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<th>Multi-Vector Model Predictive Power Control of Three-Phase Rectifiers with Reduced Power Ripples Under Nonideal Grid Conditions</th>
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Abstract—Model predictive power control (MPPC) is a promising control scheme for bidirectional AC/DC rectifiers. However, the control performance of conventional MPPC deteriorates under nonideal grid conditions. Furthermore, the one-switching-vector-per-control-interval characteristic of conventional MPPC leads to high ripples of control variables. To improve the steady-state performance of rectifiers under nonideal grid voltage conditions, a multi-vector model predictive power control (MV-MPPC) scheme is proposed. The proposed method presents a constant-switching-frequency and better steady-state control performance without increasing its sampling frequency. By selecting two active vectors and one zero vector, the range of optimal vector for active and reactive power regulation can be extended from fixed phase and magnitude to arbitrary phase and magnitude. Incorporated with a second-order generalized integrator (SOGI) for obtaining the quadrature component of grid voltage, the proposed method is applicable to the grid conditions where low-order harmonics exist. The preliminary calculation is adopted to avoid repetitive computation of the predicted values in the implementation of MV-MPPC, which reduces the calculation burden of the proposed scheme. A thorough experimental evaluation of the proposed scheme with the conventional MPPC (C-MPPC) and duty-optimal MPPC (DO-MPPC) has been conducted to validate the superiority of the proposed MV-MPPC solution.

Index Terms—Model predictive power control, three-phase rectifier, unbalanced grid voltages, power ripples reduction.

I. INTRODUCTION

THREE-PHASE pulse-width modulation (PWM) rectifiers have been widely used in hybrid microgrids [1], wind turbine generators [2], energy storage systems [3] over the last decades due to their advantages of bidirectional power flow, controllable power factor, sinusoidal line current, and good dc voltage regulation ability. To achieve the merits of PWM rectifiers, an appropriate control strategy is indispensable. Direct power control (DPC) is a simple and effective control strategy. It directly regulates active power and reactive power based on the instantaneous power theory, where the converter switching states are selected via a predefined switching table. Neither internal control loops nor modulator is required. However, its performance highly depends on the sampling frequency and the predefined switching table [4]. The discrete nature and the available model of the power converter are not fully taken into account.

Recently, the model predictive power control (MPPC) has been developed as a powerful alternative with accurate voltage vector selection [5], [6]. In MPPC, active power and reactive power are regulated by selecting a voltage vector that minimizes the cost function consisting of power errors between references and predicted ones. It has a good dynamic response and easy inclusion of nonlinearities and constraints. In [5], a conventional MPPC (C-MPPC) algorithm for rectifiers was presented, which requires no coordinate transformations or modulators. However, the C-MPPC method has relatively high ripples in control variables due to its characteristic of one-switching-vector-per-control-interval. To reduce the ripples of control variables and improve the steady-state control performance of C-MPPC, the sampling frequency has to be high, which results in a higher hardware cost and computational burden. Therefore, to improve the steady-state control performance without increasing its sampling frequency is one of the main challenges in the field of predictive control for power converters [7], [8].

To address the challenge, some improved MPPC strategies with enhanced steady-state control performance have been reported [9]–[15]. A duty-optimal MPPC (DO-MPPC) was presented in [14], where an extra zero voltage vector was added in one control interval to minimize the tracking errors. An extra control freedom is introduced since the active vector magnitude was adjustable. However, the phase of the composed voltage vector was still limited to the available fundamental active vectors. In order to achieve arbitrary phase of the active voltage vector, a two-step optimization method was presented in [11]. In [11], the phase of the optimal active voltage vector was determined through cost function minimization with two active voltage vectors, then the magnitude of the active vector synthesized in the former step was obtained by cost function minimization with an extra zero voltage vector. A better steady-state response was achieved, but the computation burden was very high. Unfortunately, all

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the aforementioned schemes are conducted under balanced grid conditions while there are always voltage unbalance issues in distribution networks, especially in rural areas and islanded grids. Although the steady-state performance of MPPC can be significantly improved with these methods under ideal grid conditions, the phase current can be highly distorted under unbalanced grid voltages.

The unbalanced grid has proven to be the great challenges for the control of converters. Positive- and negative-sequence components were extracted from the grid voltage and current using a phase-locked loop (PLL) to handle these problems [16]. Power compensations were added to the reference of active and reactive power to improve the performance using direct power control [17]. A new definition of instantaneous reactive power was proposed and analyzed in [18] for several DPC strategies, which can obtain sinusoidal grid currents and eliminate active power ripples under unbalanced grid conditions. To achieve two or three different control targets, a power compensation strategy was proposed for sliding-mode-based DPC in [19]. These DPC strategies share the same drawbacks of those methods with ideal grid voltage assumption as the discrete nature and the available model of the power converter are not considered.

To apply MPPC to unbalanced grid conditions, a new definition of instantaneous reactive power was adopted in both C-MPPC and DO-MPPC to obtain sinusoidal grid currents and eliminate active power ripples under unbalanced grid conditions. To achieve two or three different control targets, a power compensation strategy was proposed for sliding-mode-based DPC in [19]. These DPC strategies share the same drawbacks of those methods with ideal grid voltage assumption as the discrete nature and the available model of the power converter are not considered.

In this paper, a multi-vector MPPC (MV-MPPC) method is proposed for two-level three-phase rectifiers under unbalanced grid voltage conditions. The proposed method presents a constant switching frequency and better steady-state control performance without increasing its sampling frequency. The extraction of complex positive and negative sequence from the grid voltage and current is eliminated. The following list summarizes the contributions of the paper:

1) Multiple vector selection and their optimal duration calculation are proposed for better tracking of the power reference under unbalanced grid conditions. This method extends the range of the optimal voltage vector selection for active and reactive power regulation from fixed phase and magnitude to arbitrary phase and magnitude;
2) Incorporated with a second-order generalized integrator (SOGI) for obtaining the quadrature component of grid voltage, the proposed method is applicable to the grid conditions where the low-order harmonics exist;
3) Preliminary calculation is adopted to avoid repetitive computation of the predicted values in the implementation of MV-MPPC, which can reduce the calculation burden of the proposed scheme;
4) A thorough experimental evaluation of the proposed scheme with the conventional MPPC and DO-MPPC is conducted under unbalanced as well as distorted grid conditions. The experimental results confirm that the tracking errors and the quality of control variables can be considerably improved in comparison with both the C-MPPC and DO-MPPC methods.

II. SYSTEM DESCRIPTION AND MODELING

The topology of a three-phase rectifier is shown in Fig. 1, where $e_a$, $e_b$, and $e_c$ are the three-phase grid voltages; $L_s$ and $R_s$ are the filter inductance and equivalent resistance, respectively.

A. Rectifier Model

The relationship between the grid voltage and line current can be expressed in the $\alpha$-$\beta$ reference coordinate as

$$\frac{di}{dt} = -\frac{R_s}{L_s}i + \frac{1}{L_s} e - \frac{1}{L_s} v$$

(1)

where $i = i_\alpha + j \cdot i_\beta$ is the input current vector, $e = e_\alpha + j \cdot e_\beta$ is the grid voltage vector, $v = v_\alpha + j \cdot v_\beta$ is the voltage vector generated by the rectifier.

According to the instantaneous power theory [22], the output active and reactive power can be expressed as

$$P = \frac{3}{2} \Re (i^* e^*);$$

(2)

$$Q = \frac{3}{2} \Im (i^* e^*).$$

(3)

where $^*$ denotes the conjugate of a complex vector.

B. Power Analysis in Synchronous Frame

In unbalanced grid, voltage and current can be expressed as the sum of positive and negative sequence vectors [23]–[25]

$$e = e_{\alpha\beta}^+ + e_{\alpha\beta}^- = e_{dq}^+ e^{j\omega t} + e_{dq}^- e^{-j\omega t}$$

(4)

$$i = i_{\alpha\beta}^+ + i_{\alpha\beta}^- = i_{dq}^+ e^{j\omega t} + i_{dq}^- e^{-j\omega t}$$

(5)

where $e_{dq}^+ = e_\alpha^+ + j e_\beta^+$, $e_{dq}^- = e_\alpha^- + j e_\beta^-$, $i_{dq}^+ = i_\alpha^+ + j i_\beta^+$, $i_{dq}^- = i_\alpha^- + j i_\beta^-$, and $e_{\alpha\beta}^+$ and $e_{\alpha\beta}^-$ are the positive- and negative-sequence components of the grid voltage and current in the stationary frame. $e_{dq}^+$, $e_{dq}^-$, $i_{dq}^+$, and $i_{dq}^-$ are the positive- and negative-sequence components of the grid voltage and current in the synchronous frame. $\omega$ is the angular frequency.
The power supplied from the grid can be calculated as

\[
P = \frac{3}{2} \text{Re}(i^*e)
\]

\[
= \frac{3}{2} \text{Re} \left( (i_{dq}^+ e^{j\omega t} - i_{dq}^- e^{-j\omega t})(e_{dq}^+ e^{j\omega t} + e_{dq}^- e^{-j\omega t}) \right)
\]

\[
= P_0 + P_{c2} \cos (2\omega t) + P_{s2} \sin (2\omega t)
\]

(6)

\[
Q = \frac{3}{2} \text{Im}(i^*e)
\]

\[
= \frac{3}{2} \text{Im} \left( (i_{dq}^+ e^{j\omega t} - i_{dq}^- e^{-j\omega t})(e_{dq}^+ e^{j\omega t} + e_{dq}^- e^{-j\omega t}) \right)
\]

\[
= Q_0 + Q_{c2} \cos (2\omega t) + Q_{s2} \sin (2\omega t)
\]

(7)

where

\[
\begin{align*}
P_0 &= \frac{3}{2} \left( e_{q\alpha}^+ i_{q\alpha}^- + e_{q\alpha}^- i_{q\alpha}^+ + e_{q\beta}^+ i_{q\beta}^- + e_{q\beta}^- i_{q\beta}^+ \right) \\
P_{c2} &= \frac{3}{2} \left( k_1 \cos (2\omega t) + k_2 \sin (2\omega t) \right) \\
P_{s2} &= \frac{3}{2} \left( -k_4 \cos (2\omega t) + k_5 \sin (2\omega t) \right) \\
Q_0 &= \frac{3}{2} \left( e_{q\alpha}^+ i_{q\beta}^- - e_{q\alpha}^- i_{q\beta}^+ + e_{q\beta}^+ i_{q\alpha}^- - e_{q\beta}^- i_{q\alpha}^+ \right) \\
Q_{c2} &= \frac{3}{2} \left( k_3 \sin (2\omega t) + k_4 \sin (2\omega t) \right) \\
Q_{s2} &= \frac{3}{2} \left( -k_6 \cos (2\omega t) + k_7 \sin (2\omega t) \right)
\end{align*}
\]

(8)

\[
P_0 \text{ and } Q_0 \text{ are the dc component of the active and reactive power, and } P_{c2}, P_{s2}, Q_{c2} \text{ and } Q_{s2} \text{ are the ripples of active and reactive power, respectively.}
\]

C. Power Analysis in Stationary Frame

In order to simplify the calculation process, (8) can be represented in the stationary frame using the quadrature grid voltage signals.

\[
\begin{align*}
e_{dq}^+ &= \frac{1}{2} \begin{bmatrix} e_{q\alpha}^+ & j e_{q\beta}^+ \\ e_{q\beta}^- & -j e_{q\alpha}^- \end{bmatrix} e_{q\alpha}^+ \\
e_{dq}^- &= \frac{1}{2} \begin{bmatrix} e_{q\alpha}^- & j e_{q\beta}^- \\ e_{q\beta}^+ & -j e_{q\alpha}^+ \end{bmatrix} e_{q\alpha}^- \\
i_{dq}^+ &= \frac{1}{2} \begin{bmatrix} i_{q\alpha}^+ & j i_{q\beta}^+ \\ i_{q\beta}^- & -j i_{q\alpha}^- \end{bmatrix} i_{q\alpha}^+ \\
i_{dq}^- &= \frac{1}{2} \begin{bmatrix} i_{q\alpha}^- & j i_{q\beta}^- \\ i_{q\beta}^+ & -j i_{q\alpha}^+ \end{bmatrix} i_{q\alpha}^-
\end{align*}
\]

(9)

\[
e_{dq}^+ \text{ represents the quadrature signals that lag } e_{q\alpha} \text{ by 90 electrical degrees, and } i_{dq}^+ \text{ denotes the quadrature signals that lag } i_{q\alpha} \text{ by 90 electrical degrees.}
\]

Substituting (9) and (10) into (8), the active and reactive power can be denoted in the stationary frame.

\[
\begin{align*}
P_0 &= \frac{3}{4} \left( i_{q\alpha} e_{q\alpha} + j i_{q\beta} e_{q\beta} + i_{q\alpha} e_{q\alpha} + i_{q\beta} e_{q\beta} \right) \\
P_{c2} &= \frac{3}{4} \left( k_1 \cos (2\omega t) + k_2 \sin (2\omega t) \right) \\
P_{s2} &= \frac{3}{4} \left( -k_4 \cos (2\omega t) + k_5 \sin (2\omega t) \right) \\
Q_0 &= \frac{3}{4} \left( i_{q\alpha} e_{q\beta} - j i_{q\beta} e_{q\alpha} + i_{q\alpha} e_{q\beta} - i_{q\beta} e_{q\alpha} \right) \\
Q_{c2} &= \frac{3}{4} \left( k_3 \sin (2\omega t) + k_4 \sin (2\omega t) \right) \\
Q_{s2} &= \frac{3}{4} \left( -k_6 \cos (2\omega t) + k_7 \sin (2\omega t) \right)
\end{align*}
\]

(11)

\[
\begin{align*}
k_1 &= \left( i_{q\alpha} e_{q\alpha} + i_{q\beta} e_{q\beta} - i_{q\alpha} e_{q\alpha} - i_{q\beta} e_{q\beta} \right) \\
k_2 &= \left( i_{q\alpha} e_{q\alpha} + i_{q\beta} e_{q\beta} + i_{q\alpha} e_{q\alpha} + i_{q\beta} e_{q\beta} \right) \\
k_3 &= \left( i_{q\alpha} e_{q\beta} - j i_{q\beta} e_{q\alpha} - i_{q\alpha} e_{q\beta} - j i_{q\beta} e_{q\alpha} \right) \\
k_4 &= \left( i_{q\beta} e_{q\alpha} + j i_{q\alpha} e_{q\beta} + i_{q\beta} e_{q\alpha} + j i_{q\alpha} e_{q\beta} \right)
\end{align*}
\]

(12)

\[
i_{q\alpha}, i_{q\beta}, e_{q\alpha}, e_{q\beta} \text{ are the } q\alpha \beta \text{ components of grid current and voltage, respectively.}
\]

D. Power Compensation with Active Power Ripple Elimination

Under balanced grid voltage conditions, the control objective for a rectifier is to track the references of active and reactive power accurately and quickly. However, the trajectory of grid voltage is an ellipse under unbalanced grid conditions. If the active and reactive power are forced to be constant, the grid current will be highly distorted. In other words, constant active and reactive power control, balanced and sinusoidal current control cannot be achieved simultaneously under unbalanced grid voltage conditions. Therefore, various control targets have been proposed under unbalanced grid voltage conditions according to the application requirement, such as active power ripple elimination (APRE) [26], reactive power ripple elimination [27], balanced and sinusoidal current control [24].

Since the active power reference is obtained from the dc-link controller, elimination of active power oscillation helps to achieve better dc-link voltage control. In this work, the main concerns in unbalanced grid conditions are to guarantee the dc-link voltage regulation and obtain sinusoidal current control. In order to achieve this goal, the ripple of active power should be eliminated. In other words, \(P_{c2}\) and \(P_{s2}\) in (11) are zero and the dc component of the active power \(P_0\) should be equal to the active power reference \(P_{ref}\). The control objectives of the APRE can be expressed by solving the following equations.

\[
\begin{align*}
P_0 &= P_{ref} \\
Q_0 &= 0 \\
P_{c2} &= 0 \\
P_{s2} &= 0
\end{align*}
\]

(13)

Substituting (11) into (13), the current reference can be obtained as

\[
i_{q\alpha, q\beta}^{ref} = 2 \frac{P_{ref} (e_{q\beta} - j e_{q\alpha})}{3 e_{q\alpha} e_{q\beta} - e_{q\alpha} e_{q\beta}}
\]

(14)

Then, the new active and reactive power references under unbalanced grid voltage can be recalculated as

\[
P_{ref, new}^{active} = \frac{3}{2} \text{Re}(i_{q\alpha, q\beta}^{ref} e)
\]

(15)

\[
Q_{ref, new}^{active} = \frac{3}{2} \text{Im}(i_{q\alpha, q\beta}^{ref} e)
\]

(16)

Thus, the power compensation for the unbalanced grid voltage can be obtained

\[
P^{com} = P_{ref, new}^{active} - P_{ref} = 0
\]

(17)
\[ Q_{\text{com}} = Q_{\text{ref}}^{\text{new}} - Q_{\text{ref}}^{\text{f}} = e_{\alpha}^e e_{\alpha}^{e'} + e_{\beta}^e e_{\beta}^{e'} P_{\text{pref}} \]  

(18)

As shown in (17) and (18), the compensated active and reactive power for unbalanced grid conditions can be expressed by grid voltage and its quadrature signal, and the current signal is eliminated.

In order to obtain the quadrature signal of the grid voltage, a SOGI is adopted. The SOGI can generate the 90 electrical degrees lagged signal of the grid voltage and can also serve as a band-pass filter, which makes the proposed method perform well under non-ideal grid conditions with low-order harmonics (5-th, 7-th,...). The basic SOGI layout, shown in Fig. 2, has widely been used for the grid synchronization. \( e_m \) is the measured grid voltage value, \( e \) is the filtered value of \( e_m \), and \( e' \) is the quadrature signal of \( e_m \). The relation between \( e \) and \( e_m \) can be denoted using a transfer function as follows:

\[ G(s) = \frac{e}{e_m} = \frac{k\omega_o s}{s^2 + k\omega_o s + \omega_o^2} \]  

(19)

where \( k \) is the desired damping factor, and \( \omega_o \) is the frequency at which \( G(j\omega_o) \) peaks at unity gain with zero phase shift. Therefore, \( e \) is in phase with \( e_m \) at \( \omega_o \).

### E. Active and Reactive Power Prediction

The delayed grid voltage value can be expressed from (5)

\[ e' = e_{dq}^e e^{j(\omega t - \pi/2)} + e_{dq}^e e^{-j(\omega t - \pi/2)} \]  

(20)

\[ = -j e_{dq}^e e^{j\omega t} + j e_{dq}^e e^{-j\omega t} \]  

(21)

Then the time derivative of grid voltage vector can be obtained

\[ \frac{de}{dt} = j\omega e_{dq}^e e^{j\omega t} - j\omega e_{dq}^e e^{-j\omega t} = -\omega e' \]  

(22)

The dynamic of active and reactive power can be obtained from (1) and (22)

\[ \frac{dP}{dt} = \frac{3}{2L_s} \left| e \right|^2 - \text{Re}(v^* e) - \frac{R_s}{L_s} P - \frac{3}{2}\omega \text{Re}(i^* e') \]  

(23)

\[ \frac{dQ}{dt} = -\frac{3}{2L_s} \text{Im}(v^* e) - \frac{R_s}{L_s} Q - \frac{3}{2}\omega \text{Im}(i^* e') \]  

(24)

where the admissible voltage vectors \( v \), the number of which are in accord with the number of the switching states, are within a finite set. The MPPC is to select the proper switching state for active and reactive power regulation.

### III. Multi-Vector Model Predictive Power Control

After the power reference compensation, the APRE and sinusoidal current under unbalanced grid conditions can be achieved. For a grid-connected converter, the control objectives in the inner loop are active and reactive power tracking. Meanwhile, lower current ripples are required to meet the requirement of the grid code. To achieve this goal, the cost function is defined to select the optimal voltage vector for active and reactive power tracking

\[ J = (P_{\text{pref}} + P_{\text{com}} - P_{k+1})^2 + (Q_{\text{pref}} + Q_{\text{com}} - Q_{k+1})^2 \]  

(25)

where \( P_{\text{pref}} \) and \( Q_{\text{pref}} \) are the references value of active and reactive power. The active power reference is obtained by the dc-link voltage controller and the reactive power reference is given directly. \( P_{\text{com}} \) and \( Q_{\text{com}} \) are the compensated values of the active and reactive power during unbalanced grid conditions in the next control interval.

In the following, with the same cost function of (25), the C-MPPC, DO-MPPC, and the proposed MV-MPPC are introduced.

#### A. C-MPPC

In the C-MPPC [27], all the eight switching states are evaluated by the cost function (25) and the one with the minimized cost function will be applied in the next control period. As shown in Fig. 4 (a), the phase and magnitude of the voltage vector are fixed. If the optimal voltage vector is far away from these eight vectors, the power ripples will be significant.

#### B. DO-MPPC

The DO-MPPC, as presented in [20], is a variant of C-MPPC, which uses one active and one zero vector to minimize the cost function. This method gives an extra magnitude freedom of the optimal voltage vector, which, consequently leads to better steady-state performance than that of C-MPPC. As shown in Fig. 4 (b), the phase of equivalent voltage vector other than the fundamental voltage vector cannot be synthesized. Thus, the improvement of its state-state performance is limited.
C. MV-MPC

In this work, the MV-MPC is applied to the rectifier under unbalanced grid conditions. This method gives both phase and magnitude freedom to the optimal voltage vector selection. The main ideas are presented as follows. Assuming the optimal voltage vector that minimizes (25) is with arbitrary phase and magnitude within the hexagon, this vector can be synthesized by two adjacent active vectors and one zero vector. Based on this idea, the scheme is carried out by three steps: 1) two active vectors selection, 2) duration calculation of the active vectors and the zero vectors 3) duty cycle calculation. Detailed descriptions are given in the follows. The whole control diagram is depicted in Fig.3.

1) Voltage vectors selection: Two active voltage vectors are selected based on (25). One of the optimal active voltage vectors \( v_o \) is the one with the lowest value of cost function. Noted that the optimal voltage vector should lie in the sector that is adjacent to \( v_o \). Therefore, the other active voltage vector \( v_{o2} \) should be adjacent to \( v_o \). To simplify the calculation, \( v_{o2} \) is selected from the adjacent vector of \( v_o \) with a lower value of cost function.

2) Duration calculation: After determining the two active voltage vectors, the duration of each voltage vector should be calculated. The duration of each vector is obtained through power ripple root-mean-square minimization.

The derivatives of the active and reactive power (23) and (24) can be written as

\[
\begin{align*}
    s_{pi} & = \frac{dP(v_i)}{dt} \\
    s_{qi} & = \frac{dQ(v_i)}{dt}
\end{align*}
\]  

where \( i \in \{0, 1, ..., 7\} \).

The active and reactive power at the end of the control period can be expressed by the derivatives of the active and reactive power

\[
\begin{align*}
    P(k+1) & = P(k) + s_{p1}t_1 + s_{p2}t_2 + s_{p0}t_0, \\
    Q(k+1) & = Q(k) + s_{q1}t_1 + s_{q2}t_2 + s_{q0}t_0,
\end{align*}
\]

where \( s_{p1}, s_{p2}, \) and \( s_{p0} \) are the active power derivatives of voltage vectors \( v_o1, v_o2, \) and zero vector respectively, \( s_{q1}, s_{q2}, \) and \( s_{q0} \) are the reactive power derivatives of voltage vectors \( v_o1, v_o2, \) and zero vector respectively.

\[
t_0 = T_s - t_1 - t_2,
\]

where \( t_1 \) is the duration of voltage vector \( v_o1, t_2 \) is the duration of voltage vector \( v_o2, \) and \( t_0 \) is the duration of the zero vector. \( T_s \) is the sampling period.

The active and reactive power errors can be defined as

\[
\begin{align*}
    \Delta P &= P^{ref} + P^{com} - (P(k) + s_{p1}t_1 + s_{p2}t_2 + s_{p0}t_0), \\
    \Delta Q &= Q^{ref} + Q^{com} - (Q(k) + s_{q1}t_1 + s_{q2}t_2 + s_{q0}t_0).
\end{align*}
\]  

The phase and magnitude of the optimal vector is obtained in an indirect way via the calculation of the duration time of \( t_1, t_2, \) and \( t_0 \). The optimal duration time of voltage vector \( v_o1 \) and \( v_o2 \) are obtained by minimizing the sum of squared power errors [12].

\[
J(t_1, t_2) = \Delta P^2 + \Delta Q^2.
\]  

The optimal duration times should satisfy the minimum value conditions

\[
\begin{align*}
    \frac{\partial J(t_1, t_2)}{\partial t_1} &= 0, \\
    \frac{\partial J(t_1, t_2)}{\partial t_2} &= 0.
\end{align*}
\]

The solution of (34) is given by

\[
\begin{align*}
    t_1 &= \frac{t_{num1}}{t_{den}}, \\
    t_2 &= \frac{t_{num2}}{t_{den}},
\end{align*}
\]

where \( t_{num1}, t_{num2}, \) and \( t_{den} \) are denoted in (36).

3) Duty Cycle Calculation: After determining the two voltage vectors and their optimal duration time, the duty cycles of the three-phase converter are obtained. In order to further reduce the active power and reactive power ripples and obtain a fixed switching frequency, the zero voltage vector is divided into two parts: \( t_0/2 \) for \( v_0 \) and \( t_0/2 \) for \( v_7 \) to achieve a constant switching frequency. It is noted that although two adjacent active vectors and two zero vectors are adopted in this method, it is different from the space-vector pulse width modulation (SVPWM). In the SVPWM, a desired voltage vector is always obtained using a linear controller and is decomposed into basic vectors using the voltage-second equilibrium. In the proposed scheme, the duration time is directly calculated based on the power reference tracking errors minimization. Therefore, the dynamic response of MV-MPC is much faster than that of SVPWM and the controller parameters tuning process is eliminated.

4) Delay Compensation: In digital implementation of MPPC, there is one-step delay between the commanding voltage vector and the applied voltage vector [28] caused by the computation. In order to eliminate the adverse influence caused by the one-step delay, the voltage vector at \( (k+2) \) instant rather than that at \( (k+1) \) instant is applied to the converter in the cost function of (25). Based on the voltage-second equilibrium, the optimal voltage vector calculated at \( k \) instant is

\[
v_{conv} = \frac{v_o1t_1 + v_o2t_2}{T_s}.
\]

Substituting (37) into (1), the current at time instant \( (k+1) \) is obtained. The grid voltage at time instant \( (k+1) \) is equal to \( e(k+1) = e(k)e^{j\omega T_s} \). Based on this knowledge, all the signals including grid current, grid voltage, active power and reactive power at the \( (k+1) \) instant, which serve as the initial state for voltage vector selection and duty cycle calculation, can be calculated. More details regarding the computation compensation can refer to [28].

Finally, the flowchart of the proposed scheme is given in Fig. 5, where \( i \in \{1, 2, ..., 6\} \) if \( i - 1 = 0, i - 1 = 6; \) if \( i + 1 = 7, i + 1 = 1 \). In order to avoid repetitive computation of the prediction values, all the cost function values are obtained as preliminary calculation to reduce the calculation burden.
\[
\begin{align*}
 t_{\text{num}1} &= (s_{q2} - s_{q0})(P_{\text{ref}} + P_{\text{com}} - P(k)) + (s_{p0} - s_{p2})(Q_{\text{ref}} + Q_{\text{com}} - Q(k)) + (s_{q0}s_{p2} - s_{q2}s_{p0})T_s \\
 t_{\text{num}2} &= (s_{q0} - s_{q1})(P_{\text{ref}} + P_{\text{com}} - P(k)) + (s_{p1} - s_{p0})(Q_{\text{ref}} + Q_{\text{com}} - Q(k)) + (-s_{q0}s_{p1} + s_{q1}s_{p0})T_s \\
 t_{\text{dcen}} &= s_{q0}s_{p2} - s_{q1}s_{p2} - s_{q2}s_{p0} + s_{q1}s_{p0} - s_{q0}s_{p1} + s_{q2}s_{p1}.
\end{align*}
\]

(36)

TABLE I: Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filter inductance</td>
<td>10 mH</td>
<td>Dead Time</td>
<td>1 μs</td>
</tr>
<tr>
<td>Equivalent resistance</td>
<td>0.5 Ω</td>
<td>Source frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Capacitor (C_1,C_2)</td>
<td>1410 (\mu)F</td>
<td>Sampling frequency</td>
<td>20 kHz</td>
</tr>
<tr>
<td>Source voltage (RMS)</td>
<td>110 V</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

B. Steady Performance Evaluation

Fig. 7 presents the experimental results of MPPC without APRE under the unbalanced grid for (a) C-MPPC (b) DO-MPPC (c) MV-MPPC, where the MPPC without APRE represents the condition that \(Q_{\text{com}}\) and \(P_{\text{com}}\) are equal to zero in (25). A voltage dip of 20% p.u. in phase ‘c’ is imposed to the ideal grid voltage at 0.04 s. As shown in Fig. 7, all the MPPC schemes can achieve accurate tracking of power references under the balanced and unbalanced grid. However, the proposed MV-MPPC shows a drastic reduction of active and reactive power ripples. But the average switching frequency is also increased as shown in Table II where the switching frequency of MV-MPPC is constant. The average switching frequencies of C-MPPC and DO-MPPC are obtained in a way that the total numbers of the switching time \(n_f\) are recorded during a time period \(t_f = 0.2\) s and the averaging switching frequency \(f_a\) is obtained through \(f_a = \frac{n_f}{t_f}\). The use of MV-MPPC is a tradeoff between high switching frequency and high power ripples, which depends on the specific applications.

At 0.04 s, the three-phase grid voltages become unbalanced with a voltage dip of 20% p.u. in phase ‘c’, the current is distorted regardless of the type of MPPC utilized. From the results, it is easy to conclude that it is not suitable to apply the control scheme for balanced grid conditions to that of unbalanced conditions.

Fig. 8 presents the experiment results of MPPC with APRE under unbalanced grid for (a) C-MPPC (b) DO-MPPC (c) MV-MPPC, where the MPPC with APRE represents the condition that \(Q_{\text{com}}\) and \(P_{\text{com}}\) are equal to calculated values in (17) and (18). As shown in Fig. 7, all the MPPC schemes can achieve

![Flowchart of the proposed MV-MPPC.](image1)

![Experimental setup.](image2)

IV. PERFORMANCE EVALUATIONS

In this section, the effectiveness of the proposed MV-MPPC and its performance comparison with C-MPPC and DO-MPPC are evaluated with experimental results. The parameters are collected in Table I. The experimental setup is shown in Fig. 6. All the results are obtained at 20 kHz sampling rate except the data for Fast Fourier Transform (FFT) which are sampled at 1 MHz.

A. Experimental Setup

Both simulation and experimental studies are carried out to evaluate the three MPPC schemes, only experimental evaluations are presented to demonstrate the advantages of the proposed scheme since they can give a better representation of the control performance in the presence of model uncertainty, measurement noise and inverter dead time, etc. DSPACE 1006 has been used to accomplish the control algorithms. All the experimental data were sampled by the ADC board DS2004, saved by the DSPACE control desk with a sampling period of 20 kHz, and plotted by a scientific plotting software Origin. The switching sequence is generated by an up-and-down counter and a comparator, which is similar with a triangular carrier modulation method. The grid voltage unbalance is achieved by the programmable AC source CHROMA 61512. The main circuit of the rectifier is built up with discrete IGBTs (Infineon IKW40N65H5).
TABLE II: Quantitative Comparison of The Three MPPC

<table>
<thead>
<tr>
<th>Scheme</th>
<th>C-MPPC</th>
<th>DO-MPPC</th>
<th>MV-MPPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active power tracking errors (RMS, W)</td>
<td>51.7</td>
<td>25.9</td>
<td>25.3</td>
</tr>
<tr>
<td>Reactive power tracking errors (RMS, var)</td>
<td>94.5</td>
<td>42.8</td>
<td>17.4</td>
</tr>
<tr>
<td>Current THD</td>
<td>6.85</td>
<td>3.97</td>
<td>2.74</td>
</tr>
<tr>
<td>Switching frequency (kHz)</td>
<td>4.25</td>
<td>11.45</td>
<td>20</td>
</tr>
</tbody>
</table>

accurate tracking of power references under the balanced and unbalanced grid. The reactive power keeps constant under the balanced grid and oscillates at twice grid frequency under unbalanced grid while the active power keeps constant in both cases. After grid voltage becomes unbalanced, all the grid currents can keep sinusoidal. Again, the best steady-state performance is obtained by the proposed MV-MPPC with the lowest power ripples among the three schemes. From the results, it can be concluded that the MPPC with APRE can produce sinusoidal current while keeping the active power constant, and the MV-MPPC can significantly improve the current quality and reduce the power ripple, however, at the cost of the higher switching frequency. The active and reactive power tracking errors of the MPPC are shown in Fig. 9. The control performance in terms of tracking errors is evaluated using the root-mean-square (RMS) value of the tracking errors during two current periods. In the results, the MV-MPPC presents the best tracking performance especially in terms of non-constant reactive power reference tracking.

It is worthwhile to verify the control performance of the
With the control freedom of the amplitude and magnitude, the \( \text{THD} \) of C-MPPC is up to 6.85%, and that of DO-MPPC is 3.97%. As shown in Fig. 11, the current total harmonic distortion (THD) by the MATLAB with a sampling frequency of 1 MHz. As compared to (a) C-MPPC (b) DO-MPPC (c) MV-MPPC with APRE and control parameters of (a) C-MPPC (b) DO-MPPC (c) MV-MPPC with APRE, the proposed scheme under unbalanced grid conditions with low-order harmonics. Fig. 10 presents the MPPC performance under non-ideal grid conditions with 2.45% 5-th and 3.95% 7-th order harmonics for (a) C-MPPC (b) DO-MPPC (c) MV-MPPC. As shown in Fig. 10, the MPPC with the SOGI can achieve accurate tracking of power references and sinusoidal grid current under the grid conditions with low-order harmonics. However, the best steady-state performance is obtained by the proposed MV-MPPC with the lowest power ripples among the three schemes. From the results, it can be concluded that the MV-MPPC can perform well even with low-order harmonics in the grid voltage.

To further demonstrate the steady-state control performance, the harmonic spectrum of grid current for quantitative comparison of (a) C-MPPC (b) DO-MPPC (c) MV-MPPC with APRE under unbalanced grid conditions are presented in Fig. 11 at \( P_{\text{ref}} = 1500\text{W} \) and \( Q_{\text{ref}} = 0\text{var} \). The current utilized for analysis is directly measured by current probes and analyzed by the MATLAB with a sampling frequency of 1 MHz. As shown in Fig. 11, the current total harmonic distortion (THD) of C-MPPC is up to 6.85%, and that of DO-MPPC is 3.97%. With the control freedom of the amplitude and magnitude, MV-MPPC has shown that the best current quality can be achieved with a THD of 2.74%. It should be noted that for C-MPPC and DO-MPPC, using only one or two voltage vectors per one control period, the harmonics are distributed broadly, which are not easy to be filtered. On the contrary, by using the MV-MPPC scheme, the switching frequency can be constant. The current harmonics concentrate on the 400th-, 800th-, 1200th-order harmonics, which are in accordance with the sampling frequency. This brings some benefits for the design of filters.

C. Transient Performance Evaluation

Apart from the steady-state performance evaluation, the dynamic performance evaluation of the MPPC with APRE is also conducted and shown in Fig. 12, where the active power reference changes from 1500 W to 2000 W suddenly. It is clearly seen that MV-MPPC exhibits a very similar quick response to C-MPPC and DO-MPPC in spite of 90-degree delay in grid voltage when calculating \( Q_{\text{com}} \). The current of the MV-MPPC has much fewer harmonics with lower active and reactive ripples due to the freedom of the phase and magnitude that MV-MPPC offered. From the results, it can...
be concluded that the proposed MV-MPPC has no adverse effect on the dynamic response which is further verified by the zoomed figure in a small time window. The experiment results in C-MPPC, DO-MPPC, and MV-MPPC are in accordance very well with the theoretical study presented in section III, demonstrating the superiority of the proposed MV-MPPC.

V. CONCLUSION

This paper has proposed an MV-MPPC method to enhance the steady-state performance of a three-phase rectifier under nonideal grid conditions by extending the optimal voltage vector from fixed phase and magnitude to arbitrary phase and magnitude. The proposed method can achieve constant active power and sinusoidal grid currents with low power ripples and current THD under nonideal grid conditions. Compared to the prior methods, the proposed method has the following advantages. 1) In each sampling period, the proposed strategy calculates the converter switching time that minimizes the cost function defined as the sum of squared power errors, leading to minimized tracking errors. 2) By selecting two active voltage vectors and one zero voltage vector, the proposed method extends the optimal voltage vector range from fixed phase or magnitude to arbitrary one, which drastically reduces the power ripples and THD of the current. 3) By dividing the zero voltage vector time, the constant switching frequency is achieved. 4) Incorporated with a SOGI for the quadrature grid voltage acquisition, the proposed method is applicable to the grid conditions where low-order harmonics exist. It is concluded that the proposed MV-MPPC is more favorable and practical to achieve satisfactory performance of three-phase rectifiers under nonideal grid conditions in comparison with both the C-MPPC and DO-MPPC methods.

REFERENCES


