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<td>Author(s)</td>
<td>Zhang, Wenjie; Zhang, Guanglin; Zheng, Yifeng; Yang, Liwei; Yeo, Chai Kiat</td>
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Spectrum Sharing for Heterogeneous Networks and Application Systems in TV White Spaces

WENJIE ZHANG1, GUANGLIN ZHANG2, YIFENG ZHENG1, LIWEI YANG3, AND CHAI KIAT YEO4

1Key Laboratory of Data Science and Intelligence Application, School of Computer Sciences, Minnan Normal University, Zhangzhou 800084, China
2School of Information Science and Technology, Donghua University, Shanghai 201620, China
3College of Information and Electrical Engineering, China Agricultural University, Beijing 100083, China
4School of Computer Engineering, Nanyang Technological University, Singapore 639798

Corresponding author: Wenjie Zhang (zhan0300@ntu.edu.sg)

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ABSTRACT In 2010, federal communications commission released the final ruling to allow the unlicensed operation of TV white spaces, i.e., locally vacant TV channels. Since TV white spaces are open to all networks and all types of applications, it is likely that there will be multiple networks and application systems authorized to use the same TV channels at the same time. Currently, there is mechanism for different users to operate in the TV white spaces within one system, while TV spectrum sharing among heterogeneous networks and application systems is largely ignored. To address the problem, this paper designs a trading mechanism to study the TV spectrum market between Geolocation database and heterogeneous application systems. We formulate the spectrum sharing problem into a 0-1 integer optimization problem and prove that this problem is NP-complete. The proposed solution decomposes an optimal fractional solution of a NP-hard problem into a convex combination of internal solutions by randomized rounding algorithm. Simulation results show that the proposed approximation algorithm can achieve a close-to-optimal solution with far less complexity.

INDEX TERMS Spectrum sharing, heterogeneous systems, TV white spaces, NP-complete, randomized rounding algorithm.

I. INTRODUCTION

According to the report by Cisco Visual Networking Index (CVNI), worldwide mobile data traffic grew 63% in 2016, and this increase is predicted to continue for several years [1]. Available spectrum (e.g. 4G) will clearly be insufficient to support this traffic demand. Regulators have currently been searching for new spectrum - both licensed and unlicensed to meet this demand. As a result of analogue to digital transmission, a substantial amount of TV spectrum that was previously used by analog transmission will become entirely available. These newly free up spectrum along with other unused spectrum located in 50-700 MHz are called “TV white spaces”. In 2008, FCC permitted the unlicensed operation in the TV white spaces [2] so long as requirements, such as minimizing the interference to the incumbents are met. Furthermore, in 2010, FCC released the final ruling to approve TV white spaces for unlicensed use [3].

Compared to the typical WiFi bands, TV white spaces have much better propagation characteristic and excellent in-building penetration due to its low frequency. Hence, TV spectrum is the ideal candidate for covering households that are far away from the heart of the network, and connecting multiple devices separated by the walls. This in turn makes TV bands become an innovation platform for a wide range of potential important applications, including Business/home femtocell network [4], Cellular-WLAN seamless wireless services [5], Indoor localization systems [6], Smart Grids [7], and so on, as shown in Fig. 1. This could bring enormous social and economical benefits.

The successful operation of TV frequency bands strictly relies on its ability to accurately detect the presence of licensed users. Most of the early studies on incumbent detection focused on reliable spectrum sensing [8]. However, recent studies indicated that spectrum sensing alone is often
inadequate due to its high complexity and poor detection performance at low Signal to Noise Ratio (SNR) [3], [9]. As a result, many agencies (e.g., FCC and OFCOM) and organizations (e.g., IEEE) adopt Geolocation Database (GD) approach in some cases complimentary to sensing [10], [11]. Currently, GD seems to offer the best short-term solution for incumbent detection in the environments where a centralized third-party exists [12]. With a database, part of the complexity associated with spectrum sensing can be transferred to the core network. In such solution, GD maintains up-to-date channel usage information of the licensed users, and uses this information together with other specifications input by the devices to perform propagation modelling calculations, which will return a list of available TV channels at a particular location. GD has been studied extensively by many researchers [13]–[16]. However, all the above-mentioned studies on GD focused on addressing the technical issues, such as, accurate location measurement, appropriate propagation and available white space computation. In this paper, the GD is responsible for reserving the available TV spectrum from its licensee in advance, and serving heterogeneous application systems according to their bandwidth demand and payment.

As indicated in [17], the amount of TV white spaces is significantly limited in an urban area. Therefore, it is likely that multiple heterogeneous, closely-located and independently-operated networks and application systems are authorized to use the same TV channels at the same time. For each system, mechanisms have been designed to allow multiple users to operate in TV white spaces without disrupting one another [18]–[27]. However, since there is no regulatory requirement for the coexistence of heterogeneous networks and application systems in TV white spaces [28] so far, transmissions from different systems may cause unusual patterns of interference, resulting in performance degradation and inefficient use of TV spectrum. To fill in this gap, we consider in this paper, spectrum sharing for heterogeneous application systems. The detailed spectrum sharing process is illustrated in Fig.1. The GD who acts as an auctioneer reserves the TV bandwidth from the licensee in advance and then sells it to the bidders. The application systems (e.g., the Business Femtocell network, Smart Grid and university...
Cellular-WLAN) are the bidders, they will be vying to use the same usage block at the same time. In particular, we design such a trading mechanism where multiple networks and application systems compete for the use of the same TV spectrum with the goal of maximizing the ‘social welfare’ (i.e. the aggregate profit). We make the following contributions:

1) We consider the TV spectrum sharing among heterogeneous networks and application systems. To the best of our knowledge, this work is among the first that designs trading mechanism for spectrum sharing among heterogeneous networks and application systems.

2) With the objective to maximize the ‘social welfare’ (i.e. aggregate profit), we formulate the spectrum sharing problem as a 0-1 integer optimization problem. This problem has been proven to be NP-complete, which highlights the inherent challenge faced in solving it.

3) We use the randomized rounding algorithm to obtain a ρ-approximation solution. Evaluation results show that the proposed approximation algorithm can achieve a close-to-optimal solution with less complexity.

The rest of this paper is organized as follows. Some related works are briefly reviewed in Section II. The system model is introduced in Section III. The problem analysis and formulation are described in Section IV. The algorithm designs are presented in Section V. Simulation results and evaluations are given in Section VI. Finally, Section VII concludes the paper.

II. RELATED WORK

In the recent regulator’s policy [10], GD should enable users operating on TV white spaces by providing a list of available channels. Many studies have been proposed to optimize the GD design in accurate location measurement, appropriate propagation and available white space computation. For example, in [13], a detailed design of GD in TV band is proposed to protect the licensed service. In [14], Murty et al. presented a database driven white space network using a combination of an up-to-date database of the incumbents, sophisticated signal propagation modeling, and an efficient content dissemination mechanism. In [15], the design and implementation of a multi-cell infrastructure-based TV white spaces network called WhiteNet is presented, where GD is exploited to assist multi-AP coordination. In [16], Chen et al. adopted a game theoretic approach for self-organizing, white-space spectrum access network design with GD. In this paper, the GD is responsible for reserving TV bandwidth from its licensee, and sells to heterogeneous systems.

Recently, research on spectrum sharing has attracted a lot of attention. In general, prior works mainly focused on spectrum sharing within homogeneous system. For example, Chen et al. proposed a game approach for white space sharing among multiple white space devices and designed a state-based game framework to formulate the distributed AP association problem of the secondary users in [18]. In [19], Caleffi and Cacciapuoti proposed an optimal database access strategy for cognitive radio networks operating in TV white spaces with the objective to account for traffic pattern, spectrum characteristics and database access overhead. In [20], Elias et al. investigated the distributed spectrum management problem in opportunistic TV white spaces systems using a game theoretical approach where both the adjacent channel interference and spatial reuse are taken into consideration. In [21], a broker-based spectrum reservation market model for TV white space network is proposed for the database’s spectrum reservation under demand stochasticity and information asymmetry. In [22], Manickam et al. leveraged GD for interference-aware coordinated TV white spaces sharing among secondary users. In [23], a graph theory based Fairness Constrained Channel Allocation (FCCA) algorithm is proposed for spectrum sharing in TV white spaces. In addition, spectrum leasing scheme has also been investigated in the literatures. For example, in [24], a commercialized small-cell caching system which consists of a network service provider (NSP), several content providers (CPs), and multiple mobile users (MUs) is considered. The NSP leases its resources to the CPs for gaining profits, and the CPs are intended to rent the SBSs for providing better downloading services to the MUs. In [25], the problem of dynamic pricing for monopoly spectrum leasing with one primary network is investigated. The primary network acts as the spectrum seller and would like to lease its unused channels to unlicensed users. In [26], a pricing framework for spectrum leasing in mobile networks with single macrocell and multiple femtocells is proposed. Furthermore, an overview on Long-Term Evolution (LTE) spectrum sharing technologies is presented in [27], where three popular spectrums are taken into consideration, including the TV white spaces, the frequently unused service-dedicated 3.5-GHz citizens broadband radio service spectrums, and the 5-GHz unlicensed bands.

In this paper, we propose a trading mechanism to study the spectrum sharing problem for heterogeneous networks and application systems in TV white spaces. In our model, the bid submitted by each system consists of two parts: one is the price that is willing to be paid for the TV spectrum, and the other is the bandwidth demand. Thus the optimal trading needs to consider not only the economic rationality, but also the demand effectiveness. This makes our trading mechanism challenging.

III. SYSTEM MODEL

A. SYSTEM OVERVIEW

In this paper, we attempt to consider a white space network with a set of application systems, as illustrated in Fig. 1. In order to obtain a list of vacant TV channels, a geolocation database (GD) should be included. The users query the database using location information as inputs, and obtain the up-to-date TV white spaces availability. Since TV white spaces are open to all networks and all types of applications, it is likely that multiple applications are vying to use the same TV channels at the same time. Each application system may have a mechanism to permit multiple users to operate in the TV spectrum without disrupting each other.
However, for heterogeneous networks and systems, there does not exist any such regulatory requirement. Therefore, in this paper, we attempt to design a trading mechanism for spectrum sharing among heterogeneous networks and application systems. Each application system considered in this paper is assumed to be infrastructure-based, with an infrastructure serving a set of unlicensed users who cannot connect to the database directly. Fig. 2 depicts such a system: Business/home femtocell network in TV white spaces. A cognitive base station (CBS) connected to the database is required to serve each Macro Secondary User (MSU) and Femto Secondary User (FSU). The CBS may access the database periodically in order to obtain the available TV white spaces. In each access period [17]

![FIGURE 2. Illustration of Femtocell-based TV white space network.](image)

i) The CBS queries the database using its location, spectrum demand and other specific information as inputs;

ii) The database uses these information to perform a propagation modeling-type calculation, and then return a list of available white spaces at the CBS’s location;

iii) The CBS serves its connected users using the returned white spaces.

**B. TRADING MECHANISM**

In this work, we assume that any available TV spectrum must be reserved by the database from its licensee in advance. This reservation-based access mode has been considered to be more suitable for infrastructure-based systems due to its high reliability in spectrum acquisition [17], [18]. Without loss of generality, we use Fig. 2 as an example, where the interaction between the GD and CBS is taken into consideration. Inline with the access model discussed above, we propose a trading mechanism for spectrum sharing among heterogeneous application systems, which can be divided into the following four stages:

i) At the beginning of every reservation period $T$, the database will reserve bandwidth $K$ with a unit cost $c$ from the TV licensee;

ii) At each access period, the CBS collects all the bids from all its connected users within the system and submits to the database;

iii) The database returns the bidding result to the CBS;

iv) The CBS serves the users who win the bid using the received bandwidth.

Note that the bandwidth reservation period $T$ is performed on an hourly basis (e.g. two hours [17]), and the access period $t$ is done on a per-minute basis [21].

**IV. PROBLEM ANALYSIS AND FORMULATION**

In this section, we start with the problem analysis and formulate the trading-based spectrum sharing problem as a 0-1 integer optimization problem. Then, we prove that this problem is NP-complete. The key notations that will be used are listed in Table 1.

**TABLE 1. Summary of key notations.**

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
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<tr>
<td>$m$</td>
<td>Number of application systems</td>
</tr>
<tr>
<td>$n_i$</td>
<td>Number of bids for system $i$</td>
</tr>
<tr>
<td>$l$</td>
<td>Number of access period</td>
</tr>
<tr>
<td>$T$</td>
<td>Reservation period</td>
</tr>
<tr>
<td>$K$</td>
<td>Reservation bandwidth</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Set of application systems</td>
</tr>
<tr>
<td>$p_{ij}$</td>
<td>Bidding price for for system $i$</td>
</tr>
<tr>
<td>$b_{ij}$</td>
<td>Bandwidth demand for system $i$</td>
</tr>
<tr>
<td>$c$</td>
<td>Unit cost for TV spectrum</td>
</tr>
<tr>
<td>$\lambda^k$</td>
<td>Combination weighting coefficient</td>
</tr>
<tr>
<td>$X = [x_{ij}]$</td>
<td>The bidding result</td>
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**A. PROBLEM FORMULATION**

In the following, we consider the GD-to-application market, consisting of a GD and a set of application systems, denoted by $\Delta$, with $|\Delta| = m$. The GD acts as an auctioneer who reserves the TV bandwidth from the licensee in advance and then sells it to a number of application systems (bidders). Each application system $i \in \Delta$ can submit one or more bids to the database. Let $B_i$ denote all the bids submitted by system $i \in \Delta$. Each bid $(p_{ij}, b_{ij}) \in B_i$ has two elements, $p_{ij}$ specifies the price that is willing to be paid for the TV spectrum; $b_{ij} = [b_{ij}(t)]_{1 \times l}$ represents the bandwidth demand that contains entry $b_{ij}(t)$ which is the amount of bandwidth demand at access period $t$, and $l$ denotes the number of access period in each reservation period $T$.

The auction of TV bandwidth from the GD to the application systems is conducted once, at $t = 1$ based on the predicted traffic demand. Let $X = [x_{ij}]_{m \times n_i} \in \{0,1\}$, $\forall i,j$ denote an $m \times n_i$ binary matrix, describing the bidding result.
That is
\[ x_{i,j} = \begin{cases} 
1 & \text{Application system } i \text{ wins the bid } j \\
0 & \text{Otherwise} 
\end{cases} \]

Here \( n_i \) is the number of bids submitted by system \( i \). For simplicity, we assume that \( n_i \) is the same for all access periods.

By employing the XOR-bidding language in [29], we assume that only one bid can win among all the bids submitted by system \( i \in \Delta \), that is
\[ \sum_j x_{ij} \leq 1, \quad \forall i \in \Delta. \tag{1} \]

Furthermore, the total bandwidth demand for all systems at each access period cannot exceed the bandwidth reserved by the database from the spectrum licensee at the beginning of reservation period \( T \), that is
\[ \sum_{i,j} b_{ij}(t)x_{ij} \leq K, \quad \forall t. \tag{2} \]

Then the spectrum sharing among heterogeneous systems can be translated into the following optimization problem, with the objective of maximizing the aggregate profit (referred to as 'social welfare' in this paper). The Winner Determination Problem (WDP) with a given bandwidth reservation \( K \) under the above two constraints is formulated as

**Problem WDP:**
\[
\max \sum_{i,j} p_{ij}x_{ij} - cK \\
s.t. \sum_{j} x_{ij} \leq 1, \quad \forall i \in \Delta \\
\sum_{i,j} b_{ij}(t)x_{ij} \leq K, \quad \forall t \\
x_{ij} \in \{0, 1\}, \quad \forall j, \text{ and } \forall i \in \Delta 
\]

Solving the WDP is NP-complete. When the number of application systems and bids increases, the complexity to find the optimal bidding solution will grow exponentially. In the next subsection, we prove the NP-completeness of this problem.

**B. COMPLEXITY ANALYSIS OF WDP**

It is well known that the decision problem is considered to be no harder than its corresponding optimization problem. Thus the proof of NP-completeness for an optimization problem is equivalent to proving that its corresponding decision problem is NP-complete [30]–[32]. In the following, we will first define a decision problem corresponding to the WDP as follows.

**Definition 1:** WDP decision problem: Given the inputs: the application systems set \( \Delta \), all the bids submitted by system \( i \in \Delta \), \( B_i \), the total bandwidth reserved by the database \( K \), and a value of social welfare \( \alpha \). Does there exist a bidding result \( X \) that satisfies all the constraints of the WDP, and the social welfare is \( \alpha \)?

To prove the WDP decision problem is NP-complete, we need to
1) First, we prove that the WDP decision problem is an NP problem, as shown in Lemma 1.
2) Then, we prove that the WDP decision problem is an NP-hard problem, as shown in Lemma 2, by
   a) Select a known NP-complete problem. In this work, circuit satisfiability (SAT) problem is used;
   b) Map the WDP decision problem to the SAT problem in polynomial time.

**Lemma 1:** The WDP decision problem is an NP problem.

**Proof:** In the following, the proof of NP problem is done by showing that if an instance is a solution for the WDP, it can be verified in polynomial time, by checking:
1) whether \( \sum_{i,j} p_{ij}x_{ij} - cK = \alpha \);
2) whether \( \sum_{j} x_{ij} \leq 1 \) for all \( i \in \Delta \);
3) whether \( \sum_{i,j} b_{ij}(t)x_{ij} \leq K \) satisfies for all access period \( t \).

Verifying 1) and 2) takes a running time of \( O(mn_i) \), where \( |\Delta| = m \) is the number of application systems, and \( n_i \) is the number of bids for system \( i \). Furthermore, it takes a running time of \( O(mn_i l) \) to verify 3). Thus if the biding result \( X \) is a solution of the WDP decision problem, it can be verified in polynomial time. Therefore, the WDP decision problem is an NP problem.

This completes the proof.

**Lemma 2:** The WDP decision problem is NP-hard.

**Proof:** Here, we use the approach proposed in [31] to prove that the WDP decision problem is an NP-hard problem. We first define a restricted WDP decision problem, and then transform this restricted decision problem to a well known NP-hard problem in polynomial time.

**Definition 2:** The restricted WDP decision problem is a special instance of the WDP decision problem in definition 1 for small values of \( m, n_i \) and \( l \). Particularly, we set \( m = 2, n_i = 2, l = 1 \) and \( K = 2 \). For further simplification, we assume that the social welfare is 2. Regardless of which bid wins, we set \( p_{ij} = b_{ij} = 1, \forall i, j \) and \( c = 0 \).

**Definition 3:** The SAT problem is a decision problem to determine whether the output is true given a set of Boolean variables, which has been proven to be NP-complete. The Boolean formula of the SAT problem used in our work is given as
\[
(y_{1,1} \land y_{1,2} \land y_{2,1} \land y_{2,2}) \lor (y_{1,1} \land y_{1,2} \land y_{2,1} \land \overline{y_{2,2}}) \\
\lor (y_{1,1} \land y_{1,2} \land \overline{y_{2,1}} \land y_{2,2}) \lor (y_{1,1} \land y_{1,2} \land y_{2,1} \land \overline{y_{2,2}}) 
\]
where \( y_{ij}, i, j \in \{1, 2\} \), denote a set of Boolean variables for the SAT problem, and \( \overline{y_{ij}}, i, j \in \{1, 2\} \), are the complements of \( y_{ij} \), respectively. The output of the SAT problem is a Boolean value (True or False).

In order to transform the restricted WDP decision problem to the SAT problem in polynomial time, we need to show that for a given input, the restricted WDP decision problem has “yes” answer if and only if there exists a set of assignments to each variable \( y_{ij} \), such that the SAT problem defined above is true.
First, given that the bidding matrix is a “yes” answer instance for the restricted WDP decision problem. There are four possible bidding results: 1) first bids submitted by system 1 and 2 win the bid; 2) first bid of system 1 and second bid of system 2 win the bid; 3) second bid of system 1 and first bid of system 2 win the bid; 4) second bids of system 1 and 2 win the bid. Without loss of generality, we consider the first possible bidding, where the Boolean variables as \( x_{11} = x_{21} = 1 \) and \( x_{12} = x_{22} = 0 \). Then we can make the SAT problem true by setting the first Boolean variables as \( y_{ij} = 0 \), \( i, j \in \{1, 2\} \). Obviously, this transformation takes polynomial time. For other possible bidding results, we can make the transformation similarly.

On the other hand, we further show that if there exist Boolean variables that can make the SAT problem output true, then we can obtain a Yes-instance for the restricted WDP decision problem. According to the Boolean formula defined above, it can be concluded that if a set of Boolean variables can make the SAT problem output true, then the input variables satisfy

- \( y_{11} = y_{22} = 1 \) and \( y_{12} = y_{21} = 0 \), or
- \( y_{11} = y_{21} = 1 \) and \( y_{12} = y_{22} = 0 \), or
- \( y_{12} = y_{22} = 1 \) and \( y_{11} = y_{21} = 0 \), or
- \( y_{12} = y_{21} = 1 \) and \( y_{11} = y_{22} = 0 \).

Without loss of generality, we assume the first setting. In this case, if we set \( x_{ij} = y_{ij}, i, j \in \{1, 2\} \), all the constraints of the WDP can be satisfied and the social welfare is 2. Of course, this transformation can be done in polynomial time.

Therefore, we have proven that the restricted WDP can be transformed to the SAT problem in polynomial time. Thus we conclude that the WDP decision problem is NP-hard.

This completes the proof.

**Theorem 1:** The WDP is an NP-complete problem.

**Proof:** Combining Lemmas 1 and 2, we can conclude that the WDP is an NP-complete problem.

### V. ALGORITHM DESIGN

Owing to the NP-completeness of the WDP, it seems impossible to solve this problem in polynomial time. In this section, we first develop an algorithm called randomized rounding algorithm to find the \( \rho \)-approximation solution for the WDP based on a recent result in theoretical computer science. Thereafter, an efficient approximation algorithm to compute a feasible integer solution is proposed.

#### A. THE RANDOMIZED Rounding ALGORITHM

Here, we will use the randomized rounding algorithm as illustrated in Algorithm 1, which consists of three steps:

In the following, we will give a detailed description of the randomized rounding algorithm. We start with the definition of Linear Programming Relaxation (LPR) of the 0-1 Integer Programming (IP).

**Definition 4:** The Linear Programming Relaxation (LPR) of the 0-1 Integer Programming (IP) is obtained by relaxing the integrality constraint to

\[
\forall i, j \quad 0 \leq x_{ij} \leq 1 ,
\]

According to the Theorem 2.1 in [33] and [34], if \( X^* \) is the optimal solution to the LPR of WDP, then \( X^* \) dominates a convex combination of all feasible integer solutions \( X^q \) of WDP, that is,

\[
\sum_{q \in \Omega} \lambda^q X^q \geq X^* / \rho , \quad \text{and} \quad \sum_{q \in \Omega} \lambda^q = 1
\]

where \( \lambda^q \geq 0 \) for all \( q \), and \( \sum_{q \in \Omega} \lambda^q = 1 \). \( X^q \) is a feasible integer solution to the WDP and \( \Omega \) denotes the index set for all feasible integer solutions.

**Step 1 (Relaxation of the Integer Wdp):** The first step is to solve the LPR of the WDP by relaxing constraint (7) to

\[
x_{ij} \geq 0 , \quad \forall j , \text{ and } \forall i \in \Delta
\]

Note that \( x_{ij} \leq 1 \) is redundant and is ignored here. Obviously, this LPR problem is linear programmable and can be optimally solved in polynomial time.

**Step 2 (Convex Decomposition):** In the following, by using the recent convex decomposition technique [34], [35], we can decompose the optimal fractional solution into a convex combination of integral solutions each with a fractional weight that sums up to 1. This step requires an effective polynomial-time algorithm to obtain the approximation solution of the WDP, which satisfies:

\[
\sum_{i, j} p_{ij} x_{ij} - cK \geq OPT_{LPR} / \rho
\]

where \( OPT_{LPR} \) is the value of the objective function for LPR of the WDP when the solution is \( X^* \). Therefore, our goal in this step is to find the combination weighting coefficient \( \lambda^q \), such that

\[
\lambda^q = 1 , \quad \text{and} \quad \sum_{q \in \Omega} \lambda^q X^q \geq X^* / \rho
\]

**Algorithm 1 The Randomized Rounding Algorithm**

1: **Step 1. Relaxation of the WDP**
2: for all \( i, j \) do
3: \( x_{ij} \in \{0, 1\} \rightarrow x_{ij} \geq 0; \)
4: end for
5: - Calculate the optimal bidding result \( X^* = [x^*_{ij}] \) for the LPR of the WDP.
6: **Step 2. Convex decomposition**
7: - Decompose the fractional solution \( X^* \) to a convex combination of integer solutions \( X^q = [x^q_{ij}] \), that is,
8: \begin{align}
\sum_{q \in \Omega} \lambda^q X^q &\geq X^* / \rho , \quad \text{and} \quad \sum_{q \in \Omega} \lambda^q = 1 \\
\end{align}
9: - Select each feasible integer solution \( X^q \) of the WDP with probability \( \lambda^q \).
In order to obtain the combination weighting coefficient \( \lambda^q \) for each integer solution, we have to solve the following LP problem:

**Primal:**

\[
\begin{align*}
\text{max} & \quad \sum_{q \in \Omega} \lambda^q \\
\text{s.t.} & \quad \sum_{q \in \Omega} \lambda^q x^q \geq x^*/\rho \\
& \quad \sum_{q \in \Omega} \lambda^q \geq 1 \\
& \quad \lambda^q \geq 0, \quad \text{and } \forall q \in \Omega
\end{align*}
\]  

(11)

This problem is very difficult to solve because it has an exponential number of variables. Instead, we will solve its related dual problem that has an exponential number of constraints. The dual problem of (11) is defined as follows:

**Dual:**

\[
\begin{align*}
\text{min} & \quad \sum_{i,j} y_{ij} x_{ij}^*/\rho + \omega \\
\text{s.t.} & \quad \sum_{i,j} y_{ij} x_{ij}^q + \omega \leq 1, \quad \forall q \in \Omega \\
& \quad y_{ij} \geq 0, \quad \omega \geq 0 \forall i, j
\end{align*}
\]  

(15)

(16)

(17)

**Theorem 2:** Both the LP primal problem and the dual problem can be solved in polynomial time and the optimal value of objective function is 1.

**Proof:** First, it is easy to prove that \( (y_{ij} = 0, \omega = 1) \) is a feasible solution to the dual problem. In such a solution, the value of objective function is 1. Thus the optimal value for the dual problem is at least 1.

Next, we will verify that the optimal value of objective function is 1 by way of contradiction. Suppose

\[
\sum_{i,j} y_{ij} x_{ij}^*/\rho + \omega > 1. \tag{18}
\]

Then we have

\[
\sum_{i,j} y_{ij} x_{ij}^q / \rho > 1 - \omega. \tag{19}
\]

According to the first constraint in the primal LP problem, there exists at least a \( q \in \Omega \) that satisfies

\[
x_{ij}^q \geq x_{ij}^*/\rho. \tag{20}
\]

Combining (19) and (20), we have

\[
\sum_{i,j} y_{ij} x_{ij}^q > 1 - \omega. \tag{21}
\]

This violates the first dual constraint. Therefore the optimal value of the objective function is 1. Owing to the strong LP duality, the objective value of primal LP is 1 as well.

This completes the proof.

Although the dual problem has an exponential number of constraints, it can be solved by ellipsoid method in polynomial time [36]. However, in order to make the dual LP solvable, we need to design an approximation algorithm serving as a separation hyperplane for the ellipsoid method [33]. This approximation algorithm will be discussed in Algorithm 2 in next subsection. Each hyperplane corresponds to a constraint in the dual problem, providing a feasible integer solution \( X^q \) as well as its corresponding selected probability \( \lambda^q \). Hence, we can obtain the weights of the convex decomposition that sum up to 1 and solve the primal LP in polynomial time.

**Step 3 (Pick the Integer Solution With \( \lambda^q \)):** In this step, each possible integer solution is selected with a probability equal to its corresponding convex multiplier, which can be obtained in the convex decomposition in step 2. Then we have

\[
\sum_{q \in \Omega} \sum_{i,j} \lambda^q p_{ij} x_{ij}^q - cK \geq OPT_{LPR}/\rho. \tag{22}
\]

The left side represents the achievable social welfare using the approximation algorithm and the right side is the value of the objective function for LPR of the WDP when the optimal solution is \( X^* \). From the above inequality, we can note that the proposed decomposition algorithm can achieve an approximation ratio of \( \rho \).

**B. APPROXIMATION ALGORITHM FOR WDP:**

The randomized rounding algorithm requires an efficient approximation algorithm to solve the WDP. Such approximation algorithm should be able to compute a feasible integer solution efficiently. Here, we present a bidding strategy based on greedy algorithm, which is shown in the following steps:

**Step 1:** In each access period, every application system will compute the ratio of price to bandwidth demand, that is

\[
\theta_{ij} = p_{ij}/b_{ij}, \quad \forall i, j
\]

Here, \( \theta_{ij} \) represents the price for unit spectrum.

**Step 2:** The bid with maximum \( \theta_{ij} \) wins the bidding.

**Step 3:** Remove system \( i \) from the bidding. Then go back to Step 2 until either one of the following termination conditions is satisfied:

i) The GD has not enough bandwidth to support the demand;

ii) All the systems have won one bid.

The proposed greedy approximation algorithm is described in detailed in Algorithm 2, as shown in the following.

**VI. SIMULATION RESULTS**

In this section, the simulation is conducted to evaluate the proposed trading based spectrum sharing method. The number of application systems \( m \) is set as 5, in order to show the performance for a relatively large-scaled scenario, the number of bids will vary from 10 to 80. Before submitting the bids, each system needs to predict its bandwidth demand based on the traffic load in the current access period. In order to model this random characteristic, both the price \( p_{ij} \) and bandwidth demand \( b_{ij} \) are randomly generated between 1 and 10. We obtain the average social welfare by simulating 1000 times.
Algorithm 2 A Greedy Approximation Algorithm

Require: \((p_{ij}, b_{ij}) \in B_i, \forall i, j: \text{bids;}
K: \text{Reservation bandwidth.}\)

Initialization: \(\theta = [0]_{m \times n}, X = [0]_{m \times n}, \beta = 0.\)

1: for all \(i, j\) do
2: compute \(\theta_{ij} = p_{ij} / b_{ij};\)
3: end for
4: Let \(\beta = \max(\max(\theta = [\theta_{ij}]_{m \times n});\)
5: \([\text{row, col}] = \text{find}(\beta == \theta);\)
6: while \(K \geq b_{\text{row, col}}\) do
7: \(x_{\text{row, col}} = 1;\)
8: if \(x_{\text{row, col}} = 1\) then
9: for all \(j\) do
10: \(\theta_{\text{row, j}} = 0;\)
11: end for
12: end if
13: \(K = K - b_{\text{row, col}};\)
14: \(\beta = \max(\max(\theta = [\theta_{ij}]_{m \times n});\)
15: \([\text{row, col}] = \text{find}(\beta == \theta);\)
16: end while
17: Output: \(X = [x_{ij}]_{m \times n};\)
18: return rules

A. EVALUATION OF OUR PROPOSED ALGORITHM

To provide a better understanding of how our proposed greedy approximation algorithm performs, we first evaluate the performance of Algorithm 2. Fig. 3 and Fig. 4 compare the social welfare for the proposed algorithm as well as the optimal value obtained using exhaustive search. From Fig. 3 and Fig. 4, we observe that Algorithm 2 achieves an impressive performance, approaching the optimum rather closely in most cases with a maximum performance loss of 4.6% for \(K = 40\) and 4.8% for \(K = 80\). This result illustrates that our proposed greedy algorithm is reasonable and can achieve a close-to-optimal performance. Moreover, it can also be noted that the social welfare increases as the number of bids increases. This is because that, as the number of bids increases, GD can select a more favorable bid with higher price for the unit spectrum in order to receive larger social welfare.

B. EVALUATION OF PREDICTION ERROR

Furthermore, we study the performance of the proposed greedy approximation algorithm when the bandwidth demand prediction is not perfect. To model this, we assume that each application system may underestimate its near future bandwidth demand by a percentage between 5% and 40%. As illustrated in Fig. 5, the social welfare decreases linearly as the prediction error grows. The reason is that as the bandwidth demand prediction error increases, there is a corresponding reduction in the bidding price, which will result in a decrease in the social welfare. Furthermore, from Fig. 5, we can also observe that the social welfare increases with an increase in the number of bids. This is reasonable due to the fact that a larger number of bids indicates more favorable bids with higher \(\theta_{ij}\) will win the bidding.
C. PERFORMANCE COMPARISON WITH LEASING SCHEME

In the following, we compare our proposed trading based spectrum sharing strategy with spectrum leasing scheme. We use the same spectrum demand and price. Hence, in this leasing scheme, the CBS first decides the available TV spectrum amount by querying the GD, and then sets the expected spectrum leasing price for its connected-users to purchase. In this simulation, the expected price for unit spectrum is used as the leasing price. The simulation result is illustrated in Fig. 6. As a comparison, the social welfare of our method is also shown. It can be seen that, the social welfare achieved by using the spectrum leasing scheme is less than that obtained using our proposed scheme when the number of bids increases to 40.

D. COMPLEXITY EVALUATION

In the following, we will study the performance of complexity over the number of bids submitted by all the systems. Since the complexity of the greedy algorithm lies primarily in sorting $\theta$, which has been bounded by $O(N \log_2 N)$, where $N = \sum n_i$ denotes the number of bids submitted by all the systems. Thus the complexity highly depends on the number of bids. Obviously, this algorithm can be implemented in polynomial time. As illustrated in Fig.7, our proposed algorithm is much simpler than the original NP hard problem where the optimal value is obtained using exhaustive search, especially for a large number of bids. More specifically, Fig. 8 displays the time cost of the greedy approximation algorithm when the number of bidders increases from 10 to 80. These results show that the approximation algorithm can achieve a close-to-optimal solution with less complexity.

VII. CONCLUSION

In this paper, we focus on the TV spectrum sharing among heterogeneous networks and application systems, which has not been studied before. We propose a trading mechanism for the GD-to-application spectrum market with the GD acting as the auctioneer who reserves the TV bandwidth from the licensee and sells it to the application systems. Each system submits its bid containing price and bandwidth demand to the GD. With the objective to maximize the social welfare, the spectrum sharing problem is formulated as a 0-1 integer optimization problem. We prove that this problem is NP-complete and propose a randomized rounding algorithm to solve it. Evaluation results show that the proposed approximation algorithm can achieve a close-to-optimal solution with less complexity.

REFERENCES


WENJIE ZHANG received the B.E. degree in applied mathematics from the University of Electronic Science and Technology of China, Chengdu, China, in 2008, and the Ph.D. degree in computer engineering from Nanyang Technological University, Singapore, in 2014. From 2013 to 2014, he was a Post-Doctoral Research Fellow with the Department of Information Engineering, The Chinese University of Hong Kong, Hong Kong. In 2014, he joined the faculty of the Department of Computer Science and Engineering, Minnan Normal University, Zhangzhou, China. His research interests include cognitive radio networks, TV white spaces, and wireless communications. He serves as a Technical Program Committee Member for the IEEE VTC2012-Spring, ISA2017, IWWCN2016, CSA2017, and WCNA2017. He serves as an Editor on the Editorial Board of the Journal of Wireless Communication and Sensor Network.

GUANGLIN ZHANG received the B.S. degree in applied mathematics from Shandong Normal University in 2003, the M.S. degree in operational research and control theory from Shanghai University in 2006, and the Ph.D. degree in electronic engineering from Shanghai Jiao Tong University in 2012. From 2013 to 2014, he was a Post-Doctoral Research Associate with the Institute of Network Coding, Chinese University of Hong Kong. He is currently an Associate Professor and the Department Chair of the Department of Communication Engineering, Donghua University. His research interests include capacity scaling of wireless networks, vehicular networks, smart micro-grid, and energy management of data centers. He serves as a Technical Program Committee Member for the IEEE GLOBECOM from 2016 to 2017, the Mobile and Wireless Networks Symposium of the IEEE ICC 2015 2017, the Signal Processing for Communications Symposium of the IEEE ICC 2014 2017, the IEEE VTC2017-Fall, the Wireless Networking and Multimedia Symposium of the IEEE/CIC ICCC 2014, and the Future Networking Symposium and Emerging Areas in Wireless Communications of WCSP 2014, APCC 2013, and WASA 2012. He serves as the Local Arrangement Chair of ACM TURC 2017. He serves as Editor on the Editorial Board of China Communications Journal and the Journal of Communications and Information Networks. He is also an Associate Editor of the IEEE Access Journal.
YIFENG ZHENG received the B.E. degree in computer science and technology from Minnan Normal University, Zhangzhou, China, in 2004, and the M.E. degree in computer technology from the China University of Petroleum, Beijing, China, in 2016, where he is currently pursuing the Ph.D. degree. In 2004, he joined the faculty of the Department of Computer Science and Engineering, Minnan Normal University, Zhangzhou, China. His research interests include network communications, artificial intelligence, and machine learning.

LIWEI YANG received the B.E. degree in telecommunication engineering from the Chongqing University of Posts and Telecommunications, China, in 2003, and the Ph.D. degree in information and communications engineering from the Beijing University of Posts and Telecommunications, China, in 2009. From 2009 to 2011, she was a Post-Doctoral Research Fellow with the Department of Electronic Engineering, Tsinghua University, China. In 2014, she joined the faculty of the College of Information and Electrical Engineering, China Agricultural University. Her research interests include optical networks, visible light communication, and wireless communications.

CHAI KIAT YEO received the B.E. (Hons.) and M.Sc. degrees in electrical engineering from the National University of Singapore, in 1987 and 1991 respectively, and the Ph.D. degree from the School of Electrical and Electronics Engineering, Nanyang Technological University (NTU), Singapore, in 2007. She was a Principal Engineer with Singapore Technologies Electronics and Engineering Ltd., prior to joining NTU in 1993. She was the Deputy Director of the Centre for Multimedia and Network Technology, NTU, Singapore. She is currently an Associate Chair (Academic) with the School of Computer Engineering, NTU. Her research interests include ad hoc and mobile networks, overlay networks, speech processing, and enhancement.

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