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<th>Hysteresis modeling of reinforced concrete structures: state of the art</th>
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<td>Author(s)</td>
<td>Sengupta, Piyali; Li, Bing</td>
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Hysteresis modeling of reinforced concrete (RC) structures requires a constitutive relationship capable of producing requisite strength and stiffness degradation and pinching at all displacement levels. Moreover, the hysteresis model must be generic, computationally efficient, and mathematically tractable so as to perform satisfactorily with random input functions. Thus, development of hysteresis models with the entire prerequisites may become stringent considering the numerous parameters contributing to the structural behavior. Hence, an extensive literature review is conducted to comprehend the hysteresis models of RC structures developed by various researchers. Then, a comprehensive synopsis of the state of the art along with the substantive findings is reported in the present paper. Thereafter, a comparative study between the existing hysteresis models and the experimental results of RC structural components under quasi-static cyclic loading is conducted to evaluate the performance of the hysteresis models.

**Keywords:** analytical modeling; degradation; hysteresis; pinching; reinforced concrete.

**INTRODUCTION**

The devastating consequences of the natural and man-made hazards across the globe in terms of the human casualties and the huge economic loss due to massive structural damage are well documented in the past few decades. With growth of urbanization, more people and properties become potential targets of the impending hazards in urban and metropolitan areas. Consequently, performance assessment of reinforced concrete (RC) structures has gained immense popularity among researchers and engineers. Thus, for performance evaluation of RC structures under repeated cyclic loading, prediction of the structural hysteresis behavior is of utmost significance.

A hysteresis model is capable of producing a load-deformation relationship of a structural member under repeated cyclic loading. The hysteresis models can be broadly classified into polygonal hysteresis models (PHMs) and smooth hysteresis models (SHMs). In polygonal hysteresis models, stiffness changes are considered at cracking, yielding, strength, and stiffness degradation stages. In SHMs, continuous stiffness changes due to yielding and sharp changes during unloading and deteriorating are considered. This paper documents the primary characteristics and limitations of the hysteresis models reported in the existing literature. Thereafter, a comparative study between the existing hysteresis models and the experimental results of RC structural components is presented to assess the performance of the hysteresis models.

**RESEARCH SIGNIFICANCE**

Hysteresis modeling of reinforced concrete (RC) structural components is a pertinent stage in performance assessment of structures under extreme loading conditions. Hence, the primary objective of this research is to accumulate and comprehend the existing hysteresis models developed by various researchers so as to evaluate and compare their performances with respect to the past experimental results of RC structural components. The substantive findings of this research are conveyed herein so that this study can be useful to engineers and researchers for the development of new hysteresis models with further accuracy and computational efficiency in the future.

**REVIEW OF EXISTING HYSTERESIS MODELS**

The basic requirement of hysteresis models of reinforced concrete (RC) structures is the capability of producing requisite degradation and pinching at all displacement levels. Extensive research has been undertaken by various researchers to characterize hysteresis behavior of RC structural components. A brief review on the selective hysteresis models are presented in this section.

**Polygonal hysteresis model**

The elasto-plastic model by Veletsos et al., as shown in Fig. 1, is represented by an elastic curve indicating the cracked section behavior with no incremental stiffness upon yielding. The variables $V_y$, $u_y$, and $k_e$ are the yield strength, yield displacement, and elastic stiffness, respectively.

![Fig. 1—Load-deflection diagram of elasto-plastic model.](image)

The bilinear degrading stiffness model by Clough et al. operates on a bilinear primary curve with ascending post-
yielding branches and stiffness degradation at load reversals, as shown in Fig. 2. The post-yielding stiffness \( k_1 \) is defined as \( k_1 = r k_e \), where \( k_e \) is elastic stiffness and \( r \) is the stiffness ratio. Here, the reloading branch projects toward the previous unloading point of the loading history.

The trilinear primary curve of the Takeda model\(^3\) represents the uncracked, cracked, and post-yielding stages, as shown in Fig. 3. The unloading stiffness \( k_r \) is calculated in terms of yielding stiffness \( k_y \), yield deflection \( D_y \), and maximum deflection \( D_{\text{max}} \) as follows. In this model, the reloading branch projects toward the previous unloading point of the loading history.

\[
k_r = k_y \left( \frac{D_y}{D_{\text{max}}} \right)^{0.4}
\] (1)

In peak-oriented degrading bilinear (DBL) model by Imbeault and Nielsen,\(^4\) as shown in Fig. 4, stiffness \( K \) changes from primary stiffness \( K_0 \) when the prior maximum deformation is exceeded.

\[
K = K_0 \left( \frac{D_y}{D_{\text{max}}} \right)^{\alpha}
\] (2)

where \( D_y \) is yield deformation; \( D_{\text{max}} \) is maximum deformation in any direction; and \( \alpha \) is a constant.

\[
k_q = k(U_y/U_{\text{m}})^{\alpha}
\] (3)

In the bilinear primary curve of Q-hysteresis model by Saidi and Sozen,\(^5\) as shown in Fig. 5, unloading stiffness \( k_q \) in terms of elastic stiffness \( k \), yield deformation \( U_y \), maximum deformation \( U_{\text{m}} \), and constant \( \alpha \) is mentioned as follows. Reloading stiffness is determined as slope of the line \( X_0U_{\text{m}}' \), with \( U_{\text{m}}' \) being the point on the primary curve symmetric to \( U_{\text{m}} \) with respect to origin.

In the bilinear primary curve of the hysteresis model developed by Otani\(^6\) is shown in Fig. 6. The expression for unloading slope \( S_1 \) in terms of the elastic slope \( S_{\text{OY}} \), yield displacement \( U_y \), maximum displacement \( U_{\text{m}} \) and empirical constant \( \alpha \) is mentioned as follows.
\[ S_1 = S_{0Y}(U_r/U_m)^a \] (4)

The hysteresis shear model by Ozcebe and Saatcioglu\(^7\) shown in Fig. 7 was derived based on statistical analysis of experimental data. Unloading slope \( k \) from a load between cracking load \( V_{cr} \) and yield load \( V_y \) is defined in terms of cracking stiffness \( k_1 \), cracking deflection \( \Delta_{cr} \), yield deflection \( \Delta_y \), and slope of the line joining cracking and yield points \( k_2 \).

\[
k = k_1 - \frac{k_1 - k_2}{\Delta_y - \Delta_{cr}} (\Delta - \Delta_{cr})
\] (5)

When \( V_y \) is exceeded, unloading from loads above and below \( V_{cr} \) follows slopes \( k \), respectively.

\[
k = k_2 \left(1 - 0.05 \frac{\Delta}{\Delta_y}\right)
\] (6)

\[
k = 0.6k_2 \left(1 - 0.07 \frac{\Delta}{\Delta_y}\right)
\] (7)

Reloading follows the primary curve until a load higher than \( V_{cr} \) is reversed. When \( V_{cr} \) is exceeded, reloading until \( V_{cr} \) and beyond \( V_{cr} \) follow lines passing through points \( (\Delta_y, V_y) \) and \( (\Delta_{cr}, V_{cr}) \), respectively, up to primary curve, and post intersection, the reloading branch follows the primary curve.

\[
V_{p}' = V_{pe}e^{\alpha(\Delta_y/\Delta_{cr})}
\] (8)

\[
\alpha = 0.82(N/N_0) - 0.14 < 0.0
\] (9)

\[
V_{m}' = V_{me}[\beta n + \gamma(\Delta_m/\Delta_y)]
\] (10)

\[
\beta = -0.014 \sqrt{\frac{\Delta_{m}/\Delta_y}{\Delta_m/\Delta_y}}
\] (11)

\[
\gamma = -0.010
\] (12)

where \( N \) is the axial compressive force; \( N_0 \) is nominal axial load capacity; \( n \) is a counter tracing number of cycles at constant displacement; and \( \alpha, \beta, \) and \( \gamma \) are mathematical coefficients.

In pivot hysteresis model by Dowell et al.\(^8\) quadrants \( Q_1, Q_2, Q_3, \) and \( Q_4 \) of the monotonically increasing loading envelope defined by the horizontal axis and elastic loading lines represent elastic stiffness, strain-hardening stiffness, strength degradation, and residual strength (as displayed in Fig. 8). The primary pivot points \( (P_1, P_2) \) and pinching pivot points \( (P_{P2}, P_{P4}) \), as shown in Fig. 9, define softening and pinching \( \beta_i \), respectively. Positions of pinching pivot points change when maximum displacement \( d_{i\text{ max}} \) is more than strength degradation displacement \( d_{i} \).

\[
\beta_i = \frac{F_{i\text{ max}}}{F_u} \beta_i
\] (13)

where \( F_{i\text{ max}} \) and \( F_u \) are the maximum and strength degradation loads, respectively.

The energy-based hysteresis model by Sucuoglu and Erberik\(^9\) for deteriorating systems operates on a bilinear curve with elastic stiffness \( K_0 \), post-yielding stiffness \( aK_0 \), unloading stiffness \( K_r \), and reloading stiffness \( K_r \), as shown in Fig. 10. To relate the loss in energy dissipation capacity in the displacement cycle with the reduced strength, a deteriorating system with yield strength \( F_y \) and yield displacement \( u_y \) and subjected to displacement \( U_m \), as shown in Fig 11, is used. The energy dissipated in first cycle, \( E_{h1} \) and \( n\)-th cycle

Fig. 7—Load-deflection diagram of hysteresis shear model.\(^7\)

Fig. 8—Strength envelope and quadrant definition in pivot hysteresis model.\(^8\)
Eh,n -th cycle

\[ E_{h,n} = 2.5F_n(u_m - u_y) \] (14)

\[ E_{h,n} = 2.5F_d(u_m - u_y) \] (15)

\[ E_{h,n}/E_{h,1} = 0.8(F_d/F_y) \] (16)

The normalized dissipated energy \( \bar{E}_{h,n} \) at the equivalent n-th cycle number is

\[ \bar{E}_{h,n} = \alpha + (1 - \alpha)e^{\beta(n-1)} \] (17)

Here, \((1 - \alpha)\) is the loss of energy dissipation capacity for larger values of \( n \), and \( \beta \) is the rate of loss of dissipated energy.

The backbone curve of pinching hysteresis model by Ibarra et al.\(^{10}\) has three control points at yield strength \( F_y \), peak strength \( F_p \), residual strength \( F_r \) \((F_r = \lambda F_y)\), and the respective displacements \( \delta_y, \delta_c, \delta_r \), as shown in Fig. 12. \( K_e, K_s, \) and \( K_c \) are elastic stiffness, post-yielding stiffness \((K_s = \alpha_s K_e)\) and post-capping (negative) stiffness \((K_c = \alpha_c K_e)\), respectively, and \( \alpha, \alpha_s, \) and \( \lambda \) are constants. The primary curve of the pinching hysteresis model in the absence of deterioration is shown in Fig. 13. Cyclic deterioration \( \beta_i \) in excursion \( i \) is defined as follows in terms of dissipated hysteretic energy \( E_i \) in excursion \( i \).

\[ \beta_i = \left( \frac{E_i}{(E_i - \sum_{j=1}^{i-1} E_j)} \right)^c \] (18)

where \( \sum E_i \) is hysteretic energy dissipated in all previous excursions; \( E_i \) is hysteretic energy dissipation capacity

---

Fig. 9—Pivot point designation in pivot hysteresis model.

Fig. 10—Load-deflection diagram of energy-based hysteresis model.

Fig. 11—Relationship between reduced strength and dissipated energy of energy-based hysteresis model.

Fig. 12—Backbone curve of pinching hysteresis model.

Fig. 13—Load-deflection diagram of pinching hysteresis model in absence of deterioration.
$E_t = F_y \delta_t$; and $c$ is rate of deterioration rate, ranging between 1 and 2. The expressions of the basic strength, post-capping strength, unloading stiffness, and accelerated reloading stiffness degradations (shown in Fig. 14) are mentioned as follows.

\[ F_{y,i} = (1 - \beta_{s,i}) F_{y,i-1} \]  
\[ K_{s,i} = (1 - \beta_{s,i}) K_{s,i-1} \]  
\[ F_{\text{ref},i} = (1 - \beta_{c,i}) F_{\text{ref},i-1} \]  
\[ K_{u,i} = (1 - \beta_{k,i}) K_{u,i-1} \]  
\[ \delta_{t,i} = (1 + \beta_{a,i}) \delta_{t,i-1} \]

where $F_{\text{ref},i}$ is intersection of vertical axis and projected post-capping branch for excursion $i$; $\beta_{s,i}, \beta_{c,i}, \beta_{k,i},$ and $\beta_{a,i}$ are deterioration rates of basic strength, post-capping strength, unloading stiffness, and accelerated reloading stiffness, respectively, in excursion $i$; and $\delta_{t,i}$ is incremented maximum displacement after excursion $i$.

**Smooth hysteresis models**

Bouc\(^{11}\) suggested a smoothly varying hysteresis model for a single-degree-of-freedom (SDOF) system under forced vibration. Then, Baber and Wen\(^{12}\) and Baber and Noori\(^{13}\) extended the model with the inclusion of stiffness and strength degradation, and pinching effects, respectively. The equation of motion for a SDOF system, as shown in Fig. 15, is presented as follows

\[ \ddot{u} + 2\zeta \omega_0 \dot{u} + \omega_0^2 u + (1 - \alpha) \omega_0^2 z = f(t) \]  

where $u$ is relative displacement of mass $m$ with respect to ground motion; $c$ is linear viscous damping coefficient; $F_T[u(t),z(t),t]$ is non-damping restoring force consisting of linear restoring force $aku$ and hysteretic restoring force $(1 - \alpha)kz$; $\alpha$ is ratio of final asymptote tangent stiffness $k_f$ to initial stiffness $k_i$ with magnitudes 1 for a linear system and 0 for a nonlinear system; and $F(t)$ is time-dependent forcing function. Dividing both sides of Eq. (24) by $m$

\[ \ddot{u} + 2\zeta \omega_0 \dot{u} + \omega_0^2 u + (1 - \alpha) \omega_0^2 z = f(t) \]  

Fig. 14—Four deterioration modes of pinching hysteresis model\(^{10}\): (a) basic strength deterioration; (b) post-capping strength deterioration; (c) unloading stiffness deterioration; and (d) accelerated reloading stiffness deterioration.

Fig. 15—Schematic model of single-degree-of-freedom hysteretic system.

\[ \ddot{u} + 2\zeta \omega_0 \dot{u} + \omega_0^2 u + (1 - \alpha) \omega_0^2 z = f(t) \]  

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where $\xi_0$ is the linear damping ratio, $c/2\sqrt{k/m}$; $\omega_0$ is the pre-yield system natural frequency $\sqrt{k/m}$; and $f(t)$ is the mass-normalized forcing function. Hysteretic restoring force is a function of hysteretic displacement $z$ and, thus, the relationship between $z$ and $u$ is

$$\dot{z} = h(z) \frac{Au - \nu[\beta|\dot{u}|z^{-1} + \gamma\dot{u}|z^n]}{\eta}$$  \hspace{1cm} (26)$$

where $A$ is tangent stiffness; $\beta$, $\gamma$, and $n$ are hysteretic shape parameters; $\nu$ and $\eta$ are strength and stiffness degradation parameters, respectively; and $h(z)$ is pinching function. For a non-pinching and non-degrading system, hysteretic stiffness is zero at local maximum or minimum—that is, the point on the load-deflection curve where velocity changes its sign. So, at an infinitesimal distance $dz$ away from $z_{\text{max}}$, where velocity is close to but not equal to zero and $\dot{z}_{\text{max}} = \dot{z}$

$$\dot{z}_{\text{max}} = 0 = Au - \nu[\beta|\dot{u}|z^{-1} + \gamma\dot{u}|z^n]$$

$$z_{\text{max}} = \pm \{A/\nu(\beta + \gamma)\}^{1/n}$$ \hspace{1cm} (27)$$

Inclusion of variation of $A$ can contribute to the versatility of the model. However, $A$ is redundant as both hysteretic stiffness and force can be varied by stiffness ratio and hysteretic shape parameters. Thus, for simplicity, the magnitude of $A$ is set to unity.

Hysteresis shape parameters $\beta$, $\gamma$, and $n$ determine basic hysteretic shape. $\beta$ and $\gamma$ individually influence hysteretic strength and stiffness inversely while they jointly influence softening or hardening of hysteresis loops. $n$ controls sharpness of transition from initial to asymptotic slope. For $n = 1$, effects of combined $\beta$ and $\gamma$ on hysteresis loops shown in Fig. 16 are as follows.

i) $\beta + \gamma > 0$, $\gamma - \beta < 0$ weak softening

ii) $\beta + \gamma > 0$, $\gamma - \beta = 0$ weak softening on loading, mostly linear unloading

iii) $\beta + \gamma > \gamma - \beta > 0$ strong softening on loading and unloading, narrow loop

iv) $\beta + \gamma = 0$, $\gamma - \beta < 0$ weak hardening

v) $0 > \beta + \gamma$, $\beta + \gamma > \gamma - \beta$ strong hardening

Hysteretic energy $\varepsilon$, defined as irrecoverable strain energy absorbed by hysteretic element, is used to approximate structural degradation and pinching. Although the total strain energy consists of irrecoverable hysteretic energy and recoverable strain energy, the latter is quite small compared to the former. Thus, hysteretic energy is considered approximately equal to total energy and expressed as continuous integral of hysteretic force $f(t)$ over total displacement $u$.

$$\varepsilon(t) = \int_{u(0)}^{u(\infty)} f \cdot du = (1 - \alpha)\omega_0 \int_{u(0)}^{u(\infty)} z(u, t) \cdot du \cdot \frac{du}{dt}$$

$$= (1 - \alpha)\omega_0 z^\gamma(0) z(u, t) \cdot \dot{u}(t) \cdot dt$$ \hspace{1cm} (28)**
A gradual decrease in structural strength at the same displacement level is termed “strength degradation”. Progressive loss of structural stiffness in each loading cycle is defined as stiffness degradation. Strength and stiffness degradation parameters \( \nu \) and \( \eta \), respectively, are functions of hysteretic energy \( \varepsilon \), strength degradation rate \( \delta_\nu \), and stiffness degradation rate \( \delta_\eta \), as follows

\[
\nu(\varepsilon) = 1 + \delta_\nu \varepsilon \quad (29)
\]

\[
\eta(\varepsilon) = 1 + \delta_\eta \varepsilon \quad (30)
\]

Effects of change in magnitudes of \( \delta_\nu \) and \( \delta_\eta \) on hysteresis loops are shown in Fig. 17 and 18, respectively, when the input function and remaining model parameters are same. For zero magnitudes of \( \delta_\nu \) and \( \delta_\eta \), structure does not degrade. With an increase in \( \delta_\nu \), both hysteretic force and stiffness degrade while increase in \( \delta_\eta \) reduces the hysteretic force only.

The expression for pinching function \( h(z) \) is as follows

\[
h(z) = 1 - \zeta_1 e^{-(\zeta_2 - \zeta_{\text{max}})z} \quad (31)
\]

where \( \zeta_1 \) and \( \zeta_2 \) are pinching severity and pinching spread parameters, respectively; and \( q \) is pinching level as a fraction of \( z_{\text{max}} \). Both \( \zeta_1 \) and \( \zeta_2 \) vary with hysteretic energy \( \varepsilon \) as follows

\[
\zeta_1(\varepsilon) = \zeta_1 \left(1 - e^{-p\varepsilon}\right) \quad (32)
\]

\[
\zeta_2(\varepsilon) = (\psi + \delta_\psi)(\lambda + \zeta_1) \quad (33)
\]

where \( p \) is rate of initial drop in slope; \( \zeta_1 \) is total slip; \( \psi \) is total pinching; \( \delta_\psi \) is pinching spread rate; and \( \lambda \) is rate of change of \( \zeta_2 \) with change of \( \zeta_1 \). Effect of pinching on hysteresis behavior with changing \( \zeta_1 \) is shown in Fig. 19.

Based on experimental results of RC structures under repeated cyclic loading, Sengupta et al. \(^{14-17}\) modified original Bouc-Wen-Baber-Noori (BWBN) model by incorporating displacement-based stiffness ratio \( \alpha \) in terms of initial stiffness ratio \( \alpha_0 \) and absolute maximum displacement \( D_{\text{max}} \).

\[
\alpha = \alpha_0 e^{(-0.1D_{\text{max}})} \quad (34)
\]

The stiff set of ordinary differential equations involved in the mathematical expressions of the BWBN model requires a suitable solver. Because the hysteretic response is not only dependent on individual parameter magnitude, but also on
their interaction, a genetic algorithm is used in this study for systematic parameter estimation of the BWBN model.\textsuperscript{14-17}

**COMPARISON OF HYSTERESIS MODELS WITH EXPERIMENTAL RESULTS**

Based on the literature review on hysteresis models, it is observed that elasto-plastic\textsuperscript{1} and degrading bilinear models\textsuperscript{4} are unable to represent realistic structural hysteresis response. The Clough model\textsuperscript{2} does not incorporate pinching effect. Q-hysteresis\textsuperscript{5} and Otani models\textsuperscript{6} impractically consider concrete section elastic until yielding while nonlinearity initiates after appearance of cracks. Performances of Takeda model,\textsuperscript{3} pivot hysteresis,\textsuperscript{7} and hysteresis shear\textsuperscript{8} models are poor compared to the number of rules. Energy-based hysteresis model\textsuperscript{9} cannot perform satisfactorily at larger cycle numbers, as pinching effect is not included in the model. The contribution of four deterioration modes of the pinching hysteresis model\textsuperscript{10} on structural response is not defined adequately.

Thus, to assess performance of the hysteresis models with respect to the experimental load-deformation plots, a series of RC beam-column joint and wall specimens are accumulated from the literature\textsuperscript{18-34} for construction of hysteresis load-deformation plots using the existing hysteresis models. The test specimens (34 beam-column joints and 56 shear walls) are selected from the literature such that wide variations in terms of structural features, material properties, and loading characteristics are covered to enhance the effectiveness of the hysteresis model performance assessment. The experimental and hysteresis load-deformation responses of RC interior beam-column joint specimen PEER14, exterior beam-column joint specimen EJ4, wall specimen C1.5K
with rectangular cross section, and wall specimen MW3 with flanged cross section are presented in Fig. 20, 21, 22, and 23, respectively. Reinforced concrete interior beam-column joint specimen PEER14 and wall specimen MW3 exhibit pinched and thinner hysteresis loops, while exterior beam-column joint specimen EJ4 and wall specimen C1.5K exhibit non-pinched and fatter hysteresis loops. From the comparison plots, it can be observed that the BWBN hysteresis model performs reasonably well to capture both pinched and non-pinched hysteresis response. Among the polygonal hysteresis models, the pinching hysteresis model performs well with respect to the experimental load-deformation plots for both pinched and non-pinched hysteresis response. Among the polygonal hysteresis models, the pinching hysteresis model performs well with respect to the experimental load-deformation plots for both pinched and non-pinched hysteresis response. The pinching hysteresis model can capture pinched hysteresis response much better than non-pinched hysteresis response. Elasto-plastic and degrading bilinear models cannot capture pinched or non-pinched hysteresis response with requisite accuracy. The remaining models—Clough model, Takeda model, Q-hysteresis model, Otani model, pivot hysteresis model, hysteresis shear model, and energy-based hysteresis model—cannot capture pinched hysteresis response well. However, all these models can capture non-pinched hysteresis response adequately. Additionally, the Takeda model, Q-hysteresis model, Otani model, pivot hysteresis model, and hysteresis shear model can also capture structural hysteresis response with mild pinching behavior.

Because hysteretic energy is a measure to quantify the performance of the hysteresis models, experimental and model hysteretic energy are computed from experimental and hysteresis load-deformation plots of beam-column joint and wall specimens. The correlation and average deviation (in %) between the experimental and model hyster-
etic energy of beam-column joint and wall specimens are presented in Fig. 24 and 25, respectively. Figure 24 shows that the correlation between the experimental and model hysteretic energy is maximum for the BWBN model with minimum scatter, while Fig. 25 shows that average deviation in hysteretic energy obtained from the BWBN model is only 3% from experimental hysteretic energy. Average deviations in hysteretic energy obtained from pinching hysteresis model, Otani model, Takeda model, hysteresis shear model and Q-hysteresis model are 23.55%, 29.61%, 32.87%, 35.24%, and 41.90%, respectively, from experimental hysteretic energy. Similarly, average deviations in hysteretic energy obtained from degrading bilinear model, pivot hysteresis model, energy-based hysteresis model, Clough model, and elasto-plastic model are 46.28%, 76.5%, 80.52%, 81.09%, and 130.17%, respectively, from experimental hysteretic energy.

CONCLUSIONS

Based on the literature review conducted on hysteresis modeling of RC structural components in the present paper, the following conclusions are drawn:

1. Elasto-plastic and degrading bilinear models are unable to represent realistic structural hysteresis response. The Clough model does not incorporate pinching effect. Q-hysteresis and Otani models impractically consider concrete section elastic until yielding with nonlinearity initiating after the appearance of cracks. The performances of the Takeda model, pivot hysteresis model, and hysteresis shear model are poor compared to number of rules involved. The performance of the energy-based hysteresis model is not
quite satisfactory at larger cycle numbers, as the pinching effect is not included in the model. Contribution of four deterioration modes of pinching hysteresis model on the structural behavior is not defined adequately.

2. Bouc-Wen-Baber-Noori (BWBN) hysteresis model uses several parameters to capture various aspects of RC structures under repeated cyclic loading, such as structural degradation, pinching, softening and hardening. However, the stiff set of ordinary differential equations involved in the mathematical expressions of the model requires a suitable solver. Because structural hysteresis response is dependent on individual and combined magnitudes of parameters, a system identification tool is also necessary for parameter estimation. A genetic algorithm is used in this study for systematic parameter estimation of the BWBN model.

3. From the comparison study between the hysteresis models and experimental results of RC beam-column joint and wall specimens, it is quite evident that the BWBN model performs better than other hysteresis models for both pinched and non-pinched hysteresis response. The hysteresis loops obtained from BWBN Model closely match the experimental load-deformation plots, and the average difference between the experimental and model hysteretic energy is only 3%. After the BWBN model, the pinching hysteresis model can capture pinched hysteresis response much better than non-pinched hysteresis response. Elasto-plastic and degrading bilinear models cannot capture pinched or non-pinched hysteresis response with requisite accuracy. The remaining models—Clough model and Takeda

Fig. 23—Experimental and hysteresis load-deformation plots of reinforced concrete wall Specimen MW3 with flanged cross section. (Note: 1 kN = 0.225 kip; 1 mm = 0.0394 in.)
Fig. 24—Correlation study between hysteretic energy obtained from hysteresis models and experimental results. (Note: 1 kN = 0.225 kip; 1 mm = 0.0394 in.)
model, Q-hysteresis model, Otani model, pivot hysteresis Model, hysteresis shear model and energy-based hysteresis model—cannot capture pinched hysteresis response well. However, all these models can capture represent non-pinched hysteresis response adequately. Additionally, the Takeda model, Q-hysteresis model, Otani model, pivot hysteresis model and hysteresis shear model can also capture structural hysteresis response with mild pinching behavior.

4. Future research can be conducted on the simplification of the BWBN model by reducing the number of analytical parameters for predicting hysteresis behavior of RC structures. An optimum hysteresis model must be deduced such that the overall performance of the model remains satisfactory with minimum computational complicity involved in the methodology.

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