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Separation of Periodic Signals Using Non-uniform Embedding

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Abstract

In this paper, a new method to separate overlapped periodic signals is presented. The method makes use of different embedding dimensions to describe different periodic signals and a feedback structure to eliminate cross-correlation. The method is able to handle signals with non-integer periods and also quasi-periodic signals.

1 Introduction

Music signals and other types of signals are composed of several periodic or quasi-periodic signals. To automate the transcription of polyphonic music, one of the tasks is to separate the individual notes from the composite signals. Several methods have been proposed for the separation of periodic signals. M. Zou [4] proposed an algebra theory for the separation of periodic signals. D. Li [2] presented approachs using adaptive filters to separate several periodic signals. These methods require that the periods of the signals are integer multiples of sampling interval. The method proposed by Zou also imposes the additional constraint that the periods must be cross-prime. The requirements render the methods unsuitable for separating polyphonic music signals.

In this paper, a novel approach that allows for non-integer periods is presented. The method makes use of embedding, or phase space reconstruction[1]. Using different embedding dimensions to describe different signals and a feedback structure to eliminate cross-correlation, this method is able to separate periodic signals without the restriction stated above. In addition, its application can easily be extended to the separation of quasi-periodic signals with nonharmonic partials.

The proposed method is outlined in the following section. Details of the search algorithm are described in Section 3. Results of simulations are given in Section 4. Section 5 is the concluding section.

2 The Proposed Model

Let us first consider the case when a composite signal $s(n)$ comprises a linear combination of M periodic signals $s_i(n)$ with period $T_i = r_i T_s$, $i = 1, ..., M$ and sample rate $f_s = 1/T_s$. Mathematically,

$$s(n) = s_1(n) + s_2(n) + ... + s_M(n) \tag{1}$$

The periods are assumed to be a subset of a set of given periods as in the case of music signals. The periods are not required to be integer multiples of the sampling interval although such conditions will allow for faster processing. It is also not necessary to know a priori the value of M, the total number of periodic or quasi-periodic signals in the composite signal.

The model of the proposed method for M channels with each channel representing the path of one periodic signal is shown in Fig. 1.

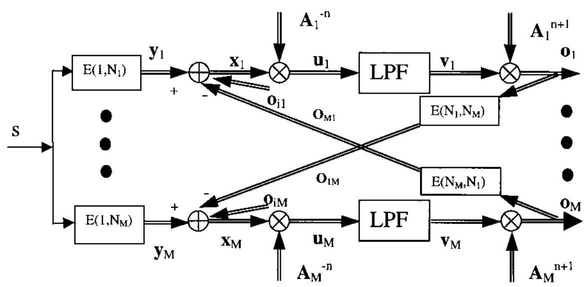

Figure 1: The proposed model

Using embedding technique, the input signals may be represented by M phase space time series $\mathbf{y}_i$. When delay lag equals to the sample interval, the operation $E(1, N_i)$ shown in Fig. 1 indicates the following embedding process,

$$\mathbf{y}_i(n) = [s(n)\ s(n+1)\ ...\ s(n+(N_i-1))]' \tag{2}$$

where N_i is obtained by truncating r_i to the nearest integers towards infinity.

The sequence $\mathbf{x}_i(n)$ of the i^{th} channel is obtained by subtracting the sum of predicted outputs $\mathbf{o}_k(n)$ where

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$k \neq i$ from all other channels from $\mathbf{y}_i(n)$. Mathematically,

$$\mathbf{x}_i(n) = \mathbf{y}_i(n) - \sum_{\substack{k=1 \\ k \neq i}}^{M} \mathbf{o}_{ki}(n) \quad (3)$$

For case when the period is integer number of samples, $\mathbf{u}_i(n)$ is obtained by rotating the sequence in $\mathbf{x}_i(n)$ backward by n samples to bring the first sample to be in line with the first sample $s_i(0)$ of the constituent signal $s_i(n)$. Mathematically, this is achieved by the following operation

$$\mathbf{u}_i(n) = \mathbf{A}_i^{-n}\mathbf{x}_i(n) \quad (4)$$

where $\mathbf{A}_i$ is a square matrix of dimension N_i given by

$$\mathbf{A}_i = \begin{pmatrix} 0 & 1 & 0\dots & 0 \\ 0 & 0 & 1\dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0\dots & 1 \\ 1 & 0 & 0\dots & 0 \end{pmatrix} \quad (5)$$

For case when the period is not integer, $\mathbf{A}_i$ shall take the following form

$$\mathbf{A}_i = \begin{pmatrix} 0 & 1 & 0\dots & 0 \\ 0 & 0 & 1\dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0\dots & \\ a_1^i & a_2^i & a_3^i\dots & a_{N_i}^i \end{pmatrix} \quad (6)$$

where a_k^i, $k = 1, ..., N_i$ are the linear prediction coefficients whose values need not be known for our method.

Let $\mathbf{A}_i$ be decomposed as $\mathbf{Q}_i\mathbf{\Lambda}_i\mathbf{Q}_i^{-1}$ where $\mathbf{\Lambda}_i$ is a diagonal matrix with the eigenvalues as the elements on the main diagonal

$$\mathbf{\Lambda}_i = \begin{pmatrix} \lambda_0^i & 0 & 0\dots & 0 \\ 0 & \lambda_1^i & 0\dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0\dots & \\ 0 & 0 & 0\dots & \lambda_{N_i-1}^i \end{pmatrix} \quad (7)$$

and

$$\mathbf{Q}_i = [\mathbf{q}_0^i \; \mathbf{q}_1^i \; ... \; \mathbf{q}_k^i \; \mathbf{q}_{N_i-1}^i] \quad (8)$$

where

$$\mathbf{q}_k^i = [q_{0k}^i \; q_{1k}^i \; ... \; q_{(N_i-1)k}^i] \quad (9)$$

Thus

$$\mathbf{A}_i\mathbf{q}_k^i = \lambda_k^i\mathbf{q}_k^i \quad k = 0, 1, ..., N_i - 1 \quad (10)$$

From Eqs. (6) and (10), we obtain

$$q_{lk} = \lambda_k^i q_{(l-1)k}^i, \quad l = 1, 2, ..., N_i - 1 \quad (11)$$

Using Eq. (11), the matrix $\mathbf{Q}_i$ can be constructed.

The next key problem is to get the eigenvalue λ_k^i. In fact, λ_k^i is determined by the harmonic frequency of the periodic signal. Thus,

$$\lambda_k^i = e^{j\omega_k^i} \quad (12)$$

where ω_k^i is acquired according to one of the harmonic frequency f_h^j of the periodic signal s_i.

$$\omega_k^i = 2\pi\frac{f_h^i}{f_s} \quad (13)$$

Note that for signals with real value, f_h^i can take both positive and negitive values.

The proposed method can readily be adapted to quasi-periodic signals such as piano music as f_h are known a priori even if the partials are non-harmonic.

For composite signals with a mixture of periodic signals with low fundamental frequencies, the periods of the signals and hence the order of the matrix $\mathbf{A}_i$ become large. This increases the burden for computing. The problem can be overcomed by increasing the delay lag of the embedding process which is equivalent to down-sample the signal to the level of Nyquist sampling rate.

The signal $\mathbf{u}_i$ is passed through a Low-Pass Filter (LPF) to remove unwanted components and noise. The estimated output of $s_i(n)$, denoted by $o_i(n)$, is then obtained by shifting the $\mathbf{v}_i$ forward to the original time interval plus one sample (as it is a predicted value of the next sample) by multiplying $\mathbf{v}_i$ by $\mathbf{A}_i^{n+1}$ to predict the next sample value of the periodic signal of the i^{th} channel.

Outputs from all channels are fed back to inputs of other channels to eliminate cross-correlation. Because each channel has different embedding dimension, the operation $E(N_i, N_j)$ is carried out to convert the dimension from N_i to N_j.

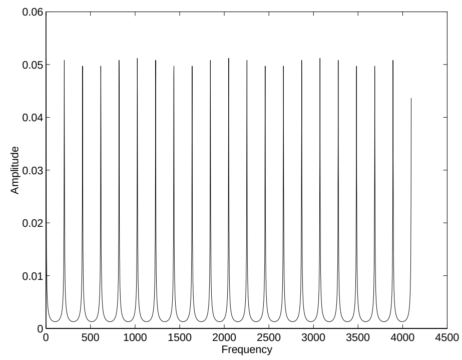

Figure 2: Spectrum of impluse response a channel without feedback

The proposed method is similar to the demodulation of AM signal. The operation $\mathbf{\Lambda}^{-n}$ moves the energy of all harmonics of the wanted periodic signal to the base-band. The function of the LPF is to remove all components outside the base-band. After which the harmonics of the desired signal are restored to their rightful frequency by the operation $\mathbf{\Lambda}^{n+1}$.

Fig. 2 is the amplitude spectrum of the impulse response of a signal channel without feedback. The period used in this case is 20. It has the structure of a comb filter. If FIR is used, the length of the filter is proportional to the period of the signal and is not desirable for signals with long periods. In this paper, a simple IIR model shown in Fig. 3 is used. For this model, the bandwidth of the filters can be varied by changing the value of μ shown in the figure.

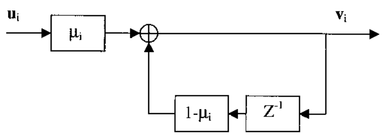

Figure 3: A simple low-pass filter

3 The Search Algorithm

In the separation problem, one must ascertain M, the number of periodic signals in the composite signal and then the periods of these signals.

In this paper, we propose an algorithm with searching time linearly proportional to K. We shall make use of the fact that the signals are obtained from a set of K periodic signals with period $\{\mathbf{T} : T_1, T_2, T3, ..., T_{K-1}, T_K\}$. This is true, for example, for notes of piano. The step by step application of the method is given below.

I. Intialization

To begin, we shall assume that the composite signal is the only periodic signal. The MSE (mean-square-error) is equal to the energy of the signal.

II. Test of additional signals

Assume next that there is an additional periodic signal and hence a new channel is added. Using the period listed in the database $\mathbf{T}$ starting from the smallest period, the process as detailed in Section 2 above is carried out to obtain the filter output $o_j(n)$. $j = N, ...$ The j^{th} MSE is calculated using

$$e_j = \sum_{n=1}^{L}(y(n) - \sum_{i=1}^{N-1} o_i(n) - o_j(n))^2 \qquad (14)$$

where j is not one of the periodic signal already identified and L is a frame-length. A value of L equals to $3 \times max(T_i), i = 1, ..., K$ is generally suitable. The least e_j is then identified. The period that gives the least MSE is taken as the targeted signal.

III. Termination

The updated MSE is compared with the previous MSE. If the MSE is larger or close to the previous one, the process is stopped else it is continued to search for additional periodic signal.

4 Applications of The Method

4.1 Separation of signals with integer periods

When the periods of the signals are integers, the algorithm is similar to the adaptive filters algorithm cited in [2] with the exception that μ in our method is not fixed but adaptable for the different channels. As signals with long periods have low fundamental frequencies, the bandwidths of the com filters shall be made narrower and hence a low value of μ shall be used.

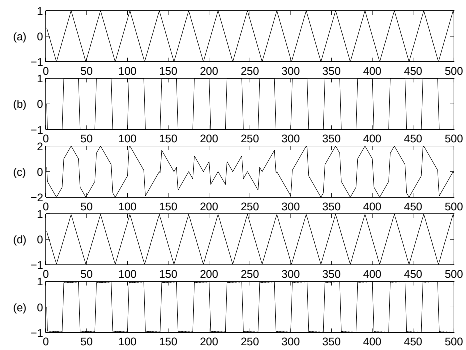

Figure 4: Separation of signals with integer periods

For ease of illustration, a case with two periodic signals: a triangular wave with period of 36 samples and a rectangle wave with period equal to 40 samples is presented. Their waveforms are shown in Fig. 4(a) and Fig. 4(b) respectively. The sum of these two signals is shown in Fig. 4(c). The corresponding outputs obtained are shown in Fig. 4(d) and Fig. 4(e). In this case, the recovery is almost perfect.

4.2 Separation of signals with periods that are non-integers

To test the effectiveness of the proposed method in separating signals with non-integer periods, the method is applied to the composite signal shown in Fig. 5(c). The signal is synthesized by adding the two periodic signals shown in Fig. 5(a) and Fig. 5(b). The two signals are generated by adding a sinusoidal signal with the fundamental frequency and a sinusoidal frequency of the first harmonic. The periodic signal of Fig. 5(a) has a period of 36.6 samples and the signal of Fig. 5(b) has a period of 40.2 samples.

The waveforms of the two separated signals are shown in Fig. 5(d) and Fig. 5(e). μ is chosen to be the same value as for separation of signals with integer periods.

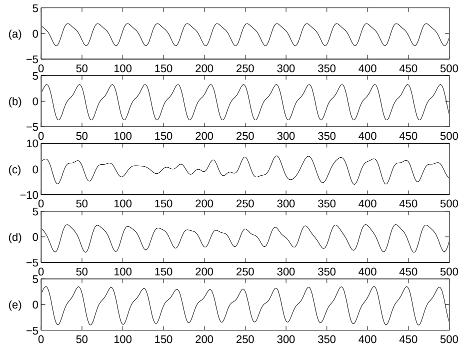

Figure 5: Separation of signals with non-integer periods

Comparing the original signal with the regenerated signal, it can be seen that there are good tracking of the waveform and phase. However, there is still some remnants of amplitude modulation in the regenerated signals. This is especially pronounced in the waveform shown in Fig. 5(d). Because the periods of the two constituent signals are very close, the partials of signal (d) at normalized frequency 0.0273 is overlapped by the 'spill-over' of frequency components of signal (e) at normalized frequency 0.0249. One way to overcome this problem is to make the bandwidth of the comb filters narrower by choosing a smaller value of μ. However, this will slow down the convergence rate of the search process.

4.3 Separation of music signal

Fig. 6(a) is a segment of a signal taken from a piano piece. It is a chord composed of three notes: g0, b0 and d2.

The algorithm proposed is applied to separate the three concurrent notes from the composite signal. The search range is the full range of 88 piano notes. As piano tones nearly always produce significant energy at either the first or second partial. The dimension required of the matrix $\mathbf{A}_i$ in Eq. (6) is 4. It shall be noted that the partials of piano notes are not at exact integer multiples of the fundemental frequency. In fact, Young[3] showed that the j^{th} partial f_j is related to the fundemental frequency f_0 by $f_j = f_0 S^{log_2 j}$ where $S = 2.0013$. This fact is used to determine the ω_k^i in Eq. (13).

In the search process, the note d2 was the first signal detected as it has the largest energy and gives the MSE. After the third note g0 is determined, further testing reveals that the MSE does not decrease significantly and the search process is terminated.

Fig. 6(c) (d) and (e) show the outputs of the channels which are eventually tuned to the notes g0, b0 and d2 respectively. For the purpose of comparison, the three signals are combined and presented in Fig. 6(b). The closeness to the original signal in Fig. 6(a) confirms the validity of the proposed method.

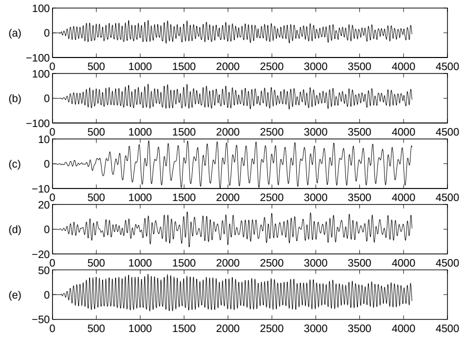

Figure 6: Separation of music signal

5 Conclusion

In this paper, a new algorithm for the separation of multi-periodic signals is presented. The algorithm makes use of non-uniform embedding technique to reconstruct the phase spaces of different signals. The method is able to handle cases where the periods of constituent signals are not integer numbers of samples. It can also deal with quasi-periodic signals with nonharmoic partials. Applications of the method for separation of periodic signals with integer and non-integer periods show excellent results. The method is readily applicable to the separation of musical signals as musical notes can be viewed as periodic or quasi-periodic time series and the embedding method provides a good way to describe the signal.

References

[1] Kevin Judd and Alistair Mees. Embedding as a modeling problem. *Physica D*, 120:273–286, 1998.

[2] Dan Li, Yi sheng Zhu, and Bin Zhu. The adaptive separation of the overlapped periodic signals,whose periods are very close. In *1992 Proceedings of the 35th Midwest Symposium on Circuits and Systems*, pages 203–206, 1992.

[3] R. Young. Inharmonicity of plain piano wire. *J. Acoust. Soc. Am.*, 24:267–273, 1952.

[4] Mouyan Zou, Zhenming Chai, and R. Unbehauen. Separation of periodic signals by using an algebraic method. In *Proceedings ISCAS*, volume 5, pages 2427–2430, 1991.