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A RAPID DESIGN EXPLORATION FRAMEWORK UNDER ADDITIVE MANUFACTURING PROCESS UNCERTAINTY

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ABSTRACT: Current computer-aided design (CAD) systems are not computationally efficient for exploring the growing design space of products manufactured with additive manufacturing (AM). A rapid method with sufficient fidelity is desired. Meanwhile, uncertainties within the additive manufacturing process should be adequately considered. This study presents a rapid design exploration framework which considers the additive manufacturing process uncertainty based on metamodels. The framework is demonstrated with a multi-material designed stiffness construct design case, with an embedded horseshoe structure, which is manufactured with projection-based stereolithography (SLA) process.

KEYWORDS: Design for additive manufacturing, computer-aided design, design space exploration, uncertainty assessment.

INTRODUCTION
State-of-the-art studies in the literature have demonstrated successful applications with additive manufacturing technologies to fabricate multi-material devices with multiple functionalities. The product design space has been dramatically enlarged by these advances and is limited by much less manufacturing constraints compared with the conventional approach. However, such a paradigm shift is not sufficiently reflected within current CAD systems which mainly suit designing products in mass production. In this design workflow, a baseline product is first created and then modified iteratively through high-fidelity analysis and simulations which are computationally expensive. The potential of additive manufacturing is not fully exploited in the current scheme. Moving towards design for additive manufacturing (DfAM), a CAD system for AM shall assist the designer to rapidly explore the entire design space with sufficient fidelity while ensuring that manufacturing constraints are not violated.

Nowadays, engineers are fully aware of additive manufacturing constraints, e.g., overhang and undercut structures which have formed as guidelines for designing AM products. However, uncertainties within the additive manufacturing process have not been adequately considered in the design phase, while an often-taken approach is to minimize the uncertainty through process improvements. To have a more realistic prediction about manufactured parts, the information about the manufacturing process, in addition to CAD representations, should be integrated into the performance analysis.

To address the above challenges, this paper presents a rapid design exploration framework under AM process uncertainty. The framework utilizes computationally efficient surrogate models, also called “metamodels,” which are trained through a set of observations of computationally expensive high-fidelity models. The framework is demonstrated with a multi-material designed stiffness construct which is manufactured with projection based stereolithography.
FRAMEWORK
The proposed framework is illustrated in Figure 1. The underlying concept of the framework is the process-structure-property relationship which stemmed from the design of hierarchically structured materials by G. B. Olson (1997). It was later introduced to DfAM by Rosen (2007). As discussed, the current CAD system explores the design space by utilizing high fidelity models to examine the performance of design variations. In other words, the time cost increases linearly with the complexity of the design space. It becomes an intractable approach to design products for additive manufacturing which can be multi-material, multi-functional, and multi-scale. In the proposed framework, only small amounts of high fidelity simulations are conducted to generate a dataset which features the mapping from the design space to the performance space. The observations are then used to train the metamodels, which are computationally efficient. Also, the propagation of the manufacturing process uncertainty is considered in the final performance prediction.

Figure 1 The proposed rapid design exploration framework under additive manufacturing process uncertainties.

High Fidelity Models
The high-fidelity model \( f_{h} : \mathbb{R}^{d}_{D} \rightarrow \mathbb{R}^{m}_{P} \) maps an n-dimensional design space \( \mathbb{R}^{n}_{D} \) to an m-dimensional performance space \( \mathbb{R}^{m}_{P} \), where n is the number of design variables and m is the number of performance metrics of interest. Computational simulation methods, e.g., finite element (FE) analysis and computational fluid dynamics (CFD) are often used as high-fidelity models. To reduce the complexity of design, design parameters are subjected to screening and sensitivity analysis to rule out those which are both insignificant to the performance and have no correlations with others. Then, a sampling plan \( X \) with k sites in the design space \( (X_{n \times k} \subset \mathbb{R}^{n}_{D}) \) is created by the design of experiment (DOE) methods, e.g., Latin Hypercube Sampling, Orthogonal Arrays. Corresponding observations \( Y \) in the performance space \( (Y_{m \times k} \subset \mathbb{R}^{m}_{P}) \) are obtained through high-fidelity simulations.

Metamodels
Metamodels \( f_{mt} \) are often used in simulation-based design to approximate the high-fidelity model \( f_{h} \) which reduces its computation expense and yet retains a sufficient level of fidelity. Widely used metamodels include response surface, radial basis function, support vector regression, and Gaussian process model. Simpson et al. (2001) offers a comprehensive review on this topic. In this study, the Gaussian process regression (GPR) model is selected because it not only allows for a rapid design exploration but also provides insights on the prediction uncertainty.

The design exploration problem, which is predicting performance responses, \( Y^{*} \), on unexplored designed sites, \( X^{*} \), based on \( f_{mt} \), can be expressed as the conditional probability, \( \hat{Y}^{*} = f_{mt}(X^{*}) = P \)
(Y*|Y, X, X*) (denoted as P(Y*|Y)). The response in the performance space is assumed to have its finite-dimensional distribution as Gaussian, Y~GPS(μY, ΨY), and Y*~GPS(μY*, ΨY*), where μ is the mean value and Ψ is the covariance function which takes different kernels, e.g., squared exponential and kriging functions. Subscripts denote their specific values respect to Y and Y*. Based on the Bayes’ Theorem, P(Y*|Y) = P(Y, Y*)/P(Y), the Gaussian distributed P(Y*|Y) is expressed as:

\[
 f_{\text{mt}}(X^*) = P(Y^*|Y) \sim GPS(\mu_{Y^*} + \Psi_{Y,Y^*}^{-1}(Y - \mu_Y), \Psi_{Y^*} - \Psi_{Y,Y^*}^{-1}\Psi_{Y,Y^*}) 
\]  (1)

The set of mean values, μ, can be represented using a polynomial regression model ȕ+H. H is a set of basis functions for the design parameters which can take the 0th, 1st, or higher order forms; ȕ is the corresponding coefficient vector and its prior is Gaussian,  \( \mu \sim GPS(0, \Psi) \). The best prediction  \( \hat{Y}^* \) in (2) is updated based on the expression proposed by Rasmussen and Williams (2006):

\[
 \mu_1 = E(f_{\text{mt}}(X^*)) = H_{Y^*}^T \beta + \Psi_{Y,Y^*}^{-1}(Y - H_Y^T \beta). 
\]  (2)

The metamodel approximation uncertainty is:

\[
 \sigma_1^2 = \text{Var}(f_{\text{mt}}(X^*)) = \Psi_{Y^*} - \Psi_{Y,Y^*}^{-1}\Psi_{Y,Y^*}^{-1} + R^T(B^{-1} + H_Y K_Y^{-1} H_Y^T)^{-1} R, 
\]  (3)

where, \( \beta = (B^{-1} + H_Y K_Y^{-1} H_Y^T)^{-1}(H_Y K_Y^{-1} Y + B^{-1} b) \) and \( R = H_Y - H_Y K_Y^{-1} H_Y^T \).

In addition to the metamodel approximation uncertainty, the manufacturing uncertainty exists which results in the actual input becoming \( X^* q \) rather than the nominal design input, \( X^* \). The manufacturing uncertainty, \( q \), is assumed Gaussian, \( q \sim GPS(\mu_q, \sigma_q) \). Sources of uncertainties in the additive manufacturing process are plenty, e.g., the uncontrollable oxygen inhibition in the SLA process, the stochastically distributed powder size in the selective laser melting process, etc. Recent efforts attempt to quantify process uncertainties using multiscale computational models. An alternative approach is to characterize process uncertainty with an empirical statistical model based on experiments. Without considering the metamodel approximation uncertainty and using the best prediction, \( \mu_1 \), Equation (2) is updated to include the uncertainty \( q \) as:

\[
 \mu_2 = E(\mu_1(X^* + q)) = \int \mu_1(X^* + q) p(q) dq, 
\]  (4)

and its variance due to manufacturing uncertainty only is:

\[
 \sigma_2^2 = \text{Var}(\mu_1(X^* + q)) = \int(\mu_1(X^* + q) - \mu_2(X^* + q))^2 p(q) dq, 
\]  (5)

where, \( p(q) \) is the probability density function.

Considering both metamodel approximation and manufacturing uncertainties, the best prediction \( Y^* \) for the design inputs \( X^* \) is given as Zhou et al. (2017):

\[
 \mu_3 = \mu_2, 
\]  (6)

and the uncertainty about this prediction is:

\[
 \sigma_3^2 = \sigma_2^2 + \int \text{Var}(f_{\text{mt}}(X^* + q)) p(q) dq. 
\]  (7)

**CASE STUDY**

The design case of interest is a multi-material construct with designed stiffness characteristics. The idea is to selectively stiffen an elastomer, hydrogel, or other low-modulus material with an embedded design structure composed of a material with a much higher modulus. Horseshoe structures were selected as the embedded structure since they have a high area coverage ability and...
large stretchability which are desired characteristics for target applications. As shown in Figure 2, a simple horseshoe structure is selected as a demo case in this study. The horseshoe structure is controlled by three design parameters, radius (R), length (L), and beam width (W), which all have their units in mm. The stress-strain curve is the performance characteristic of interest in this study.

**FEM Modeling**

To characterize the stress-strain relation, the horseshoe structure is modeled with FE models using ABAQUS/Standard. The model is meshed with two-node 3D beam elements (B31). One end of the beam is fixed and the other end is applied with a point load. To automate the data generation process for parameter sets defined by DOE, a MATLAB centered simulation framework has been constructed. Parametric studies have been conducted to examine the influence of changes in design parameters on the stress-strain curve, as shown in Figure 2. The stress-strain curve is influenced by different design parameters in different strain ranges, e.g., width variations have a more pronounced effect on the strain under 200%, as shown by the curve family marked in blue. A dataset (X, Y) was generated using FE simulations for a sampling plan $X_{3 \times 125}$ that contains 125 designs with the ranges $R=[1.5, 2.5]$, $L=[9, 11]$, and $W=[1, 2]$. The Gaussian process model ($f_{\text{mul}}$) was trained with the dataset using the Statistics and Machine Learning Toolbox in MATLAB.

![Figure 2](image)

Figure 2 (a) a horseshoe structure and its design parameters; (b) stress-strain curves obtained from FE models for different design variations. R2L10W1.5 is a baseline design and abbreviated for the design comprising dimensions of radius=2 mm, length=10 mm and width=1.5 mm.

**Dimensional Uncertainty**

The process uncertainty affects the performance in different aspects, e.g., geometric dimension, material property and it depends on various conditions, e.g., processes, machines, and materials. Here, only the dimensional uncertainty on the width of the structure is studied to demonstrate the methodology feasibility. Autodesk Ember, which is a projection based SLA machine, was used to manufacture the horseshoe structure. The photopolymer used was a composition of Vero Clear (60%) and Agilus 30 (40%). Three design sets (5 sample per set) with different widths, Set 1:1.3 mm, Set 2:1.5 mm, and Set 3:1.7 mm, were printed while keeping other design parameters constant (L=10 mm, R=2 mm, thickness t=2 mm, $X^*$ is therefore a 3x3 matrix).

To characterize the uncertainty in widths, a GOM ATOS Triple Scanner was used to obtain the 3D shapes which are represented by point clouds. In-house MATLAB scripts are used to calculate the width of printed parts along the structure. The point cloud is first projected on the horizontal plane. The 2D boundary contour can be then derived to represent the geometry of the manufactured part and it is later converted into a bitmap image. By using the medial axis transform, the loci of the centers of circles that are tangent to the boundary contours can be obtained. The radius of these circles is the half width of the printed structure. The resolution of point clouds is below 1 μm;
however, this analysis will take a day just to calculate the width for one part. To speed up the post-processing, the resolution is adjusted to 10 μm. Figure 3 shows the measurement results for all three sets. The uncertainty \( q \) is fitted with a normal distribution \( q \sim N(-20,270) \), where the unit is in μm.

Figure 3 (a) Image showing the printed parts in three sets which have different widths; (b) width measurements for three different sets with five samples in each set. The means and standard deviations are compared with the nominal value (marked in circles) on the right.

**Results**

By using the trained metamodel \( f_m \), three stress-strain curves \( Y^* \) and their prediction intervals can be obtained for the designed three printed sets, \( X^* \), using Equation (6) and (7), respectively, as shown in Figure 4(a). Similarly, it is possible to predict the stress-strain curve for any design that is within the same design parameter range as the training dataset \( X \). The uncertainty band width is configured to \( \sigma_3 \) which covers 68% of possible stress-strain curves under the discussed uncertainties. For a more conservative design, \( 2\sigma_3 \) can be used which bounds 95% of the possibilities. In robust design optimization, \( f = u_3 + 2\sigma_3 \) is typically used as the objective function to be minimized.

To validate the framework, tensile tests were conducted for printed samples using MTS Criterion Model 43. As shown in Figure 4(b), measurement results suggest a trend that the horseshoe structure with a smaller width has a steeper corner in the J-shaped stress-strain curve which matches with metamodel predictions. It should also be noted that the behavior of stress-strain curves at large elongation range (above 325%) was not properly predicted by the metamodel, since the high-fidelity FE model was constructed using a linear elastic material model which is not capable of simulating large deformations where advanced models are needed. The performance of the metamodel is limited by the accuracy of our high-fidelity models. Meanwhile, the uncertainty band for printed samples with the same design is within the prediction interval even with a confidence level of one standard deviation which might be due to the small number of tested samples. As a result, the proposed framework not only offers rapid predictions about the performance but also gives prediction intervals which reflect both the metamodel and manufacturing uncertainties.

It should also be noted that the uncertainty band of the measured stress-strain curves also includes the measurement uncertainty of the tensile test. Meanwhile, the manufacturing uncertainty used as an input to metamodels includes also the measurement uncertainty of the GOM scanner. A large number of measurements needs to be conducted to characterize the uncertainty of both
measurements. These two measurement uncertainties have opposite effects and are assumed to be at the same level; therefore, they are considered canceled by each other in this study.

Figure 4 (a) Predictions using the proposed framework with one σ interval. (b) Tensile test results for printed horseshoe parts.

CONCLUSIONS
A rapid design exploration framework which incorporates manufacturing uncertainty has been presented. This framework utilizes metamodels, specifically a Gaussian process regression model, as a tool to rapidly navigate from the design space into the performance space. Also, the prediction interval can be estimated with such a model, which provides a confidence level to the designer. To demonstrate the proposed framework, a horseshoe structure has been manufactured using SLA. The geometric dimension uncertainty has been experimentally characterized and considered to be within the performance predictions. The tensile test results show that the measured stress-strain curves match with the model prediction well. Current work is based on the process-structure-property relation and focuses on the structure-property relation with the process uncertainty as inputs. The future work will also cover the process-structure relationship.

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