvious that $\Delta_i$ is a random variable. Denote

$$E_i = \begin{pmatrix} \theta_i p_i & (1-\theta_i)(1-p_i) & (1-\theta_i)p_i \\ \theta_i(1-q_i) & (1-\theta_i)q_i & (1-\theta_i)(1-q_i) \\ \theta_i p_i & (1-\theta_i)(1-p_i) & (1-\theta_i)p_i \end{pmatrix}.$$

**Lemma 4.3** *The probability distribution of $\Delta_i$ is given by*

$$\lambda_{i,h} \triangleq P(\Delta_i = h) = \begin{cases} \theta_i q_i, & h = 1, \\ (\theta_i(1-q_i)\ (1-\theta_i)q_i\ (1-\theta_i)(1-q_i))(E_i)^{h-2}(\theta_i(1-p_i)\ \theta_i q_i\ \theta_i(1-p_i))^{\mathrm{T}}, & h \geq 2. \end{cases}$$

*Proof* We first introduce the following augmented Markov chain $\zeta_i(k)$:

$$\zeta_i(k) = \begin{cases} 1, & \text{the } i\text{-th channel is reliable and the } i\text{-th subsystem is chosen at time instant } k, \\ 2, & \text{the } i\text{-th channel is unreliable and the } i\text{-th subsystem is chosen at time instant } k, \\ 3, & \text{the } i\text{-th channel is reliable and the } i\text{-th subsystem is not chosen at time instant } k, \\ 4, & \text{the } i\text{-th channel is unreliable and the } i\text{-th subsystem is not chosen at time instant } k. \end{cases}$$

Then we can introduce the following matrices:

$$A_i = \begin{pmatrix} \theta_i q_i & \theta_i(1-q_i) & (1-\theta_i)q_i & (1-\theta_i)(1-q_i) \\ \theta_i(1-p_i) & \theta_i p_i & (1-\theta_i)(1-p_i) & (1-\theta_i)p_i \\ \theta_i q_i & \theta_i(1-q_i) & (1-\theta_i)q_i & (1-\theta_i)(1-q_i) \\ \theta_i(1-p_i) & \theta_i p_i & (1-\theta_i)(1-p_i) & (1-\theta_i)p_i \end{pmatrix}$$

and

$$B = \begin{pmatrix} 0\ 0\ 0\ 0 \\ 0\ 1\ 0\ 0 \\ 0\ 0\ 1\ 0 \\ 0\ 0\ 0\ 1 \end{pmatrix},$$

where $A_i(m,n) = P(\zeta_i(k+1) = n | \zeta_i(k) = m)$.

When $h = 1$, it is easy to have $P(\Delta_i = 1) = \theta_i q_i$. When $h \geq 2$, note that $(1\ 0\ 0\ 0)(A_i B)$ collects the transition probability from state 1 to states 2, 3, 4, $BA_iB$ collects the transition probability from states 2, 3, 4 to states 2, 3, 4 and $(BA_i)(1\ 0\ 0\ 0)^{\mathrm{T}}$ collects the transition probability from state 2, 3, 4 to state 1. Therefore, $P(\Delta_i = h) = (1\,0\,0\,0)(A_iB)(BA_iB)^{h-2}(BA_i)(1\,0\,0\,0)^{\mathrm{T}} = (\theta_i(1-q_i)\ (1-\theta_i)q_i\ (1-\theta_i)(1-q_i))(E_i)^{h-2}(\theta_i(1-p_i)\ \theta_i q_i\ \theta_i(1-p_i))^{\mathrm{T}}$. ∎

After that, we can estimate $E(J_i^*(x_i(\Delta_i))|x_i(0))$ by considering two different cases: One is that $\Delta_i \leq N_i$, which means that the predicted control sequence is not used up so that the $i$-th subsystem does not run in a completely open-loop manner; the other one is that $\Delta_i > N_i$, which means that the predicted control sequence is used up and the $i$-th subsystem runs in a completely open-loop manner for a few steps. The expectation is just a weighted average of the two different cases.

We first consider the situation when $\Delta_i \leq N_i$. Suppose that at $k = 0$ the optimal solution of Problem 3.1 is $U_i^*(0) = (u_i^{*\mathrm{T}}(0|0), \cdots, u_i^{*\mathrm{T}}(N_i - 1|0))^{\mathrm{T}}$. At $k = \Delta_i$ we construct a feasible solution $\widetilde{U}_i(\Delta_i) \triangleq (\widetilde{u}_i^{\mathrm{T}}(\Delta_i|\Delta_i), \cdots, \widetilde{u}_i^{\mathrm{T}}(\Delta_i + N_i - 1|\Delta_i))^{\mathrm{T}} = (u_i^{*\mathrm{T}}(\Delta_i|0), \cdots, u_i^{*\mathrm{T}}(N_i - 1|0), \kappa_i^{\mathrm{T}}(\widetilde{x}_i(N_i|\Delta_i)), \cdots, \kappa_i^{\mathrm{T}}(\widetilde{x}_i(\Delta_i + N_i - 1|\Delta_i)))^{\mathrm{T}}$, where $\widetilde{x}$ is defined by

$$\widetilde{x}_i(m+1|\Delta_i) = f_i(\widetilde{x}_i(m|\Delta_i), \widetilde{u}_i(m|\Delta_i), 0),$$
$$\widetilde{x}_i(\Delta_i|\Delta_i) = x_i(\Delta_i), \quad m = \Delta_i, \Delta_i + 1, \cdots, \Delta_i + N_i - 1.$$

**Lemma 4.4** *If Assumption* 2.1 *holds, then*

$$\|\widetilde{x}_i(\Delta_i + m|\Delta_i) - x_i^*(\Delta_i + m|0)\|^s \leq L_{f_i}^m \frac{1 - L_{f_i}^{\Delta_i}}{1 - L_{f_i}} L_{w_i} d_i^s, \quad m = 0, 1, \cdots, N_i - \Delta_i.$$

*Proof* When $m = 0$, to show the inequality, we take the difference between the following equations:

$$x_i(l+1) = f_i(x_i(l), u_i^*(l|0), w_i(l)),$$
$$x_i^*(l+1|0) = f_i(x_i^*(l|0), u_i^*(l|0), 0), \quad l = 0, 1, \cdots, \Delta_i - 1.$$

It leads to that

$$\|x_i(l+1) - x_i^*(l+1|0)\|^s \leq L_{f_i}\|x_i(l) - x_i^*(l|0)\| + L_{w_i} d_i^s,$$

which implies that

$$\|\widetilde{x}_i(\Delta_i|\Delta_i) - x_i^*(\Delta_i|0)\|^s = \|x_i(\Delta_i) - x_i^*(\Delta_i|0)\|^s \leq \frac{1 - L_{f_i}^{\Delta_i}}{1 - L_{f_i}} L_{w_i} d_i^s.$$

When $m > 0$, we take the difference between the following equations:

$$\widetilde{x}_i(\Delta_i + l + 1|\Delta_i) = f_i(\widetilde{x}_i(\Delta_i + l|\Delta_i), \widetilde{u}_i(\Delta_i + l|\Delta_i), 0),$$
$$x_i^*(\Delta_i + l + 1|0) = f_i(x_i^*(\Delta_i + l|0), u_i^*(\Delta_i + l|0), 0), \quad l = 1, 2, \cdots, N_i - \Delta_i - 1.$$

It leads to that

$$\begin{aligned}\|\widetilde{x}_i(\Delta_i + m|\Delta_i) - x_i^*(\Delta_i + m|0)\|^s &\leq L_{f_i}^m \|\widetilde{x}_i(\Delta_i|\Delta_i) - x_i^*(\Delta_i|0)\|^s \\ &\leq L_{f_i}^m \frac{1 - L_{f_i}^{\Delta_i}}{1 - L_{f_i}} L_{w_i} d_i^s.\end{aligned}$$

The proof is finished. ∎

**Lemma 4.5** *Suppose that Assumptions* 2.1*,* 4.1 *and* 4.2 *hold, and* $\Delta_i \le N_i$*. Then* $J_i^*(x_i(\Delta_i)) - J_i^*(x_i(0)) \le \widetilde{\psi}_{\Delta_i,N_i} d_i^s - \sum_{m=0}^{\Delta_i-1} l_i(x_i^*(m|0), u_i^*(m|0))$*, where* $\widetilde{\psi}_{\Delta_i,N_i} = \psi_{\Delta_i,N_i} + L_{V_i} L_{f_i}^{N_i-\Delta_i} L_{w_i} \frac{1-L_{f_i}^{\Delta_i}}{1-L_{f_i}}$ *and* $\psi_{\Delta_i,N_i} = \frac{(1-L_{f_i}^{\Delta_i})(1-L_{f_i}^{N_i-\Delta_i})}{(1-L_{f_i})^2} L_{l_i} L_{w_i}$.

*Proof* According to the definitions of $J_i$ and $J_i^*$, one can write that

$$
\begin{aligned}
& J_i(x_i(\Delta_i), \widetilde{U}_i(\Delta_i)) - J_i^*(x_i(0)) \\
= & -\sum_{m=0}^{N_i-1} l_i(x_i^*(m|0), u_i^*(m|0)) - V_i(x_i^*(N_i|0)) + \sum_{m=0}^{N_i-1} l_i(x_i^*(\Delta_i+m|\Delta_i), u_i^*(\Delta_i+m|\Delta_i)) \\
& + V_i(x_i^*(\Delta_i+N_i|\Delta_i)) \\
= & \sum_{m=\Delta_i}^{N_i-1} (l_i(\widetilde{x}_i(m|\Delta_i), u_i^*(m|\Delta_i)) - l_i(x_i^*(m|0), u_i^*(m|0))) + \sum_{m=N_i}^{N_i+\Delta_i-1} l_i(\widetilde{x}_i(m|\Delta_i), \kappa_i(\widetilde{x}_i(m|\Delta_i))) \\
& - \sum_{m=0}^{\Delta_i-1} l_i(x_i^*(m|0), u_i^*(m|0)) + V_i(\widetilde{x}_i(\Delta_i+N_i|\Delta_i)) - V_i(x_i^*(N_i|0)).
\end{aligned}
\tag{4}
$$

By using Lemma 4.4, we have

$$
\sum_{m=\Delta_i}^{N_i-1} (l_i(\widetilde{x}_i(m|\Delta_i), u_i^*(m|\Delta_i)) - l_i(x_i^*(m|0), u_i^*(m|0))) \le \psi_{\Delta_i,N_i} d_i^s.
$$

Assumption 4.2 implies that $V_i(\widetilde{x}_i(\Delta_i+N_i|\Delta_i)) \le V_i(\widetilde{x}_i(\Delta_i+N_i-1|\Delta_i)) - l_i(\widetilde{x}_i(\Delta_i+N_i-1|\Delta_i), \kappa_i(\widetilde{x}_i(\Delta_i+N_i-1|\Delta_i))) \le \cdots \le V_i(\widetilde{x}_i(N_i|\Delta_i)) - \sum_{m=N_i}^{N_i+\Delta_i-1} l_i(\widetilde{x}_i(m|\Delta_i), \kappa_i(\widetilde{x}_i(m|\Delta_i)))$.

By substituting the above two inequalities to (4) one can obtain that

$$
\begin{aligned}
& J_i(x_i(\Delta_i), \widetilde{U}_i(\Delta_i)) - J_i^*(x_i(0)) \\
\le & \psi_{\Delta_i,N_i} d_i^s + V_i(\widetilde{x}_i(N_i|\Delta_i)) - V_i(x_i^*(N_i|0)) - \sum_{m=0}^{\Delta_i-1} l_i(x_i^*(m|0), u_i^*(m|0)) \\
\le & \widetilde{\psi}_{\Delta_i,N_i} d_i^s - \sum_{m=0}^{\Delta_i-1} l_i(x_i^*(m|0), u_i^*(m|0)),
\end{aligned}
$$

where we use the inequality $V_i(\widetilde{x}_i(N_i|\Delta_i)) - V_i(x_i^*(N_i|0)) \le L_{V_i} \|\widetilde{x}_i(N_i|\Delta_i) - x_i^*(N_i|0)\|^s$ and Lemma 4.4.

Finally, noticing that $J_i^*(x_i(\Delta_i)) \le J_i(x_i(\Delta_i), \widetilde{U}_i(\Delta_i))$, the desired result is proved. ▮

Then we consider the situation when $\Delta_i > N_i$. In this case, after $N_i$ steps, the control input fed into the $i$-th subsystem is 0.

**Lemma 4.6** *Suppose that Assumptions* 2.1*,* 4.1 *and* 4.3 *hold, and* $\Delta_i > N_i$*. Then* $J_i^*(x_i(\Delta_i)) - J_i^*(x_i(0)) \le (\gamma_i^{\Delta_i-N_i} - 1) V_i(x_i^*(N_i|0)) + \widetilde{\phi}_{\Delta_i,N_i} d_i^s - \sum_{m=0}^{N_i-1} l_i(x_i^*(m|0), u_i^*(m|0))$*, where* $\widetilde{\phi}_{\Delta_i,N_i} = \phi_{\Delta_i,N_i} + \frac{1-L_{f_i}^{N_i}}{1-L_{f_i}} L_{V_i} L_{w_i} \gamma_i^{\Delta_i-N_i}$ *and* $\phi_{\Delta_i,N_i} = c_i \frac{1-\gamma_i^{\Delta_i-N_i}}{1-\gamma_i}$.

*Proof* By Lemma 4.2 we have $J_i^*(x_i(\Delta_i)) \le V_i((x_i(\Delta_i))$. By using Assumption 4.3, we can obtain that

$$
V_i((x_i(\Delta_i)) \le \gamma_i V_i((x_i(\Delta_i - 1)) + c_i d_i^s
$$

$$
\begin{aligned}
&\le \gamma_i^2 V_i((x_i(\Delta_i-1)) + (\gamma_i+1)c_i d_i^s \\
&\vdots \\
&\le \gamma_i^{\Delta_i-N_i} V_i(x_i(N_i)) + c_i d_i^s \frac{1-\gamma_i^{\Delta_i-N_i}}{1-\gamma_i}.
\end{aligned}
$$

Therefore, one can write that

$$
\begin{aligned}
&\quad J_i^*(x_i(\Delta_i)) - J_i^*(x_i(0)) \\
&\le V_i((x_i(\Delta_i)) - J_i^*(x_i(0)) \\
&\le \gamma_i^{\Delta_i-N_i} V_i(x_i(N_i)) + \phi_{\Delta_i,N_i} d_i^s - V_i(x_i^*(N_i|0)) - \sum_{m=0}^{N_i-1} l_i(x_i^*(m|0), u_i^*(m|0)) \\
&\le (\gamma_i^{\Delta_i-N_i}-1) V_i(x_i^*(N_i|0)) + \phi_{\Delta_i,N_i} d_i^s + \gamma_i^{\Delta_i-N_i} L_{V_i} \|x_i^*(N_i|0) - x_i(N_i)\|^s \\
&\quad - \sum_{m=0}^{N_i-1} l_i(x_i^*(m|0), u_i^*(m|0)) \\
&\le (\gamma_i^{\Delta_i-N_i}-1) V_i(x_i^*(N_i|0)) - \sum_{m=0}^{N_i-1} l_i(x_i^*(m|0), u_i^*(m|0)) + \widetilde{\phi}_{\Delta_i,N_i} d_i^s.
\end{aligned}
$$

Now we can state our main result. Define $\mu_i = \inf_{x_i\in\mathbb{R}^{n_i}, u_i\in\mathbb{U}_i} \frac{l_i(x_i,u_i)}{V_i(x_i)}$. Note that the eigenvalues of $E_i$ are 0 and $\frac{s_i\pm\sqrt{s_i^2-4l_i}}{2}$, where $s_i = p_i + q_i - q_i\theta_i$, $l_i = (p_i + q_i - 1)(1 - \theta_i)$, and $|\frac{s_i-\sqrt{s_i^2-4l_i}}{2}| \le |\frac{s_i+\sqrt{s_i^2-4l_i}}{2}| < 1$. Define $\Theta = \{(\theta_1, \theta_2, \cdots, \theta_M) : 0 < \theta_i < 1, \sum_{i=1}^M \theta_i = 1, \gamma_i \frac{s_i+\sqrt{s_i^2-4l_i}}{2} < 1, i = 1, 2, \cdots, M\}$ and we assume that $\Theta$ is not empty.

**Lemma 4.7** *Suppose that Assumptions* 2.1, 4.1, 4.2 *and* 4.3 *hold. The probability $\theta_i$ is chosen such that $\gamma_i \frac{s_i+\sqrt{s_i^2-4l_i}}{2} < 1$, $\forall i \in \mathcal{N}$, $\sum_{i=1}^M \theta_i = 1$. Then there exists sufficiently large prediction horizon $N_i$ for each subsystem such that there exist positive constants $\rho_i < 1$ and $\Phi_{N_i}$ such that $E(J_i^*(x_i(\Delta_i))|x_i(0)) - J_i^*(x_i(0)) \le \Phi_{N_i} d_i^s - \rho_i J_i^*(x_i(0))$, $\forall i \in \mathcal{N}$.*

*Proof* By Lemmas 4.3, 4.5 and 4.6 we obtain that

$$
\begin{aligned}
&E(J_i^*(x_i(\Delta_i))|x_i(0)) - J_i^*(x_i(0)) \\
&\le \sum_{j=0}^{N_i} \lambda_{i,j} (\widetilde{\psi}_{j,N_i} d_i^s - \sum_{m=0}^{j-1} l_i^*(x_i^*(m|0), u_i^*(m|0))) + \sum_{j=N_i+1}^{+\infty} \lambda_{i,j} ((\gamma_i^{j-N_i}-1) V_i(x_i^*(N_i|0)) \\
&\quad + \widetilde{\phi}_{j,N_i} d_i^s - \sum_{m=0}^{N_i-1} l_i(x_i^*(m|0), u_i^*(m|0))).
\end{aligned}
$$

Since $\gamma_i \frac{s_i+\sqrt{s_i^2-4l_i}}{2} < 1$, $\sum_{j=N_i+1}^{+\infty} \lambda_{i,j} \widetilde{\phi}_{j,N_i}$ is finite and we denote

$$
\sum_{j=0}^{N_i} \lambda_{i,j} \widetilde{\psi}_{j,N_i} + \sum_{j=N_i+1}^{+\infty} \lambda_{i,j} \widetilde{\phi}_{j,N_i}
$$

as $\Phi_{N_i}$. Denote $\beta_{N_i,i} \triangleq \sum_{j=N_i+1}^{+\infty} \lambda_{i,j} (\gamma_i^{j-N_i} - 1)$.

Then one can write that

$$\begin{aligned}&E(J_i^*(x_i(\Delta_i))|x_i(0)) - J_i^*(x_i(0))\\ &\leq \Phi_{N_i} d_i^s + \beta_{N_i,i} V_i(x_i^*(N_i|0)) - l_i(x_i(0), u_i^*(0|0)).\end{aligned}$$

Note that $l_i(x_i(0), u_i^*(0|0)) + V_i(x_i^*(N_i|0)) \leq J_i^*(x_i(0)) \leq V_i(x_i(0))$. It leads to that

$$\begin{aligned}&E(J_i^*(x_i(\Delta_i))|x_i(0)) - J_i^*(x_i(0))\\ &\leq \Phi_{N_i} d_i^s + \beta_{N_i,i} V_i(x_i(0)) - (1 + \beta_{N_i,i}) l_i(x_i(0), u_i^*(0|0))\\ &\leq \Phi_{N_i} d_i^s + (\beta_{N_i,i} - (1 + \beta_{N_i,i})\mu_i) V_i(x_i(0)).\end{aligned}$$

Therefore, to derive the desired results, one needs to make $\rho_i \triangleq -(\beta_{N_i,i} - (1 + \beta_{N_i,i})\mu_i) > 0$, which implies that we need to chose $N_i$ such that $\beta_{N_i,i} < \frac{\mu_i}{1-\mu_i}$. Note that both $\sum_{j=1}^{\infty} \lambda_{i,j} \gamma_i^j$ and $\sum_{j=1}^{\infty} \lambda_{i,j}$ are finite, which implies that as $N_i \to \infty$, $\beta_{N_i,i} \to 0$. Therefore, such $N_i$ exists though no explicit solution can be found. Finally, note that $J_i^*(x_i(0)) \leq V_i(x_i(0))$. This completes the proof. ▮

**Theorem 4.8** *Suppose that Assumptions* 2.1*,* 4.1*,* 4.2 *and* 4.3 *hold, and $\theta_i$ and $N_i$ are chosen to satisfy conditions in Lemma* 4.7*. Then there exist positive constants $C_{1,i}$ and $C_{2,i}$ such that $\forall i \in \mathcal{N},\ j \in \mathbb{N}$,*

$$\max_{k \in \{k_i^j, k_i^j+1, \cdots, k_i^{j+1}-1\}} E(\|x_i(k)\|^s) \leq C_{1,i}(1-\rho_i)^j \|x_i(0)\|^s + C_2^i d_i^s.$$

*Proof* We first consider the time instants $0, 1, \cdots, \Delta_i - 1$ and check the quantity

$$\sum_{j=0}^{\Delta_i - 1} E(\|x_i(j)\|^s | x_i(0)).$$

By taking conditional expectation on $\Delta_i$ we have

$$\begin{aligned}&\sum_{j=0}^{\Delta_i - 1} E(\|x_i(j)\|^s | x_i(0))\\ &= \sum_{l=1}^{+\infty} \lambda_{i,l} \sum_{j=0}^{l} E(\|x_i(j)\|^s | \Delta_i = l, x_i(0))\\ &\leq \sum_{l=1}^{+\infty} \lambda_{i,l} \sum_{j=0}^{N_i - 1} E(\|x_i(j)\|^s | \Delta_i = N_i, x_i(0)) + \sum_{l=N_i}^{+\infty} \lambda_{i,l} \sum_{j=N_i}^{l} E(\|x_i(j)\|^s | \Delta_i = l, x_i(0))\\ &= \sum_{j=0}^{N_i - 1} E(\|x_i(j)\|^s | \Delta_i = N_i, x_i(0)) + \sum_{l=N_i}^{+\infty} \lambda_{i,l} \sum_{j=N_i}^{l} E(\|x_i(j)\|^s | \Delta_i = l, x_i(0)).\end{aligned}$$

By using Lemma 4.4, it can be obtained that

$$\sum_{j=0}^{N_i - 1} E(\|x_i(j)\|^s | \Delta_i = N_i, x_i(0))$$

$$\leq 2^s \sum_{j=0}^{N_i-1} E(\|x_i^*(j|0)\|^s|\Delta_i = N_i, x_i(0)) + 2^s \sum_{j=0}^{N_i-1} E(\|x_i(j) - x_i^*(j|0)\|^s|\Delta_i = N_i, x_i(0))$$
$$\leq \frac{2^s}{\alpha_{L_{l_i}}} J_i^*(x_i(0)) + \widetilde{C}_i d_i^s,$$

where $\widetilde{C}_i = 2^s(\frac{L_{f_i}(L_{f_i}^{N_i-1}-1)}{(L_{f_i}-1)^2} - \frac{N_i-1}{L_{f_i}-1})L_{w_i}$.

Assumption 4.3 implies that

$$\sum_{j=N_i}^{l} E(\|x_i(j)\|^s|\Delta_i = l, x_i(0))$$
$$\leq \frac{1}{\alpha_{V_i}} \sum_{j=N_i}^{l} V_i(x_i(j))$$
$$\leq \frac{1}{\alpha_{V_i}}(\widetilde{C}_{1,i}V_i(x_i(N_i)) + \widetilde{C}_{2,i}d_i^s)$$
$$\leq \frac{1}{\alpha_{V_i}}\left(\widetilde{C}_{1,i}V_i(x_i^*(N_i|0)) + \left(\widetilde{C}_{2,i} + \widetilde{C}_{1,i}L_{V_i}L_{w_i}\frac{1-L_{f_i}}{1-L_{f_i}^{N_i}}\right)d_i^s\right)$$
$$\leq \frac{1}{\alpha_{V_i}}\left(\widetilde{C}_{1,i}J_i^*(x_i(0)) + \left(\widetilde{C}_{2,i} + \widetilde{C}_{1,i}L_{V_i}L_{w_i}\frac{1-L_{f_i}}{1-L_{f_i}^{N_i}}\right)d_i^s\right),$$

where $\widetilde{C}_{1,i} = \frac{1-\gamma_i^{l-N_i+1}}{1-\gamma_i}$ and $\widetilde{C}_{2,i} = c_i(\frac{l-N_i+1}{1-\gamma_i} - \frac{1-\gamma_i^{l-N_i+1}}{(1-\gamma_i)^2})$.

Note that $\gamma_i\frac{s_i+\sqrt{s_i^2-4l_i}}{2} < 1$. We can conclude that

$$\sum_{j=0}^{\Delta_i-1} E(\|x_i(j)\|^s|x_i(0)) \leq C_{3,i}J_i^*(x_i(0)) + C_{4,i}d_i^s, \tag{5}$$

where $C_{3,i} = \frac{2^s}{\alpha_{L_{l_i}}} + \frac{1}{\alpha_{V_i}}\sum_{l=N_i}^{+\infty}\lambda_{i,l}\widetilde{C}_{1,i}$ and $C_{4,i} = \widetilde{C}_i + \frac{1}{\alpha_{V_i}}\sum_{l=N_i}^{+\infty}\lambda_{i,l}(\widetilde{C}_{2,i} + \widetilde{C}_{1,i}L_{V_i}L_{w_i}\frac{1-L_{f_i}}{1-L_{f_i}^{N_i}})$.

Note that Lemma 4.7 and (5) can be extended to all $k_i^j$. We have $E(J_i^*(x_i(k_i^{j+1}))|x_i(0)) \leq (1-\rho_i)^j J_i^*(x_i(0)) + \Phi_i d_i^s \sum_{l=0}^{j-1}(1-\rho_i)^l$ and $\sum_{l=k_i^j}^{k_i^{j+1}-1} E(\|x_i(l)\|^s|x_i(0)) \leq C_3^i E(J_i^*(x_i(k_i^j))|x_i(0)) + C_{4,i}d_i^s$. Combining the two inequalities together leads to

$$\sum_{l=k_i^j}^{k_i^{j+1}-1} E(\|x_i(l)\|^s|x_i(0))$$
$$\leq C_3^i(1-\rho_i)^{j-1}J_i^*(x_i(0)) + \Phi_i d_i^s \sum_{l=0}^{j-2}(1-\rho_i)^l + C_{4,i}d_i^s$$
$$\leq L_{V_i}C_3^i(1-\rho_i)^{j-1}\|x_i(0)\|^s + \Phi_i d_i^s \sum_{l=0}^{j-2}(1-\rho_i)^l + C_{4,i}d_i^s.$$

By letting $C_{1,i} = \frac{L_{V_i}C_{3,i}}{1-\rho_i}$ and $C_{2,i} = \frac{\Phi_i}{\rho_i} + C_{4,i}$, we obtain the desired result. ∎

**Remark 4.9** From Lemma 4.7, by noting that $\frac{\partial(s_i+\sqrt{s_i^2-4l_i})}{\partial\theta_i}<0$ and $\frac{\partial(s_i+\sqrt{s_i^2-4l_i})}{\partial p_i}>0$ we can see that for a subsystem with larger $\gamma_i$ and/or with larger $p_i$ a larger probability should be assigned to its actuator so that the actuator can communicate with the controller more frequently. By letting $\theta_i=1$ we obtain the well known fundamental inequality $\gamma_i p_i<1$ for mean square stability of networked control system[17]. This is consistent with our intuition that a more unstable system should be manipulated more frequently and more transmissions are needed for a more unreliable communication link. More tedious calculation could lead to that $\theta_i>1+\frac{\gamma_i p_i-1}{\gamma_i^2(\frac{q_i}{\gamma_i}-(p_i+q_i-1))}$.

**Remark 4.10** In this work we mainly consider external noises. In practice, variable time delay will also be introduced by the communication network. Variable delays can be equalized by introducing a buffer at the receiver, where data packets can be held so that all packets appear to have the same delay from the perspective of the NCS. However, all packets will appear to have a delay as large as the worst case. More detailed discussion can be found in [18].

**Remark 4.11** Recall the quantity

$$\beta_{N_i,i}\triangleq\sum_{j=N_i+1}^{+\infty}\lambda_{i,j}(\gamma_i^{j-N_i}-1).$$

First we consider the situation when there is only one plant to be controlled. In this case, $\theta_i=\theta=1,\ \lambda_{i,j}=\lambda_j=(1-q)(p^{j-2}-p^{j-1})$ and $\beta_{N_i,i}=\beta=\frac{(1-q_i)(\gamma_i-1)p_i^{N_i-1}}{1-\gamma_i p_i}$. By letting $\beta<\frac{\mu}{1-\mu}$ we rediscover the result in [9].

Then we consider the situation when there is no packet dropout. In this case, we let $p_i=1-q_i$, $p_i\to 0$ and $q_i\to 1$. We can obtain that $\lambda_{i,j}=\frac{\theta_i(1-\theta_i)^j}{1-\theta_i}$ and $\beta_{N_i}^i=\frac{(1-\theta_i)^{N_i}(\gamma_i-1)}{1-\gamma_i(1-\theta_i)}$. By letting $\beta_{N_i}^i<\frac{\mu_i}{1-\mu_i}$ we obtain our previous result in [19].

## 5 Numerical Example

Consider three open-loop unstable subsystems with dynamics:

$$\begin{aligned}x_{i,1}(k+1)&=x_{i,2}(k)+u_{i,1}(k)+w_{i,1}(k),\\x_{i,2}(k+1)&=-\mathrm{sat}(x_{i,1}(k)+a_i x_{i,2}(k))+u_{i,2}(k)+w_{i,2}(k),\end{aligned}$$

where $w_{i,1}(k),w_{i,2}(k)\in[-0.01,0.01]$, $i=1,2,3,\ k\in\mathbb{Z}$, $a_1=0,\ a_2=1,\ a_3=1.5$,

$$\mathrm{sat}(x)=\begin{cases}-1, & \text{if } x\le -1,\\ x, & \text{if } -1<x\le 1,\\ 1, & \text{if } 1<x.\end{cases}$$

Denote $x_i=(x_{i,1},x_{i,2})^{\mathrm{T}}$ and $u_i=(u_{i,1},u_{i,2})^{\mathrm{T}}$. The constraint for control input is $-0.9\le(0,1)u_i\le 0.9$. The stage and terminal cost functions are chosen as $l_i(x_i,u_i)=\|x_i\|$ and $V_i(x_i)=2\|x_i\|$. The terminal controller $\kappa_i$ is chosen as $\kappa_i(x_i)=(-x_{i,2},0.9\mathrm{sat}(x_{i,1}+a_i x_{i,2}))^{\mathrm{T}}$. The parameter $\gamma_i$ in Assumption 4.3 can be calculated as $\gamma_1=1,\ \gamma_2=1.618,\ \gamma_3=2$. Then the probability assigned to each actuator are $\theta_1 = 0.01,\ \theta_2 = 0.43,\ \theta_3 = 0.56$. The failure rate is $1 - q_i = 0.1$ and recovery rate is $1 - p_i = 0.9$. The prediction horizons are chosen as $N_1 = 5,\ N_2 = 9,\ N_3 = 7$ to satisfy the condition in Lemma 4.7. Then the initial states for each subsystem are given as $x_1(0) = (1, 2)^{\mathrm{T}},\ x_2(0) = (3, 4)^{\mathrm{T}},\ x_3(0) = (5, 6)^{\mathrm{T}}$.

In Figure 2, the states of subsystems are shown. In Figure 3 a realization of the stochastic scheduling is shown. Since subsystem 1 is marginally stable, based on our results, very little attention is paid on it to ensure its stability while for the other two subsystems, more communication and computation resources are used.

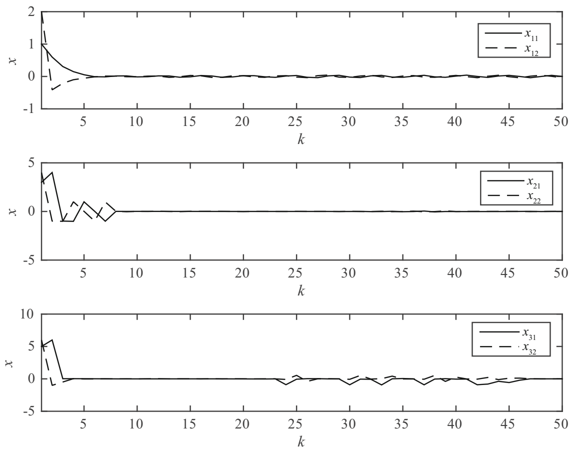


**Figure 2** State of each subsystem

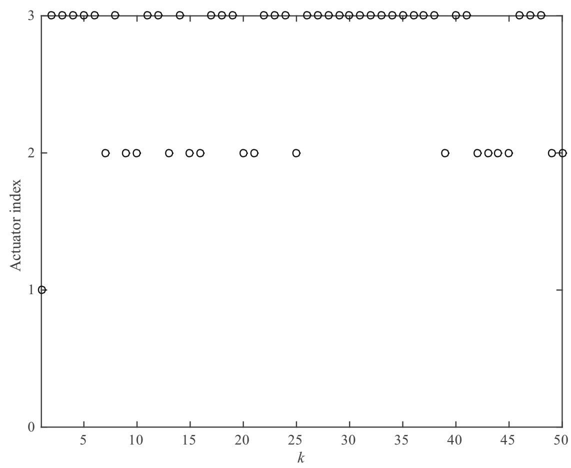


**Figure 3** A sample path of the schedule

## 6 Conclusions

In this paper, we have studied the channel scheduling problem for a group of dynamically decoupled nonlinear systems with Markovian packet dropout. A stochastic scheduling approach has been proposed to allocate the usage of the communication channels to each subsystem. By using the predictive control idea and a buffer scheme in the actuator, a packetized protocol has been designed to improve system performance during the open-loop period. Sufficient conditions on the assigned probability and prediction horizon to ensure stochastic stability have been derived. A numerical example has been given to illustrate our results. Future research could include power constraint[20] and data rate limit[21] in this framework.

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