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Compensation of MR Head Non-Linearities Using a Saturable Transfer Function

Maria Yu Lin, Say Wei Foo, George Mathew, J. W. M. Bergmans, and Weichun Ye

Abstract—It is well known that the nonlinear behavior of Magneto-Resistive (MR) heads result in the isolated pulses having asymmetry and saturation. These nonlinear effects can be quantified by measuring the positive and negative isolated pulses. In this paper, we propose a saturable model for estimating the MR nonlinearity from read-back signals. It models the nonlinearity using bias and saturation parameters. The proposed model can be used to compensate the nonlinearity in the read-back signal, thus providing a way to linearize the channel. Simulations done using measured data demonstrate the effectiveness of the proposed model.

Index Terms—Magnetic recording, channel characterization, MR heads, nonlinear distortion, MR asymmetry, MR saturation.

I. INTRODUCTION

MAGNETO-RESISTIVE (MR) heads have been widely used in digital magnetic recording systems due to their high signal-to-noise ratio (SNR) and high resolution. However, the MR heads exhibit nonlinear behavior that results in asymmetry and saturation in the isolated pulses. These nonlinearities cause degradation in the detection performance of read channels. Hence, it is important to estimate the extent of these nonlinearities and develop techniques to compensate them in the playback signal. The main factors that influence the amount of nonlinearity are biasing current, media fields and sensor structure [1]. In the recent years, several researchers reported results on how the performance of different read channels degrade due to MR nonlinearity [1]–[4]. The models used for the MR transfer curve in these studies are obtained either by doing direct input-output measurements on the head or by using a polynomial model with parameters chosen based on experimental observations. Use of a 3rd order polynomial for estimation of MR transfer curve is considered in [2], [4]. On the other hand, the use of 3rd order polynomial for compensation of nonlinearities is considered in [5], [6].

In this paper, we propose a saturable transfer function, namely, the hyperbolic tangent function, to model the MR transfer characteristics and a method to adaptively estimate the model parameters from the read-back signal. The nonlinear compensation of the read-back waveform can be done using the inverse of the proposed model. We also describe a method for adaptive estimation of the parameters of the 3rd order polynomials, which are used, for modeling the MR transfer curve and its inverse. Experimental results on measurement and compensation of MR nonlinearity using the proposed model, the 3rd order polynomial model and their inverse models are also presented.

II. HYPERBOLIC TANGENT NONLINEAR MODEL

Inspired by the similarity between the hyperbolic tangent and measured MR head transfer characteristics, we choose it to model the MR transfer curve. That is, the MR head is modeled as a linear channel in cascade with a hyperbolic tangent type nonlinear function. Mathematically, the output of MR head is modeled as

\[ y(t) = f \left( \sum a(k)h(t - kT) + b \right) + c \quad (1) \]

where \( h(t) \) is the dibit response of the linear channel, \( T \) is the duration of one bit, \( a(k) \in \{-1, 1\} \) is the magnetization of \( \sigma_n^2 \) bit, and \( b \) represents the bias that causes the asymmetry. The \( c \) parameter is used to suppress the DC caused by asymmetry. The function \( f(\cdot) \) is a customized hyperbolic tangent function, defined as

\[ f(v) = g \frac{1 - \exp(-2v/g)}{1 + \exp(-2v/g)} \quad (2) \]

where \( g \) is a positive constant that denotes the saturation effect of the transfer function. The above form of \( f(\cdot) \) is different from the conventional hyperbolic tangent function given by \( f_c(v) = \frac{1 - \exp(-v/\alpha)}{1 + \exp(-v/\alpha)}, \alpha > 0 \). This customization is done to ensure that \( f(\cdot) \) satisfies the following properties:

- \( f(\infty) = g, f(-\infty) = -g, f(0) = 0 \)
- \( \frac{df}{dv}|_{v=0} = 1 \). Unlike the conventional hyperbolic tangent function that saturates to ±1 with a variable slope of \( \alpha/2 \) at the origin, the customized bipolar hyperbolic tangent function has unity slope at the origin and has variable saturation limits. Thus, the nonlinear function given in (2) mimics the nature of the MR transfer curve, i.e., a linear operating region with positive and negative saturating regions. By setting the slope at origin to 1, the model only characterizes the nonlinear effects in the read-back waveform without considering the influence of the gain. Ideal bias, \( b = 0 \), correspond to the center point of the transfer curve. Asymmetry is modeled as an improperly chosen bias. By changing this bias, one can obtain waveforms that suffer different levels of saturation for top and bottom halves of the waveform, thus resulting in asymmetry.
III. MR NONLINEARITY COMPENSATION

If the channel is linear, then one can develop optimal equalization and detection algorithms for obtaining the best performance. Once we have a good parametric model of the MR transfer curve, we may use it to linearize the channel by compensating for the nonlinearity before equalization. Below, we show how this linearization can be done using the proposed model. Assume that we have already identified the model parameters using the read-back waveform (described in Section IV). It follows from (1) that the read-back signal $y(t)$ can be written as $y(t) = f(v(t) + b) + c$ where $v(t)$ is the output of the linear channel without the bias term $b$. The compensation process is that of recovering the linear waveform $v(t)$ from the distorted waveform $y(t)$. Mathematically, this amounts to finding the inversion of the hyperbolic tangent model. Doing this inversion, we get

$$v(t) = -\frac{g}{2} \log_e \frac{g - y(t) + c}{g + y(t) - c} b.$$  (3)

This nonlinear transfer function could be implemented by table look up.

IV. CHARACTERIZATION OF LINEAR CHANNEL AND MR NON-LINEARITY

In this section, we explain the experimental/simulation set-up and algorithms used for doing the estimation of the parameters of the nonlinear channel model, i.e., the combination of linear channel and the MR transfer function model.

A. Experimental Setup

Fig. 1 shows the block schematic of the system setup used for characterizing the nonlinear channel. The ‘real channel’ represents the effective channel resulting from write and read heads and disk medium. Channel characterization involves estimating the linear channel response $h(t)$ and the model parameters $\alpha$, $\beta$, and $c$ for the 3rd order polynomial or $g$, $b$, and $c$ for the hyperbolic tangent model. The data $a(k)$ is written at the rate 100 Mbits/second and the read-back signal $y(t)$ is sampled at 1 Gsamples/second. A low pass filter is used to eliminate the out of band noise in $y(t)$. The linear channel model is initialized using the well-known Lorentzian response. For long data lengths, it is necessary to use a timing loop to synchronize the model output $\hat{y}(t)$ with the read-back signal $y(t)$ while doing the characterization. Initial synchronization is done by cross-correlating the read-back signal with model output. This is done in two stages. First, a coarse synchronization is done to obtain an accuracy of $\pm T$ using $T$-spaced samples and then the fine synchronization to an accuracy of $\pm 0.1T$ is done using the 10-times over-sampled data. Experiments revealed that noise and other distortions might limit this accuracy to $\pm 0.3T$. This accuracy can be achieved for reasonable SNR’s and typical channel densities. The timing loop is started after this initial synchronization. A relatively large loop gain is used during acquisition of the phase and a small loop gain is used during tracking. A numerically controlled oscillator (NCO) is used to control the sampling frequency/phase. Estimation of the channel parameters (linear and nonlinear) using an adaptive algorithm is started after the acquisition is over. The channel estimation is done using 2-times over-sampled read-back data.

Fig. 2 shows the set-up used for adaptively estimating the parameters of the inverse model. In this paper, this is used for obtaining the 3rd order polynomial inverse. The linear channel parameters are fixed to that obtained from the channel estimating done using Fig. 1. The adaptive algorithm used for estimating the parameters in Figs. 1 and 2 are described below.

B. Adaptive Algorithm for Parameter Estimation

The parameters of the 3rd order polynomial model, hyperbolic tangent model, linear model and 3rd order inverse polynomial model are estimated using the well-known LMS (least mean square) adaptation algorithm. This algorithm updates the parameters in the direction of the negative gradient of the squared error between the measured signal and the model output.

The general form of LMS adaptation algorithm is given by

$$p_k = p_{k-1} - \mu \nabla p_k \quad \nabla p_k = \frac{\partial^2 e_k}{\partial p_k^2}$$  (4)

where $p_k$ denotes the estimated value of the parameter $p$ at instant $k$, $\nabla p_k$ is the corresponding instantaneous gradient, and $e_k$ is the error between measured data and model output. The linear portion of the channel model is assumed to be Lorentzian with the parameters $A$ (amplitude of the step-response) and $PW50$ (pulse width at 50% amplitude). The adaptation equations for
the linear channel parameters \((A, PW50)\) and hyperbolic tangent model parameters \((g, b, c)\) are as given below.

\[
A_k = A_{k-1} + \mu c f_{k-1} \sum a_k h(t-T)/A_{k-1} \\
PW50_k = PW50_{k-1} + \mu c A_{k-1} f_{k-1} \sum a_k b_{k-1} \\
g_k = g_{k-1} + \mu c (f_{k-1} - v_{k-1} f_{k-1})/g_{k-1} \\
b_k = b_{k-1} + \mu c f_{k-1} \\
c_k = c_{k-1} + \mu c
\]

where

\[
f_{k-1} = 4 \exp(-2v_{k-1}/g_{k-1})/(1 + \exp(-2v_{k-1}/g_{k-1})^2, \\
f_{k-1} = f v_{k-1}, \quad \text{and} \\
b_{k-1} = 8(p(t) - p(t-T))/PW50_k^{3}
\]

with

\[
p(t) = (t-T)^2[1 + (2(t-T)/PW50_{k-1})^2]^{-2}.
\]

The adaptation rule for the other models can be derived in similar manner.

Since the convergence speed of the adaptation algorithm is different for each parameter, we used the following ad-hoc rule to speed up the convergence process. The step-size \(\mu\) is multiplied by the factor

\[
K_p = \sqrt{\sum_k \nabla^2_{\text{model}, k} / \sqrt{\sum_k \nabla^2_{f_{k-1}}}}
\]

where \(\nabla_{f_{k-1}}\) is the gradient of \(f_{k-1}\) with respect to the parameter \(p\) at instant \(k\) and \(\nabla_{\text{model}, k}\) is the absolute maximum among the gradients corresponding to all the parameters.

### V. RESULTS AND DISCUSSION

We now present experimental results to illustrate the performance of the proposed hyperbolic tangent model for estimation and compensation of MR-head nonlinearities and the adaptive method for estimating the model parameters. In the experiments, the read biasing current was chosen to result in a read-back signal that suffers from significant MR asymmetry and saturation. Input data is constructed such that the resulting waveform contains sufficient low-density isolated transitions and high frequency bursts.

For the sake of comparison, the following experiments were conducted for the adaptive modeling case: LM—linear model where \(f(v) = v + c\), HM—hyperbolic tangent model (2), and 3M—3rd order polynomial model where \(f(v) = \alpha v^3 + \beta v^2 + v + c\). The measure used for comparing these three models is the MSE (mean-squared-error) between the model output and the measured output of the channel. The MSE for the three models are given in Table I. Observe that the hyperbolic tangent model gives 18.5% improvement in MSE compared to the linear model. Further, it performs comparable to the order model. The identified coefficients of these models are also shown in Table I.

The following experiments were conducted for the adaptive compensation case: HC—inverse hyperbolic tangent model (3) and 3C—3rd order inverse polynomial model \(f^{-1}(v) = \alpha v^3 + \beta v^2 + v + c\). Table II shows the resulting MSE. Observe that the HC-method performs comparable to the 3C-method. The slightly better performance provided by 3C-method is because the adaptation minimizes the error between the compensated output and the linear model output, thus minimizing noise enhancement. If the bias term \(c\) is dropped from the 3C-model, the MSE would worsen significantly.

### VI. CONCLUSION

A hyperbolic tangent model has been proposed to model the MR head nonlinearity in magnetic recording. An estimation and compensation of the MR head nonlinearity method is developed and described in detail. Efficiency of the proposed method and model has been demonstrated using simulations performed on read-back data measured from spin-stand. The results show that the proposed method and model can linearize the read-back data effectively.

### REFERENCES


