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<td><strong>Author(s)</strong></td>
<td>Liu, Aifei; Baker, Christopher J.; Teh, Kah Chan; Sun, Hongbo; Gao, Caicai</td>
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Eigensubspace method for space–time adaptive processing in the presence of non-i.i.d. clutter and array errors

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Abstract: This study examines space–time adaptive processing in the presence of non-independent and identically distributed (i.i.d.) clutter and array errors. The authors propose a clutter rank estimation method by exploring the spatial–temporal steering vectors of clutter. The proposed method is independent of clutter statistics and direction-independent array errors. They prove that when the proposed clutter rank estimation is used, the estimate of the clutter subspace is asymptotically independent of clutter statistics. This enables an eigensubspace method to acquire the asymptotic independence on clutter statistics. In addition, they prove that the eigensubspace method can suppress the clutter regardless of direction-independent array errors. They also suggest a geometrical non-homogeneity detector for the eigensubspace method. Simulation and experimental results with multi-channel airborne radar measurement (MCARM) data confirm that the eigensubspace method can suppress non-i.i.d. clutter such as discrete clutter as well as correlated clutter regardless of array gain-phase errors. The ability to suppress clutter regardless of clutter statistics and direction-independent array errors makes the eigensubspace method unique and feasible to the practical scenario when clutter is non-i.i.d. and the direction-independent array errors are present.

1 Introduction

Space–time adaptive processing (STAP) is an effective solution for airborne radar to suppress the significant clutter and thus better indicate the presence of slowly moving targets [1]. Conventional statistical STAP methods such as sample matrix inversion (SMI) and modified SMI (MSMI) methods require that the samples/secondary data are independent and identically distributed (i.i.d.) and the number of the secondary data is not less than twice the dimension of the AP [2]. In practice, it is generally difficult to satisfy this requirement and the analysis of measured data shows that the airborne radar environment can be quite non-homogeneous [3]. For example, urban areas and land–sea interfaces result in the problem of large variations in terrain over a relatively short distance. In [4], the clutter non-homogeneity is analysed. Thus, partially adaptive SMI-based STAP algorithms (for example, the MSMI method in the element-space post-Doppler (ESPD) domain called the MSMI-ESPD method for short) [1] are preferred compared with fully adaptive SMI-based STAP algorithms. In addition, the eigenanalysis-based methods, for example, the principal component-inverse (PCI) method [5] and the minimum norm eigencanceller [6], were developed to reduce the required sample support.

The clutter rank estimation is important for the eigenanalysis-based method. The clutter rank can be predicted by Brennan’s rule [7]. However, Brennan’s rule underestimates the clutter rank in some cases, e.g. when the crab angle (the angle between the antenna reference direction and the flight direction of the platform) is not equal to zero. In [8], a clutter rank calculation method based on the first-order Taylor expansion was proposed; however, it provides an accurate clutter rank estimate only for the angle bins near the centre of mainbeam. Neither the minimum description length (MDL) nor the Akaike information criterion (AIC) [9] is suitable for the clutter rank estimation in the STAP application since the estimated clutter-plus-noise covariance matrix is usually rank deficient. In [1], the clutter rank can be estimated based on the sharp drop of the eigenspectra of the clutter-plus-noise covariance matrix, and thus it is dependent on clutter statistics.

Moreover, reported research also examined the non-homogeneity detector (NHD), which chooses the i.i.d. secondary data for estimating the clutter-plus-noise covariance matrix more accurately [10–12]. The general inner product (GIP) was proposed in [10] as an NHD. In [11], the output of the joint domain localised method in the localised processing region was used as an NHD. In [12], an NHD was proposed using the subaperture smoothing technique. In [13], the devised covariance matrix based on the geometric median is used to exclude the outliers from the secondary data. In [14], the information of the terrain segmentation is explored to select the appropriate training data and different STAP processing tailored to each region of interest. In [15, 16], the prior knowledge of the terrain and a data-adaptive training selector were utilised together to eliminate the non-homogeneous clutter.

We focus on the detailed investigation on the PCI method. Herein, we rename it as the eigensubspace method since we use our proposed clutter rank estimation for estimating the clutter subspace and we obtain two new insights which are directly related to the eigensubspace of the sampled clutter-plus-noise covariance matrix. It is a well-recognised conclusion that the eigensubspace method requires that clutter in both azimuth and range domains are i.i.d. This assumption strictly limits the practical usage of the eigensubspace method because clutter is commonly non-i.i.d.

We first propose a clutter rank estimation method based on system parameters. Our new method is independent of clutter statistics and direction-independent array errors. Following that, we prove that the clutter subspace estimation from the sampled clutter-plus-noise covariance matrix is asymptotically independent of clutter statistics. On the basis of this proof and the proposed clutter rank estimation, we obtain a new insight that the eigensubspace method is asymptotically independent of clutter statistics. As a result, it can suppress the correlated and discrete clutter in the primary range bin at the same time when the number of samples is sufficiently large. Moreover, it can safely use the non-i.i.d. secondary data. Then, we propose a geometrical non-homogeneous clutter detector (GNHD) to remove the geometrical non-homogeneous clutter but remain the non-i.i.d. clutter. Moreover, we observe that the eigensubspace method can suppress the clutter.
regardless of the direction-independent array errors. Overall, these two new insights broaden the application scenarios of the eigensubspace method. It should be emphasised that these two new insights are valid when the proposed clutter rank estimation is embedded in the eigensubspace method.

It was shown in [17] that the PCI method can suppress discrete clutter via simulation results. However, its ability of discrete clutter suppression was not investigated in detail. In addition, in the PCI method, the clutter rank is estimated by Brennan’s rule.

2 Background

2.1 STAP model

We consider $N$ antennas and $K$ pulses in the STAP model. The spatial–temporal steering vector of one range bin $r$ can then be denoted as

$$s(\theta, f(r)) = b(f(r)) \otimes a(\theta(r))$$  \hspace{1cm} (1)

where $b(f(r)) = [1, e^{i2\pi f(r)}, e^{i4\pi f(r)}, \ldots, e^{i2(\pi f(r)/\lambda)}]^{T}$ is the temporal steering vector, $a(\theta(r)) = [1, e^{i2\pi \theta}, e^{i4\pi \theta}, \ldots, e^{i2(\pi \theta/\lambda)}]^{T}$ is the spatial steering vector, $f(r)$ and $\theta(r)$ are the normalised temporal and spatial frequencies, respectively, and $\otimes$ denotes the Kronecker product.

We assume that the radar operates in the side-looking model and that there is no intrinsic clutter motion. Under these assumptions, $\theta(r)$ and $f(r)$ are independent of $r$. Thus, we omit the range bin $r$ in $\theta(r)$ and $f(r)$ in the subsequent analyses. According to (1), the spatial–temporal steering vector of the $i$th clutter patch can be written as

$$s(\theta_{ci}, f_{ci}) = b(f_{ci}) \otimes a(\theta_{ci})$$ \hspace{1cm} (2)

where $f_{ci}$ and $\theta_{ci}$ are the normalised temporal and spatial frequencies of the $i$th clutter patch, respectively. $f_{ci}$ and $\theta_{ci}$ can be computed according to the system parameters. For example, for a linear array, $f_{ci}$ and $\theta_{ci}$ can be calculated as

$$f_{ci} = \frac{2v_{ci}T_{c}}{\lambda} \cos \theta_{ci}$$ \hspace{1cm} (3)

$$\theta_{ci} = \frac{d}{\lambda} \cos \theta_{ci}$$ \hspace{1cm} (4)

where $v_{ci}$ is the platform velocity, $d$ is the inter-element spacing of the linear array, $T_{c}$ is the pulse repetition interval (PRI), $\lambda$ is the wavelength, and $\theta_{ci}$ is the cone angle between the flight direction and the direction of the $i$th clutter patch.

Since both $f_{ci}$ and $\theta_{ci}$ can be expressed as functions of $\theta_{ci}$, we use $s(\theta_{ci})$ instead of $s(\theta_{ci}, f_{ci})$. For simplicity, we do not consider range ambiguities. The received clutter in the range bin $r$ can then be expressed as [1]

$$x_{r}(r) = \sum_{i=1}^{N_{c}} a_{r}(r) s(\theta_{ci})$$ \hspace{1cm} (5)

where $N_{c}$ is the number of clutter patches and $a_{r}(r)$ is the reflection of the $i$th clutter patch. We denote the clutter covariance matrix as

$$R_{c}(r) = E[x_{r}(r)x_{r}^{H}(r)]$$ \hspace{1cm} (6)

It can be rewritten as

$$R_{c}(r) = A\Sigma(r)A^{H}$$ \hspace{1cm} (7)

where

$$A = [s(\theta_{c1}), s(\theta_{c2}), \ldots, s(\theta_{cN_{c}})]$$ \hspace{1cm} (8)

is an $NK \times N_{c}$ matrix of clutter spatial–temporal vectors and

$$\Sigma(r) = \text{diag}(\tilde{\zeta}_{1}(r), \tilde{\zeta}_{2}(r), \ldots, \tilde{\zeta}_{N_{c}}(r))$$ \hspace{1cm} (9)

where $\tilde{\zeta}_{i}(r)$ denotes the power of the $i$th clutter patch and $\text{diag}(\tilde{\zeta}_{1}(r), \tilde{\zeta}_{2}(r), \ldots, \tilde{\zeta}_{N_{c}}(r))$ denotes a diagonal matrix with diagonal elements $\tilde{\zeta}_{1}(r), \tilde{\zeta}_{2}(r), \ldots, \tilde{\zeta}_{N_{c}}(r)$.

In the presence of the clutter and noise, the received signal $x(r)$ in the range bin $r$ can be expressed as

$$H_{c}: x(r) = x_{c}(r) + n(r)$$ \hspace{1cm} (10)

$$H_{1}: x(r) = x_{1}(r) + x_{c}(r) + n(r)$$ \hspace{1cm} (10)

where $H_{c}$ and $H_{1}$ correspond to the target absence and presence hypotheses, respectively, $x_{1}(r) = \kappa s(\theta_{c1}, f_{1})$, $\kappa$ is the amplitude of the moving target; $s(\theta_{c1}, f_{1})$ is the spatial–temporal steering vector of the moving target, $f_{1}$ and $\theta_{c1}$ are the normalised temporal and spatial frequencies of the moving target, respectively, and $n(r)$ is a Gaussian noise vector with zero mean and $E[n(r)n^{H}(r)] = \sigma_{n}^{2}I_{NK}$.

Note that $\sigma_{n}^{2}$ is the noise power and $I_{NK}$ is an $NK \times NK$ identity matrix. $n(r)$ is independent of $x_{c}(r)$ and $x_{1}(r)$.

Denote $R_{c+1}(r)$ as the clutter-plus-noise covariance matrix under $H_{0}$ in the range bin $r_{0}$. Then $n(r_{0})$ is independent of $x_{c}(r_{0})$, and according to (10), $R_{c+1}(r)$ can be expressed as

$$R_{c+1}(r) = R_{c}(r) + \sigma_{n}^{2}I_{NK}$$ \hspace{1cm} (12)

SMI-based methods use the clutter-plus-noise covariance matrix $R_{c+1}(r)$ to suppress the clutter. $R_{c+1}(r)$ needs to be estimated by the secondary data. We denote the number of guard cells as $2g$ and the set of the guard cells plus the primary cell as $G = \{r_{0} - g, \ldots, r_{0} + g\}$. The clutter-plus-noise covariance matrix can be estimated as

$$\hat{R}_{c+1}(r_{0}) = \frac{1}{M} \sum_{r = r_{0} - g}^{r = r_{0} + g} E[x(r)x^{H}(r)]$$ \hspace{1cm} (13)

where $M$ is the number of the secondary data.

2.2 Clutter non-homogeneity

$\hat{R}_{c+1}(r)$ is the maximum-likelihood estimate of $R_{c+1}(r)$ under the assumption that $R_{c+1}(r) = R_{c}(r)$. According to (12), $R_{c+1}(r) = R_{c}(r)$ means that $R_{c}(r) = R_{c}(r_{0})$. Thus, we define $x(r)$ as homogenous if $R_{c}(r) = R_{c}(r_{0})$; otherwise, $x(r)$ is non-homogenous. From (7) to (9), we observe that the non-homogeneity is caused by two items: (a) the matrix of clutter spatial–temporal vectors and (b) clutter power distribution. Since the first item is related to the geometry of the radar system, e.g. the non-zero angle, the non-homogeneity caused by the first term is named as the geometrical non-homogeneity. In contrast, the second item is related to clutter statistics; therefore, the non-homogeneity caused by the second term is named as the statistical non-homogeneity. For clarification, we call the statistical non-homogeneous clutter as non-i.i.d. clutter.

In practise, the secondary data $x(r)$ may be corrupted by a moving target with a strong reflectivity. In this case, $\hat{R}_{c}(r) \neq R_{c}(r)$ and thus $x(r)$ is non-homogenous. Since a moving target has a different spatial–temporal steering vector from those of clutter, the non-homogeneity caused by a moving target can be considered as the geometrical non-homogeneity. Note that in [18], it has been shown that the contribution of weak moving targets in the secondary data can be ignored when estimating the clutter-plus-noise covariance matrix.
The existing clutter rank estimation methods either incorrectly continue along the azimuth in one range bin. To calculate the Moreover, the eigensubspace method assumes that the clutter assumption strictly limits the practical usage of the eigensubspace method. In Section 4, we have proved that the clutter rank does not change in the presence of array errors, receiver taper, and identical radiation pattern. We consider that array errors include the direction-independent errors such as array gain-phase errors and the mutual coupling matrix [20] and denote the $N \times N$ matrix of array errors as $\Gamma_r$. We denote the $N \times N$ matrix of the array receiver taper as $\Gamma_t$, which is a diagonal matrix with its $(i, i)$th diagonal element equal to the taper for the $i$th receiver antenna, and denote the $N_r \times N_c$ matrix of the identical radiation pattern as $\Gamma_p$, which is a diagonal matrix with its $(i, j)$th diagonal element representing the transmit gain at the direction of the $i$th clutter patch. Note that we did not consider the nulls in the radiation pattern and thus $\Gamma_p$ is full rank. In the case of the nulls in the radiation pattern at some angles, we should exclude the angles corresponding to the nulls when we construct the matrix $A$ in (17), which ensures that $\Gamma_p$ is full rank.

Let $\Gamma_1 = I_k \otimes \Gamma_r$ and $\Gamma_2 = I_k \otimes \Gamma_t$. In the presence of array errors, receiver taper, and radiation pattern, the matrix comprising the practical clutter steering vectors can be written as

$$\tilde{A} = \Gamma_1 \Gamma_2 A \Gamma_p$$.

(19)

For any two matrices $X$ and $Y$, we have

$$\text{rank}(X \otimes Y) = \text{rank}(X)\text{rank}(Y).$$

(20)

According to (20) and due to the fact that $\Gamma_1$ and $\Gamma_2$ are full rank in practise, we obtain that $\Gamma_1$ and $\Gamma_2$ are full rank. Since $\Gamma_p$, $\Gamma_1$, and $\Gamma_2$ are full rank, we can obtain

$$\text{rank}(\tilde{A}) = \text{rank}(A).$$

(21)

On the basis of (21), we know that direction-independent array errors, receiver taper, and radiation pattern do not change the clutter rank and thus the rank of $A$ comprising the practical steering vectors is the same as that of $A$ consisting of the assumed steering vectors. The proposed clutter rank estimation method is shown below:

1. Calculate the assumed clutter steering vector $s(\theta_i)$, $i = 1, 2, ..., N_c$ using system parameters. For example, for the ULA, $s(\theta_i)$ can be calculated according to (2)-(4).
2. Calculate the matrix $A$ by (17).
3. Compute the rank of the matrix $A$, which can be taken as the estimate of the clutter rank in the presence of array errors, receiver taper, and radiation pattern according to (21).

Since Step 1 is based on system parameters, the proposed clutter rank estimation method is independent of clutter statistics. In addition, according to (21), the proposed method is applicable to the practical scenario when direction-independent array errors, receiver taper, and radiation pattern are present. It is worth noting...
that from (32) and (33), it is seen that the $\tilde{x}(r_n)$ is always larger than $\sigma^e$. Therefore, once $p$ is known, the first $p$ largest eigenvalues and their eigenvectors can always be successfully identified, and thus the clutter subspace can be estimated based on the $p$ eigenvectors.

4 New insights of eigensubspace method

Lemma 1: The clutter subspace estimation from the sampled clutter-plus-noise covariance matrix is asymptotically independent of clutter statistics.

The proof of Lemma 1 is given in the Appendix.

The observation of Lemma 1 about the asymptotic independence of the clutter subspace estimation from $\tilde{R}_{c,+}(r_n)$ on clutter statistics may first seem to be unconventional from the perspective of statistical analysis since the clutter subspace matrix asymptotically independent of clutter statistics. However, from the aspect of the subspace theory, the clutter subspace is spanned by the clutter steering vectors which are independent of the clutter reflectivity. Thus, Lemma 1 is sensible. Lemma 1 is verified by the simulation and experimental results of the discrete clutter suppression by the eigensubspace method in Section 6.

In the following, we use the proposed clutter rank estimation for Step 1 of the eigensubspace method to estimate the clutter rank. Thus, Step 1 of the eigensubspace method is independent of clutter statistics. Although Step 2 depends on clutter statistics, from Lemma 1, we know that the estimated clutter subspace $P_c$ is asymptotically independent of clutter statistics. Therefore, we obtain the first new insight about the eigensubspace method below:

New insight 1: The eigensubspace method is asymptotically independent of clutter statistics but depends on the clutter steering vectors. Thus, the secondary data can be non-i.i.d. or i.i.d. but must be geometrically homogeneous.

Since the eigensubspace method is asymptotically independent of clutter statistics, it can suppress the discrete clutter as well as the correlated clutter in the primary data when the number of samples is sufficiently large. In addition, it can safely use the non-i.i.d. secondary data for the clutter-plus-noise covariance matrix estimation. To the best of our knowledge, this is the first work that proves the asymptotic independence of the eigensubspace method on clutter statistics. This new insight is very useful in the STAP application which commonly encounters the problem of non-i.i.d. clutter.

In the presence of array errors, we rewrite the received signal as

$$H_0 \tilde{x}(r_n) = \tilde{x}(r_n) + n(r_n)$$

$$H_1 \tilde{x}(r_n) = \tilde{a}(f_1, \theta_1) + \tilde{x}(r_n) + n(r_n)$$

(22)

where $\tilde{x}(r_n) = \Gamma_1 \tilde{x}(r_n)$ and $\tilde{a}(f_1, \theta_1) = \Gamma_1 v(f_1, \theta_1)$. Correspondingly, we denote the estimated projection matrix of the complementary/}

noise subspace from the estimated practical clutter-plus-noise covariance matrix as $\tilde{P}_c^\perp$. Thus, we can obtain

$$\tilde{P}_c^\perp \tilde{x}(r_n) \approx 0$$

(23)

Equation (23) is obtained based on the fact that the clutter subspace is estimated from the estimated practical clutter-plus-noise covariance matrix and thus it is composed of the practical clutter steering vectors which include array errors. Correspondingly, as shown in (23), the complementary subspace of the estimated clutter subspace is orthogonal to the practical clutter steering vectors. The weight vector of the eigensubspace method is accordingly changed as

$$\tilde{w}(\theta_n, f_n) = \tilde{P}_c^\perp \tilde{x}(f_n, \theta_n)$$

(24)

Recall that the proposed clutter rank estimation used in the eigensubspace method requires that the array errors are direction independent. Thus, according to (23), we obtain the second new insight as follows:

New insight 2: The eigensubspace method can suppress clutter regardless of direction-independent array errors.

In [6], it was proved that the eigensubspace method is robust to array errors under the constraint of one clutter patch/interference. This constraint is not practical because the number of clutter patches is far from one in the practical environment. New insight 2 removes this constraint. It is useful in practice because the direction-independent array errors are commonly unavoidable.

5 Geometrical NHD (GNHD) for eigensubspace method

As shown in Section 4, the eigensubspace method can use the non-i.i.d. secondary data. On the other hand, the geometrical non-homogeneous clutter such as moving targets in the secondary data will degrade the performance of the eigensubspace method. Thus, the requirement on the secondary data in the eigensubspace method is that the secondary data need to be geometrically homogenous but can be non-i.i.d. Therefore, the NHD for the eigensubspace method should be designed to remove the geometrical non-homogeneous secondary data and retain the non-i.i.d. secondary data. Hence, we propose that the target declaration result of the eigensubspace method rather than GIP is chosen as its GNHD. For clarity, the scheme of the eigensubspace method with GNHD is shown in Fig. 2, where the target declaration can be implemented by constant false-alarm rate (CFAR) algorithms [21]. In Fig. 2, the GNHD declares the non-homogeneity for each range bin according to the output of the eigensubspace method. Following that, the eigensubspace method simply excludes the clutter subspace from the secondary data for each primary range bin to estimate the clutter-plus-noise covariance matrix. Since the GNHD is implemented just once before the STAP, the computation complexity of GNHD is $O\left(\frac{NK}{c}\right)^2$ due to the decomposition of the covariance matrix.

Note that since the GIP excludes the secondary data including the geometrical and non-i.i.d. secondary data, the GIP for the eigensubspace method wastes the secondary data which are non-i.i.d. but geometrically homogeneous. Hence, the eigensubspace method with the GIP fails in the practical scenario, where most clutters are non-i.i.d. because of ground variants.

6 Results

6.1 Clutter rank estimation

In this section, we compare the proposed clutter rank estimation method with the MDL and AIC methods [9]. There are $N=10$ elements and $K=10$ pulses in one coherent processing interval (CPI). The other radar system parameters are given in Table 1, where PRF denotes the pulse repetition frequency. We consider that the clutter is distributed from $0^\circ$ to $180^\circ$ in azimuth and $N_s = 9000$
The clutter rank is calculated as 19. From Fig. 3, we observe that application, the number of available samples is generally less than proposed method and the MDL and AIC methods versus the CNR

Table 1 Radar system parameters

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<th>Value</th>
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<td>Operating wavelength ($\lambda$)</td>
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<tr>
<td>Velocity ($v_a$)</td>
<td>150 m/s</td>
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<tr>
<td>PRF</td>
<td>$\frac{4v_a}{\lambda}$ Hz</td>
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<tr>
<td>Number of elements ($N$)</td>
<td>10</td>
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<tr>
<td>Number of pulses ($K$) in one CPI</td>
<td>10</td>
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<tr>
<td>Inter-element spacing</td>
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In this section, we verify the performance of the eigensubspace method using the MCARM data set. The MCARM radar and platform parameters are listed in Tables 2 and 3, respectively. A detailed description of the MCARM system can be found in [22, 23]. The data of 5-575 is used for the methods versus the CNR in Fig. 4, we observe that both AIC and MDL methods estimate the clutter rank incorrectly when the CNR is <25 dB. This is because they estimate the clutter rank by using the eigenvalues which are close to the noise variance in the case of a low CNR. On the other hand, the proposed method uses the steering vectors to estimate the clutter rank and thus it performs independently of CNR.

6.2 Eigensubspace method

In this section, we illustrate the performance of the eigensubspace method in terms of clutter suppression and moving target detection, and compare it with that of the MSMI method.

6.2.1 Simulated data: A discrete clutter with a power level of 60 dB and a moving target with a power level of 10 dB are injected in the primary data. The other simulation settings are the same as those in Section 6.1.

The spatial–temporal spectra of correlated clutter, discrete clutter, and moving target are shown in Fig. 5. In Fig. 5, the correlated clutter lies on the ridge. The spatial and normalised Doppler frequencies of the discrete clutter and moving target are located at (0.2, 0.2) and (0, 0.2), respectively. The mainbeam clutter is the correlated clutter located at the same direction as the moving target. The discrete clutter and moving target have the same Doppler frequency. The array gain and phase errors are generated by two uniform processes with variances 0.01 and 25, respectively.

The angle–Doppler maps of the eigensubspace and MSMI methods in the presence of array gain-phase errors are given in Figs. 6 and 7, respectively. In Fig. 6a, we can see that the output powers along the clutter ridge are low. Furthermore, from Fig. 6b, we observe that the eigensubspace method suppresses the discrete clutter in the presence of array gain-phase errors because of (23); otherwise, the output power at the position of the discrete clutter will be much higher since the discrete clutter has higher power than the correlated clutter around it. On the other hand, Fig. 7 shows that the MSMI method cannot suppress the discrete clutter because it depends on clutter statistics and the discrete clutter in the primary range bin is not included in the secondary data.

6.2.2 Experimental data: In this section, we verify the performance of the eigensubspace method using the MCARM data set. The MCARM radar and platform parameters are listed in Tables 2 and 3, respectively. A detailed description of the MCARM system can be found in [22, 23]. The data of #5-575 is used for the
analysis presented here. For the following results, we use one row from the received elements \((N=11)\) and all available pulses \((K=128)\). We investigate the range–Doppler map from the 250th range bin to the 350th range bin.

Owing to the practical sample limitation, instead of the MSMI method, we use the MSMI-ESPD method for comparisons. For the MSMI-ESPD method, we use \(K_t=3\) Doppler bins and employ \(6NK_t\) samples excluding 20 guard cells adjacent to the range bin of interest. For the eigensubspace method, we get the rank of \(A\) equal to 154 considered as the clutter rank according to Section 3. In [24], it has been shown that the eigenanalysis-based methods require the number of samples to be at least two times the clutter rank. Herein, we adopt 449 samples from the 160th range bin to the 630th range bin excluding 20 guard cells adjacent to the range bin of interest.

(a) Injected discrete clutter suppression and moving target detection: In this section, we investigate the performance of the eigensubspace and MSMI-ESPD methods in terms of suppressing a discrete clutter and detecting a moving target, which are injected in the MCARM data. The injected discrete clutter has amplitude equal to 0.01. It is in the 281st range bin and the 45th angle bin (there are 129 angle bins in azimuth according to the steering array indicated in [22]). The Doppler of the discrete clutter is −477.43 Hz, which is calculated according to the angle–Doppler relationship of clutter. The parameters of injected moving target are set as: target amplitude = 0.01; range bin = 282; angle bin = 65 (broadside); and Doppler = -294.5 Hz. In addition, in order to

![Fig. 6 Eigensubspace method in the presence of array gain-phase errors](image1)

(a) Angle–Doppler map, (b) Principal cuts at target spatial and Doppler frequencies

![Fig. 7 MSMI method in the presence of array gain-phase errors](image2)

(a) Angle–Doppler map, (b) Principal cuts at target spatial and Doppler frequencies

<table>
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<th>Table 3 MCARM platform parameters</th>
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guarantee that the injected discrete clutter and moving target are consistent with the true environment of the MCARM data, we use the measured spatial steering vectors for generating the injected discrete clutter and moving target. In contrast, we use the assumed spatial steering vector at broadside in the two methods. Thus, we can treat the following obtained results as the performance of the methods in the presence of array errors.

The range–Doppler maps of the eigensubspace and MSMI-ESPD methods without NHD are shown in Figs. 8a and b, respectively. From Fig. 8a, we observe that the injected discrete clutter can be suppressed by the eigensubspace method. On the other hand, from Fig. 8b, we observe that the injected discrete clutter cannot be suppressed by the MSMI-ESPD method. In addition, the eigensubspace and MSMI-ESPD methods can indicate several similar possible moving targets as described in Section 6.2.2.(c). This implies that the eigensubspace and MSMI-ESPD methods can suppress correlated clutter since the possible moving targets are somehow masked by correlated clutter before STAP processing. On the other hand, we observe that neither the eigensubspace method nor the MSMI-ESPD method has clear output at the position of the injected moving target. This is because, in the absence of NHD, the return in the 299th range bin, which has the same Doppler as the injected moving target, is included in the secondary data for the clutter-plus-noise covariance matrix estimation. Thus, this return is considered as clutter in the eigensubspace and MSMI-ESPD methods, causing the suppression of the injected moving target.

(b) NHD: In this section, we investigate the effect of NHD on the detection of the injected moving target. The NHD for the MSMI-ESPD method is implemented according to [11]. Figs. 9a and b show the range–Doppler maps of the eigensubspace method with GNHD and the MSMI-ESPD method with NHD, respectively. By comparing Figs. 8 and 9, we can observe that both the eigensubspace and MSMI-ESPD methods with NHD have a stronger activity at the position of the injected moving target than those without NHD. This is due to the fact that the NHDs of the eigensubspace and MSMI-ESPD methods exclude the return, which is in the 299th range bin and has the same Doppler as the injected moving target, from the secondary data.

(c) Target declarations: Finally, we set $P_{fa} = 10^{-6}$ and show the target declaration results of Fig. 9 in Fig. 10. From Fig. 10a, we observe that the eigensubspace method with GNHD can detect the injected moving target. At the same time, it can eliminate the injected discrete clutter. From Fig. 10b, we observe that the MSMI-ESPD method with NHD can also detect the injected moving target. However, it cannot remove the injected discrete clutter. In
addition, in Fig. 10b, the false alarms 1–3 in the red circles are declared according to their velocities as −36.11, −43.61, and −34.24 m/s, respectively, which are obviously higher than the speed limit of 29.1 m/s [25] in Delaware illuminated by the MCARM radar mainbeam [26]. The rest of the declared targets in Figs. 10a and b have velocities falling in between two intervals [−20.98 m/s, −11.38 m/s] and [12.62 m/s, 27.02 m/s]. It is thought that these are moving targets as their velocities are lower than the speed limit of 29.1 m/s. This corresponds to vehicles on the several highways that are illuminated by the MCARM radar mainbeam and it is consistent with previously reported results [27].

7 Conclusion

This paper aims to address the problem of clutter suppression in the scenario of non-i.i.d. clutter and array errors. To accomplish this aim, we have first proposed a clutter rank estimation method. The proposed method does not require the estimate of the clutter-plus-noise covariance matrix and it is independent of clutter statistics. Moreover, it estimates the clutter rank correctly regardless of direction-independent array errors, receiver taper, and radiation pattern. Following that, we have proved that, when the proposed clutter rank estimation is used, the clutter subspace estimation from the estimated clutter-plus-noise covariance matrix is asymptotically independent of clutter statistics. Consequently, we have obtained the first new insight, i.e. the eigensubspace method is asymptotically independent of clutter statistics but it depends on clutter steering vectors. Correspondingly, for the eigensubspace method, we have developed a GNHD to remove the geometrical non-homogeneous secondary data but keep the non-i.i.d. secondary data. Moreover, we have obtained the second new insight, i.e. the eigensubspace method can suppress clutter regardless of direction-independent array errors.

Simulation results have been provided to verify the effectiveness of the proposed clutter rank estimation and the capability of the eigensubspace method to suppress discrete and correlated clutter, and show that this capability is independent of direction-independent array errors. Experimental results with the MCARM data further confirmed the above-mentioned capability. Moreover, experimental results demonstrated the target gain obtained from the GNHD for the eigensubspace method. Since the eigensubspace method can suppress clutter regardless of clutter statistics and direction-independent array errors, it is a practical solution for the STAP application when non-i.i.d. clutter and direction-independent array errors are present.

8 References

independent of and (26), we can express determined by the spatial–temporal steering vectors of clutter.

According to (25), we have


The clutter subspace is defined as the subspace spanned by the clutter steering vectors. It can be estimated by the rank estimation method proposed in Section 3. Define the clutter subspace as \( \mathcal{S}_c \). Note that the rank of \( \mathcal{S}_c \) is \( p \). According to (5), we can obtain

\[ \mathcal{S}_c = \text{span}\{s(\theta_1), s(\theta_2), \ldots, s(\theta_{N_{\text{cl}}})\}. \]  

Assume that one orthogonal basis of clutter subspace \( \mathcal{S}_c \) is \( \{u_1, u_2, \ldots, u_p\} \). According to (25), we have

\[ \text{span}\{s(\theta_1), s(\theta_2), \ldots, s(\theta_{N_{\text{cl}}})\} = \text{span}\{u_1, u_2, \ldots, u_p\} \]  

We denote \( U = [u_1, \ldots, u_p] \). According to (26), \( U \) can be determined by the spatial–temporal steering vectors of clutter patches and is independent of clutter statistics. According to (5) and (26), we can express \( x(r) \) by the orthogonal basis \( U \) and the corresponding coefficients \( \beta(r) \) as

\[ x(r) = \sum_{i=1}^{p} \beta_i(r) u_i = U \beta(r) \]  

where \( \beta(r) = [\beta_1(r), \beta_2(r), \ldots, \beta_p(r)]^T \). By substituting (27) into \( H_c \) in (10) and (13), we can estimate the clutter-plus-noise covariance matrix as

\[
\hat{R}_{c+n}(r) = \frac{1}{M} \sum_{r} \beta(r) \beta_h^H(r) U H \sum_{r} \beta(r) \beta_h^H(r) U H + \frac{1}{M} \sum_{r} n(r) n_h^H(r) U H + \frac{1}{M} \sum_{r} n(r) n_h^H(r)
\]  

where \( M \) is the number of the secondary data. Note that \( \beta(r) \) is independent of \( n(r) \) and \( n(r) \) is zero-mean. Therefore, when \( M \rightarrow \infty \), we can rewrite (28) as

\[
\hat{R}_{c+n}(r) = \frac{1}{M} U R_h(r) U H + \sigma_n^2 I
\]  

where

\[ R_h(r) = \sum_{r} \beta(r) \beta_h^H(r) \]  

and its dimension is \( p \times p \). Denote

\[ B = \begin{bmatrix} \beta(r_0 - g - M/2) & \ldots & \beta(r_0 - g - 1) \beta(r_0 + g + 1) \\ \ldots & \ldots & \ldots \end{bmatrix} \]  

We then rewrite \( R_h(r) \) as \( R_h(r) = B B^H \). In practise, it is realistic to assume that \( \beta(r) \) and \( \beta(r) \) for \( i \neq j \) are independent since \( \beta(r) \) and \( \beta(r) \) are caused by different ground patches. This means that rank(B) = \( p \) when \( M \geq p \). Note that the independence of the statistics for two different clutter patches was also employed in [1]. In practise, \( M \) can be larger than \( p \) because \( p \) is the clutter rank and it is much smaller than the spatial–temporal dimension \( NK \). Since rank\( [R_h(r)] = \text{rank}(B) = p \), we obtain that \( R_h(r) \) is full rank. Decomposing \( U R_h(r) U^H \) leads to

\[ \frac{1}{M} U R_h(r) U^H = \sum_{i=1}^{p} \xi_i(r_0) h_i(r_0) h_i^H(r_0) \]  

where \( \xi_i(r_0) \) and \( h_i(r_0) \) are the \( i \)th eigenvalue and its corresponding eigenvector, respectively. 

By decomposing \( \sigma_n I \), we have

\[ \sigma_n^2 I = \sigma_n^2 \sum_{i=1}^{p} h_i(r_0) h_i^H(r_0) + \sigma_n^2 \sum_{i=1}^{NK-p} e_i(r_0) e_i^H(r_0) \]  

where \( e_i(r_0) \), \( i = 1, 2, \ldots, NK - p \), are the eigenvectors which are orthogonal to \( h_i(r_0) \), \( i = 1, 2, \ldots, p \). According to (30) and (31), \( \hat{R}_{c+n}(r) \) in (29) can be decomposed as

\[ \hat{R}_{c+n}(r) = \sum_{i=1}^{p} \xi_i(r_0) h_i(r_0) h_i^H(r_0) + \sigma_n^2 \sum_{i=1}^{NK-p} e_i(r_0) e_i^H(r_0) \]  

where

\[ \xi_i(r_0) = \xi_i(r_0) + \sigma_n^2 \]  

According to (29), (32), and the subspace theory, we obtain

\[ \text{span}\{h_i(r_0), h_j(r_0), \ldots, h_p(r_0)\} = \text{span}\{u_1, u_2, \ldots, u_p\} \]  

According to (25), (26), and (34), we have

\[ \text{span}\{h_1(r_0), h_2(r_0), \ldots, h_p(r_0)\} = S_c \]  

From (32) and (35), we can observe that the clutter subspace \( S_c \) can be calculated from the eigendecomposition of \( \hat{R}_{c+n}(r) \). Since \( M \rightarrow \infty \) is required to get (35) and \( S_c \) is determined by the clutter steering vectors regardless of clutter statistics, (35) implies that the clutter subspace calculated from \( \hat{R}_{c+n}(r) \) is asymptotically independent of clutter statistics. Note that different clutter statistics may lead to different orthogonal bases of the same clutter subspace \( S_c \).