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7th International Conference on Fluid Mechanics, ICFM7

Electric field enhances mixing in micro circular pipes

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Abstract

Mixing of two miscible flows with a radial conductivity stratification in a micro-pipe can be enhanced by an axial electric field. The mechanism is interpreted by a linear stability analysis. The effect of ionic diffusion on rapid mixing between the two liquids is discussed. A case study shows that, when the conductivity in the inner layer is larger, the growth rate becomes smaller as the electrical Schmidt number increases; when the conductivity in the outer layer is larger, the growth rate either decreases or increases as the electrical Schmidt number increases.

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Peer-review under responsibility of The Chinese Society of Theoretical and Applied Mechanics (CSTAM)

Keywords: Electrohydrodynamic instability; mixing; stratification flows.

1. Introduction

Flow in small scale devices is usually laminar and liquids mixing due to turbulence becomes an impossible task. Many approaches have been proposed to facilitate mixing in small-scale devices, such as using meandering microchannels to increase the diffusion time. Applying an outer electric field is found to be a potential way to enhance the mixing in microfluidic devices because the mechanical techniques are not suited to obtaining mixing for the requirement of prohibitive amounts of power\cite{1}. The external electric field may trigger a circulation flow in the system whence the mixing is enhanced. Such a method is useful when there exists a spatial change in the electrical properties of the liquids. Generally, previous studies focus on two problems: mixing in immiscible and miscible liquids.

In the former case, studies were concentrated on the instability of liquid interface of multifluid systems\cite{2,3}. The electric field is found to either stabilize or destabilize the liquid-liquid interface. For two immiscible fluids between two parallel plates, Ozen et al. proposed a criteria: \((\sigma^2 - \epsilon)(\sigma - 1) < 0\) under which the electric field is stabilizing\cite{2}. Here, \(\sigma\) and \(\epsilon\), respectively, is the electrical conductivity ratio and electrical permittivity ratio of the upper layer to the lower layer. Ding et al. investigated two immiscible fluids coflowing in an annulus channel wherein the criteria proposed by Ozen et al.\cite{2} was modified by them\cite{3}. However, Ding et al. found that, such a criteria in the annulus flow system was only valid to explain the effect of the normal Maxwell stress on suppressing or enhancing the deformation of the interface. When the electrical charges are free to accumulate at the liquid-liquid interface, the tangential Maxwell stress could enhance the interfacial instability caused by viscosity stratification\cite{3}.

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The latter study of electric field enhancing mixing between two miscible liquids was pioneered by Lin et al.[4]. Two electrolyte solutions with different ionic concentrations were pumped into a micro channel by a syringe pump. Then, the pressure gradient was removed and a tangential electric field was imposed along the channel which initiated a weak electroosmosis flow. When the strength of the applied electric field exceeds a critical value, the system becomes unstable and strong mixing phenomenon was observed[4]. Ding and Wong investigated the mixing of a single liquid layer with a conductivity gradient in an annulus pipe[5]. When an electric field is applied on an ionic liquid, the local ionic concentration, pH value, and conductivity gradients developed along the axis of the imposed electric field and the local conductivity could vary by more than an order of magnitude over a length of 1mm. However, the flow would become unstable as the strength of the electric field exceeds a threshold value. Ding and Wong observed that, when the conductivity gradient was small, the critical unstable mode is given by three-dimensional oblique wave; when the conductivity gradient is large, it would switch to the stationary mode rather than the oscillatory mode[5]. Recently, Ding and Wong investigated two-miscible flows with a conductivity stratification in an axial electric field wherein the basic flow was driven by a constant pressure gradient[6]. Studies by them showed that the basic flow stabilizes the system[6].

In this paper, we revisit mixing in two miscible liquids in a micro circular pipe with an electrical conductivity stratification[6]. The effect of ionic diffusion on the rapid mixing is examined.

2. Mathematical formulation

The radius of the micro pipe is $b$. The two liquids are miscible dilute electrolyte solutions. The liquids are Newtonian and the density $\rho$, kinematic viscosity $\nu$, and dynamic viscosity $\mu = \rho \nu$ of the two liquids are assumed to be the same. There is a sharp change in the ionic concentration where the two liquids meet at $r = a$. Therefore, a sharp change in the electrical conductivity occurs at $r = a$. A constant electric field of strength $E_0$ is imposed in the axial direction. In this paper, we assume that the electrical current density $J_e$ as well as the induced current density $\partial \epsilon \partial \tau$ are modest, such that the induced magnetic field is negligible. Therefore, the electrostatic problem is considered. For detailed mathematical formulation, we refer the reader to Ref.[6].

2.1. Dimensionless system

Following the work of Ding and Wong[6], the basic state of the system is described as

$$ u = (0, 0, \frac{\partial \bar{p}}{4\mu} (r^2 - b^2)), \quad \phi = \phi_0 - E_0 z, \quad \bar{\rho}_c = 0. $$

(1)

The profile of the conductivity is approximated by the error function as suggested by Ding and Wong[6]:

$$ \bar{\sigma} = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_2 - \sigma_1}{2} \text{erf}(\frac{r - a}{\delta}), $$

(2)

Taking the velocity scale $W = -\frac{\partial \bar{p} \bar{b}^2}{4\mu}$, the length scale $b$, the time scale $b/W$, the pressure scale $\mu W/b$, the electrical potential scale $E b$, the conductivity scale $\sigma_2 - \sigma_1$, the non-dimensional governing equations of the system read:

$$ \nabla \cdot v = 0, $$

(3)

$$ Re \frac{Dv}{Dt} = -\nabla p + \nabla^2 v + \frac{Q}{Re \sigma} \nabla^2 \phi \nabla \phi, $$

(4)

$$ \frac{1}{Re} \frac{D(\nabla^2 \phi)}{Dt} + \nabla \cdot \left\{ [(\eta - 1)\sigma + 1] \nabla \phi \right\} = 0, $$

(5)

$$ \frac{D\sigma}{Dt} = \frac{1}{Re \sigma} \nabla^2 \sigma, $$

(6)

where $Re = \frac{\rho W b}{\mu}$ is the Reynolds number. $\eta = \frac{\sigma_2}{\sigma_1}$ denotes the conductivity ratio. $Q = \frac{e E_0^2 b^2}{\mu \kappa_{eff}}$ is the scaled electrical energy and is defined as the electrical number in this paper; $\eta = \frac{\sigma_2}{\sigma_1}$ denotes the conductivity ratio; and
where $L^2$ indicates that the ionic diffusion becomes weak and the dielectrophoretic effect becomes stronger. Therefore, the electric time is quite smaller than the fluid time. Then, Eq.(5) can be simplified to:

$$[(\eta - 1)\sigma + 1]\nabla^2 \phi + (\eta - 1)\nabla \sigma \cdot \nabla \phi = 0.$$  

(7)

2.2. Linearized system

The linear stability analysis of the flow system is implemented by perturbing the base state with infinitesimal disturbances:

$$[u, v, w, p, \sigma, \phi] = [0, 0, 1 - r^2, \bar{p}, \bar{\sigma}, \bar{\phi}] + [\hat{u}, \hat{v}, \hat{w}, \hat{\bar{p}}, \hat{\bar{\sigma}}, \hat{\bar{\phi}}] \exp(i(m\theta + kz) + \lambda t).$$

(8)

Here we have considered the normal mode analysis. $\bar{\sigma} = \frac{1}{2} + \frac{i}{2}\text{erf}(\frac{r - 1}{\sqrt{2}})$ and $\bar{\phi} = \phi_0 - z$. $[\hat{u}, \hat{v}, \hat{w}, \hat{\bar{p}}, \hat{\bar{\sigma}}, \hat{\bar{\phi}}]$ is the Fourier amplitude, $m$ is the azimuthal wave number, $k$ is the streamwise wave number, $\lambda$ is the complex temporal growth rate. Substituting Eq.(8) into Eqs. (3), (4), (5), (6) and after linearizing, we obtain the governing equations of the eigenvalue problem:

$$\hat{D} \hat{u} + \frac{\hat{u} + i m \hat{v}}{r} + ik \hat{w} = 0,$$

(9)

$$\lambda \Re \hat{u} = -\Re \hat{p} - ik \Re \hat{w} + \mathcal{L} \hat{u} - \frac{\hat{u} + 2i \hat{v}}{r^2},$$

(10)

$$\lambda \Re \hat{v} = -\frac{im \hat{p}}{r} - ik \Re \hat{w} + \mathcal{L} \hat{v} - \frac{\hat{v} - 2i \hat{u}}{r^2},$$

(11)

$$\lambda \Re \hat{w} = -ik \hat{p} - ik \Re \hat{w} - \Re \mathcal{D} \hat{u} + \mathcal{L} \hat{w} - \frac{Q}{Pe} \mathcal{L} \hat{\phi},$$

(12)

$$Pe \lambda \hat{\sigma} = -Pe \Re \hat{\sigma} - ik \Re \hat{\phi} + \mathcal{L} \hat{\sigma},$$

(13)

$$[(\eta - 1)\hat{\sigma} + 1]\mathcal{L} \hat{\phi} + (\eta - 1)\Re \mathcal{D} \Re \hat{\phi} - ik(\eta - 1)\hat{\sigma} = 0,$$

(14)

where $\mathcal{L} = D^2 + \frac{1}{r}D - \frac{m^2}{r^2} - k^2$, $D = \frac{d}{dr}$. Note that, replacing Re by Pe does not change the governing equations, which indicates that the ionic diffusion and basic shear flow should have a similar effect on the instability. Boundary conditions for the eigenvalue problem are discussed by Ding and Wong in their recent work[6].

A Chebyshev collocation method is implemented to solve the eigenvalue problem. In order to modify the computation near the interface $r = a$, the Chebyshev collocation points are clustered in the mixing region at $r = a$[6]. It is found that 51 Chebyshev collocation points are sufficient to provide satisfying accuracy[6].

3. Results and discussions

Two critical unstable modes are illustrated by figure 1. It was shown by Ding and Wong[6] that the critical instability is dominated by the corkscrew mode when the conductivity in the inner layer is larger, while it is dominated by the axisymmetric mode when the conductivity in the outer layer is larger. The instability is caused by the dielectrophoretic force[4–6]. Although it is mentioned that the ionic diffusion effect may have a dual effect on the stability of the flow, how the ionic diffusion influences the two unstable modes is not elaborated. The Schmidt number is defined as $Sc = \frac{k_r}{D_{ij}}$ which measures the ratio between viscous diffusion and ionic diffusion. The value of $Sc$ can be increased by reducing $k_{eff}$ or increasing $\nu$. The ionic diffusion has a tendency to remove the conductivity stratification, therefore, a stabilizing effect. If we fix the viscosity of the fluid $\nu, Sc$ can be increased by decreasing $k_{eff}$ which implies that the ionic diffusion becomes weaker and the dielectrophoretic effect becomes stronger. Therefore, the flow would become more unstable. If we fix the value of $k_{eff}, Sc$ can be increased by increasing the fluid viscosity $\nu$. It implies that
the viscous retardation becomes stronger. As is well know that viscosity dissipates the kinetic energy which plays a stabilizing role in the system, the flow will become stable as $Sc$ increases.

Here, we presented a detailed investigation of the ionic diffusion. Figure 2 shows the real temporal growth rate $\lambda_r$ versus the streamwise wave number $k$. Two cases were investigated. When the conductivity in the inner layer is larger, it is found that, with increasing the value of $Pe$ (the value of $Pe$ can be increased by increasing $Sc$ where the Reynolds number is held fixed), the growth rate $\lambda_r$ becomes smaller. It indicates that the corkscrew mode is suppressed by increasing the value of $Sc$. However, our results showed that, when the conductivity in the outer layer is larger, the growth rate reduces as $Pe$ increases as observed in right panel of Fig.2. Results in Fig.2 indicate that, the effect of ionic diffusion on the linear stability of the system depends on the conductivity ratio between the two layers. When the conductivity in the inner layer is larger, the flow becomes more stable as $Sc$ increases because the viscous dissipation effect becomes stronger, while when the conductivity in the outer layer is larger, the viscous dissipation mechanism is not the key factor causing the system to be more stable. As indicated by Ding and Wong[6], when the conductivity in the outer layer is larger, the Reynolds stress plays an important role. The dissipation effect due to Reynolds stress becomes stronger while the viscous dissipation becomes weaker as $Sc$ increases[6] when $Sc$ is moderate (For large $Sc$, the system is always becoming more stable as $Sc$ increases). Therefore, the flow can be more unstable as $Sc$ increases due to the enhancement of the dielectrophorectic effect.

Acknowledgements

This work is supported by Singapore Ministry of Education, Academic Research Fund Tier 2 research grant MOE2011-T2-1-036.

References


