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Achievable PID Performance using Sums of Squares Programming

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Abstract

In addition to process elements like time delay, the PID structure of the controller can pose fundamental limitations on the achievable control performance. A key difficulty in characterizing the limitations due to controller structure is the non-convexity of the resulting optimization problem. In this paper, we present a global lower bound on the achievable PID performance, defined in terms of output variance, by solving a series of convex programs using sums of squares programming. This result is also extended to minimize the weighted sum of the variances of input rate and output. The tightness of the proposed bounds is demonstrated using several benchmark examples drawn from literature.

Key words: Achievable performance; Minimum variance; PID controllers; Sums of squares programming.

1 Introduction

Proportional integral derivative (PID) controllers are widely used in process industries. During the past few decades, a number of methods have been proposed for tuning PID controllers; see *e.g.* [2; 3; 15]. A related problem, which has received less attention, involves characterization of the limitations imposed by the PID structure of the controller on achievable control performance. For stable linear discrete time systems with time delay, which have no finite zeros on or outside the unit circle, minimum variance (MV) benchmark denotes the least achievable output variance [5]. When the controller is restricted to be a PID controller, however, MV benchmark may not be achievable.

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The key difficulty in characterizing the achievable performance for a restricted structure controller is the non-convexity of the resulting optimization problem. During the past few years, some researchers have suggested the use of gradient based methods [1; 10; 11] and stochastic optimization [4] to solve the non-convex optimization problem. These methods do not guarantee global optimality and hence can only provide upper bounds on the achievable PID performance. Furthermore, for analyzing fundamental limitations arising due to the controller structure, it is more useful to derive a lower bound. Recently, Kariwala [9] proposed an analytical lower bound on the achievable PID performance by considering the first $(2d-1)$ impulse response coefficients of the closed-loop transfer function between disturbance and output, where d is the process delay. Clearly, the lower bound in [9] can be loose in general due to the neglected impulse response coefficients.

In this paper, we show that the impulse response coefficients of the closed-loop transfer function between disturbance and output can be represented as polynomials in unknown controller parameters. Subsequently, the non-convex optimization problem related to minimization of output variance is solved using sums of squares (SOS) programming, which guarantees a lower bound on the global optimal solution [12]. The SOS programming technique also provides sub-optimal tuning parameters of the PID controller, which can be used to compute a tight upper bound on the achievable PID performance.

The minimization of output variance only may lead to excessive input variation. To overcome this difficulty, we extend the SOS programming approach to find bounds on the weighted sum of the variances of input rate and output. Similar to the linear quadratic gaussian (LQG) benchmark for unrestricted controllers [5], this result can be used to find the trade-off between the variances of input rate and output for PID control. We demonstrate the tightness of proposed bounds using several benchmark examples drawn from literature.

2 Sums of Squares Programming

In this section, we provide a brief overview of the SOS programming technique for polynomial minimization; see [12] for further details. A polynomial $r(x)$ is called SOS if it can be decomposed as

$$r(x) = \sum_{i=1}^{n} f_i^2(x) \tag{1}$$

where $f_i(x)$ are polynomials in x. As an SOS polynomial is nonnegative for all $x \in \mathbb{R}^{n_x}$, it follows that the polynomial $r(x)$ is SOS if there exist a positive semidefinite matrix Q such that

$$r(x) = z(x)^T \, Q \, z(x) \tag{2}$$

In (2), $z(x)$ is a vector of monomials of the independent variables of $r(x)$ up to degree $m/2$, where m is the degree of $r(x)$. Now, the problem of minimizing $r(x)$ can be posed as

$$\max_{Q} \; p \tag{3}$$
$$\text{s.t. } r(x) - p = z(x)^T \, Q \, z(x) \tag{4}$$
$$Q \geq 0 \tag{5}$$

By comparing the coefficients of the polynomials on both sides, (4) can be expressed as a set of linear equality constraints. Hence the optimization problem in (3)-(5) can be seen as a semidefinite program (SDP). The p^* obtained by solving the SDP does not necessarily correspond to the minimum value of $r(x)$, but is guaranteed to be its global lower bound [12].

To find x^*, which solves the optimization problem in (3)-(5), we note that $r(x^*) - p^* = z(x^*)^T \, Q \, z(x^*) = 0$. Since $Q \geq 0$, it admits Cholesky decomposition, where $Q = L^T L$ and $L \, z(x^*) = 0$. As the first element of the vector $z(x^*)$ is unity, $z(x^*)$ and L can be conformably partitioned to get

$$\begin{bmatrix} L_1 & L_2 \end{bmatrix} \begin{bmatrix} 1 & \tilde{z}(x^*)^T \end{bmatrix}^T = 0 \quad \Leftrightarrow \quad L_2 \tilde{z}(x^*) = -L_1 \tag{6}$$

Due to numerical errors, it may not be possible to satisfy (6) exactly, but a least squares estimate of $\tilde{z}(x^*)$ can be obtained as

$$\tilde{z}(x^*) = -L_2^{\dagger} L_1 \tag{7}$$

where $\dagger$ denotes the pseudo-inverse. Now, $x^* \in \mathbb{R}^{n_x}$ can be taken as the first n_x elements of $\tilde{z}(x^*)$ in (7). In this paper, we use Sedumi [14] interfaced with Matlab through Yalmip [8] for solving the SDPs.

3 Lower bound on Achievable Performance

We consider the closed-loop system shown in Figure 1, where $y(t)$, $u(t)$, and $a(t)$ denote the controlled output, manipulated variable, and disturbance, respectively. For this process,

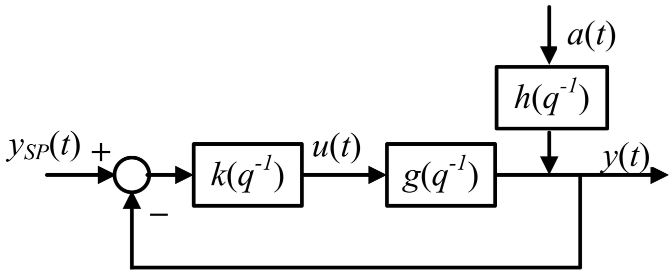


Fig. 1. Block diagram of closed loop system

$$y(t) = g(q^{-1})\, u(t) + h(q^{-1})\, a(t) \tag{8}$$

Here $g(q^{-1})$ and $h(q^{-1})$ are process and disturbance transfer functions, respectively. We make the following assumptions:

(1) $g(q^{-1})$ and $h(q^{-1})$ are stable, causal and contain no zeros on or outside the unit circle except at infinity (due to time delay).
(2) $a(t)$ is a random noise sequence with unit variance.

Note that there is no loss of generality in assuming that $a(t)$ has unit variance, as $h(q^{-1})$ can always be scaled to satisfy this assumption. For notational simplicity, we drop the arguments q^{-1} and t in subsequent discussion.

The closed-loop transfer function between a and y can be expressed as

$$y = s\, a; \quad s = (1 + g\, k)^{-1}\, h \tag{9}$$

Our objective is to find the least achievable variance of y, *i.e.*

$$J = \min_k \ \mathrm{Var}(y) = \min_k \ \|s\|_2^2 \tag{10}$$

where $\|.\|_2$ denotes the $\mathcal{H}_2$-norm. When no restrictions are imposed on the controller, the least achievable value of $\|s\|_2^2$ is known as minimum variance (MV) benchmark, where [5]

$$J_{mv} = \min_k \ \|s\|_2^2 = \sum_{i=0}^{d-1} h_i^2 \tag{11}$$

In (11), h_i is the i^{th} impulse response coefficient of h and d is the time delay associated with g. When the controller is restricted to have PID structure, however, MV benchmark may not be achievable. The primary difficulty in characterization of achievable PID performance is the non-convexity of the optimization problem in (10), when k is restricted to have PID structure. A key observation to overcome this difficulty is that

$$\|s\|_2^2 = \sum_{i=0}^{\infty} s_i^2 \geq \sum_{i=0}^{n} s_i^2 \tag{12}$$

for any finite n. Thus, a lower bound on the achievable PID performance can be found by minimizing $\sum_{i=0}^{n} s_i^2$. In the following discussion, we relate s_i with the unknown parameters of the PID controller given as

$$k_{PID} = k_P + \frac{k_I}{\Delta} + k_D\,\Delta \tag{13}$$

where k_P, k_I and k_D are proportional, integral and derivative gains, respectively, and $\Delta = 1 - q^{-1}$. By defining $c_0 = (k_P + k_I + k_D)$, $c_1 = -(k_P + 2k_D)$ and $c_2 = k_D$, the PID controller in (13) can be alternately represented as

$$k_{PID} = \frac{1}{\Delta}\,c; \qquad c = c_0 + c_1\,q^{-1} + c_2\,q^{-2} \tag{14}$$

We note that

$$s = (1 + g\,k)^{-1}\,h = (1 + \hat{g}\,c)^{-1}\,h \tag{15}$$

where $\hat{g} = g/\Delta$. When the closed-loop system is stable, (15) can be expanded using Taylor series expansion to get

$$s = \left[\sum_{i=0}^{\infty}(-1)^i(\hat{g}\,c)^i\right] h \tag{16}$$

For given n, we define the following $n \times n$-dimensional Hankel matrices

$$\hat{G}_H = \begin{bmatrix} \hat{g}_0 & 0 & \cdots & 0 \\ \hat{g}_1 & \hat{g}_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \hat{g}_{n-1} & \cdots & \hat{g}_1 & \hat{g}_0 \end{bmatrix}; \qquad C_H = \begin{bmatrix} c_0 & 0 & 0 & 0 & \cdots & 0 \\ c_1 & c_0 & 0 & 0 & \cdots & 0 \\ c_2 & c_1 & c_0 & 0 & \cdots & 0 \\ 0 & c_2 & c_1 & c_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & c_2 & c_1 & c_0 \end{bmatrix} \tag{17}$$

and the following n-dimensional vectors

$$h_v = \begin{bmatrix} h_0 & h_1 & \cdots & h_{n-1} \end{bmatrix}^T \tag{18}$$

$$s_v = \begin{bmatrix} s_0 \; s_1 \; \cdots \; s_{n-1} \end{bmatrix}^T \tag{19}$$

Based on (16)-(19), s_v can be compactly written as

$$s_v = \left[\sum_{i=0}^{n} (-1)^i (\hat{G}_H \, C_H)^i \right] h_v = \left[\sum_{i=0}^{\text{round}(n/d)} (-1)^i (\hat{G}_H \, C_H)^i \right] h_v \tag{20}$$

where the equality in (20) follows as $(\hat{G}_H \, C_H)^i = 0$ for $i > \text{round}(n/d)$. Now, a lower bound on the achievable PID performance can be found by solving

$$J_{PID} = \min_{c_0, c_1, c_2} \sum_{i=0}^{\infty} s_i^2 \geq \min_{c_0, c_1, c_2} \; s_v^T s_v \tag{21}$$

for any finite n. For $n = (2d-1)$, we note from (20) that $s_v = (I - \hat{G}_H \, C_H) \, h_v$. Then, s_v depends linearly on the unknown controller parameters and a lower bound on the achievable PID performance can be found in a least squares fashion [9]. The bound proposed in [9] can be loose, however, as the contribution of s_i, $i > (2d-1)$ towards $\|s\|_2^2$ is not accounted for.

We note that though nonlinear, s_v in (20) and thus $s_v^T s_v$ in (21) is a polynomial in terms of c_0, c_1 and c_2 for every finite n. Thus, the optimization problem in (21) can be solved using SOS programming approach discussed in Section 2. In comparison with [9], the use of SOS programming provides a tighter lower bound, when $n > (2d-1)$. For any value of the controller parameters

$$\sum_{i=0}^{n+1} s_i^2 \geq \sum_{i=0}^{n} s_i^2 \tag{22}$$

Thus, a tight lower bound on achievable PID performance can be obtained by solving (21) with n being increased sequentially until convergence. An upper bound on the achievable PID performance can be computed by finding c_0, c_1 and c_2, which solve (21), and evaluating $\|s\|_2^2 = \|(1 + \hat{g}\, c)^{-1} \, h\|_2^2$ using these parameters. For sufficiently large n, $s_v^T \, s_v \approx \|(1 + \hat{g}\, c)^{-1} \, h\|_2^2$ and thus the lower and upper bounds can be expected to be close to each other.

Example 1 *The proposed method is applied to find the lower and upper bounds on the achievable PI(D) performance for numerous case studies taken from literature. These case studies and the corresponding bounds are shown in Tables 1 and 2, respectively*[1] *. A few salient observations for these case studies are as follows:*

[1] The Matlab code used for solving these case studies is available at `http://www.ntu.edu.sg/home/Vinay/pub.htm`

Case	g	h	Reference
1	$\frac{0.2\,q^{-5}}{1-0.8\,q^{-1}}$	$\frac{1}{\Delta\,(1+0.4\,q^{-1})}$	[1]
2	$\frac{0.08919\,q^{-12}}{1-0.8669\,q^{-1}}$	$\frac{0.08919}{1-0.8669\,q^{-1}}$	[4]
3	$\frac{0.5108\,q^{-28}}{1-0.9604\,q^{-1}}$	$\frac{0.5108}{1-0.9604\,q^{-1}}$	[4]
4	$\frac{q^{-6}}{1-0.8\,q^{-1}}$	$\frac{1+0.6\,q^{-1}}{(1-0.5\,q^{-1})(1-0.6\,q^{-1})(1+0.7\,q^{-1})}$	[10]
5	$\frac{q^{-6}}{1-0.8\,q^{-1}}$	$\frac{1-0.2\,q^{-1}}{\Delta\,(1-0.3\,q^{-1})(1+0.4\,q^{-1})(1-0.5\,q^{-1})}$	[10]
6	$\frac{q^{-6}}{1-0.8\,q^{-1}}$	$\frac{1+0.6\,q^{-1}}{\Delta\,(1-0.5\,q^{-1})(1-0.6\,q^{-1})(1+0.7\,q^{-1})}$	[10]
7	$\frac{0.1\,q^{-5}}{1-0.8\,q^{-1}}$	$\frac{0.1}{\Delta\,(1-0.3\,q^{-1})(1-0.6\,q^{-1})}$	[10]
8	$\frac{0.1\,q^{-3}}{1-0.8\,q^{-1}}$	$\frac{1}{\Delta}$	[11]
9	$\frac{0.1\,q^{-6}}{1-0.8\,q^{-1}}$	$\frac{0.1}{\Delta(1-0.7\,q^{-1})}$	[11]
10	$\frac{0.1\,q^{-3}}{1-0.8\,q^{-1}}$	$\frac{\sqrt{0.001}}{\Delta\,(1+0.2\,q^{-1})}$	[13]

Table 1
Case studies

(1) *In comparison with time delay, the additional limitations on achievable performance imposed by controller structure can be as high as* 100%, e.g. *for Cases 6 and 7 (PI control).*

(2) *For all cases, the lower bound computed using SOS programming on achievable PI(D) performance is tight, as can be verified through comparison with the upper bound. The maximum relative difference between the lower and upper bounds is about* 7%, *which is seen for Case 6 (PID control).*

(3) *For Cases 2, 3, and 4, the nearly-optimal controllers do not have integral action, i.e. the integrating poles of the controllers are cancelled by zeros on the unit circle. We recall that a stabilizing controller k must have at least as many poles at $q = 1$ as the disturbance model h. For Cases 2, 3, and 4, h does not have any integrating poles and thus a stable closed-loop system is obtained without having integral action in the controllers.*

(4) *For Case 1, the lower bound on achievable PI performance calculated using the result in [9] is 2.9726, which is close to the MV benchmark implying no significant limitations due to controller structure. In comparison, the SOS programming method correctly identifies the additional limitations on achievable performance due to controller structure.*

(5) *To characterize the limitations due to the PID structure of the controller, Huang [7] suggested using an integrator as the disturbance model. We note that for Cases 4, 5, and 6, though the process model g is the same, the limitations due to controller structure differ significantly, as the disturbance models are different for each case. This observation highlights that the use of an arbitrary disturbance model (*e.g. *integrator) may lead to incorrect conclusions regarding the limitations on achievable performance due to controller structure.*

Case	J_{mv}	PI Controller				PID Controller		
		UB	SOS Programming			SOS Programming		
		(Literature)	LB	UB	Controller	LB	UB	Controller
1	2.9427	3.5961	3.5154	3.5186	$\frac{1.1209-0.9930\,q^{-1}}{\Delta}$	3.0730	3.0730	$\frac{2.8327-4.3950\,q^{-1}+1.7478\,q^{-2}}{\Delta}$
2	0.0306	0.0314	0.0313	0.0314	0.3092	0.0310	0.0310	$1.7433 - 1.4495\,q^{-1}$
3	3.0112	3.8349	3.1703	3.1706	0.0305	3.0492	3.0495	$0.4796 - 0.4493\,q^{-1}$
4	3.4004	3.4410	3.4408	3.4408	0.0234	3.4065	3.4065	$0.1338 - 0.1148\,q^{-1}$
5	11.9528	17.7460	17.7044	17.7477	$\frac{0.2118-0.1896\,q^{-1}}{\Delta}$	13.6341	13.8243	$\frac{0.7251-1.2072\,q^{-1}+0.5200\,q^{-2}}{\Delta}$
6	58.3406	123.5400	122.4089	123.6037	$\frac{0.2304-0.2070\,q^{-1}}{\Delta}$	83.5605	89.6983	$\frac{0.8989-1.5413\,q^{-1}+0.6926\,q^{-2}}{\Delta}$
7	0.2978	0.6511	0.5856	0.5884	$\frac{2.6044-2.3248\,q^{-1}}{\Delta}$	0.4278	0.4278	$\frac{8.6911-14.3626\,q^{-1}+6.2116\,q^{-2}}{\Delta}$
8	3.0000	3.7044	3.7002	3.7050	$\frac{3.4416-3.0074\,q^{-1}}{\Delta}$	3.2093	3.2093	$\frac{6.6909-9.5975\,q^{-1}+3.6106\,q^{-2}}{\Delta}$
9	0.3144	0.6028	0.5949	0.5968	$\frac{2.2945-2.0588\,q^{-1}}{\Delta}$	0.4288	0.4288	$\frac{8.7008-14.6895\,q^{-1}+6.4323\,q^{-2}}{\Delta}$
10	0.0023	-	0.0027	0.0027	$\frac{3.5074-3.1188\,q^{-1}}{\Delta}$	0.0024	0.0025	$\frac{5.1681-6.6294\,q^{-1}+2.0130\,q^{-2}}{\Delta}$

Table 2
Lower (LB) and upper (UB) bounds on achievable PI and PID performance

(6) For Cases 1, 3, 7 and 9, the upper bound on achievable PI performance found using SOS programming is smaller than reported in the literature. This is likely due to the use of local optimal values of c_0, c_1, *and* c_2 *found using gradient based methods [1; 10; 11] or stochastic optimization [4] for the non-convex optimization problem.*

(7) For each case, the proposed method takes less than 100 seconds on a PC with Intel® Dual Core$^{\mathrm{TM}}$ 2.40GHz, 2GB RAM using MATLAB® 2007a showing computational efficiency, even for processes with large time delay (Case 3).

In summary, these case studies demonstrate that SOS programming can be used to find tight bounds on achievable PI(D) performance efficiently and reliably.

4 Weighted Input and Output Performance

Controller tuning through minimization of output variance only (cheap control) may lead to excessive input variation. This difficulty can be overcome by penalizing the rate of input change, *i.e.* minimizing a weighted sum of the variances of output and input rate defined as

$$J_w = \min_k \; (\mathrm{Var}(y) + \rho\, \mathrm{Var}(\Delta\, u)) \tag{23}$$

where ρ is the penalty factor. When there is no restriction on the controller, the optimal solution to the problem in (23) can be found using the subspace method [6]. When a PID controller is used, similar to cheap control, *i.e.* $\rho = 0$, the bounds on J_w found using subspace method may not be achievable. To find the achievable value of J_w with PID controller, let $\Delta\, u = w\, a = -\Delta\, k\, s\, a$. For given n, define

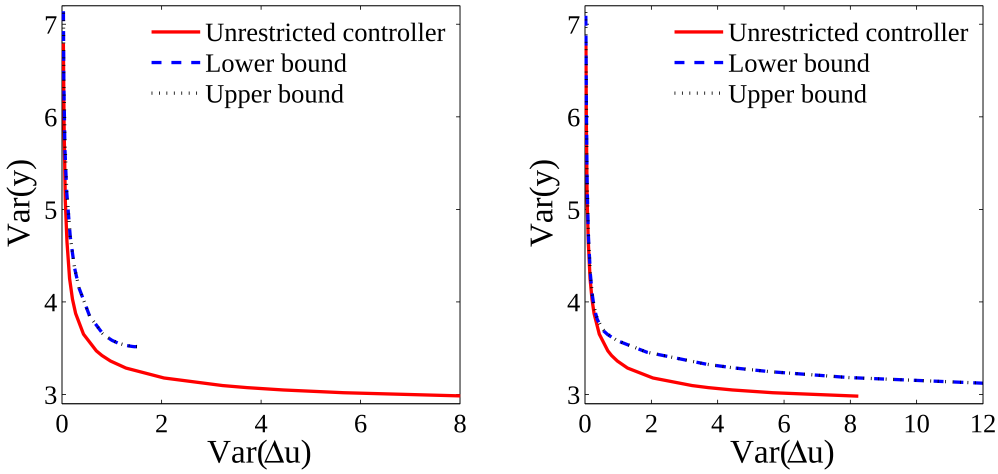
$$w_v = \begin{bmatrix} w_0 \; w_1 \; \cdots \; w_{n-1} \end{bmatrix}^T \tag{24}$$

It follows that $w_v = -C_H\, s_v$. Then

$$J_{w,PID} = \min_{c_o,c_1,c_2} \sum_{i=0}^{\infty} (s_i^2 + \rho\, w_i^2) \tag{25}$$

$$\geq \min_{c_o,c_1,c_2} \sum_{i=0}^{n} (s_i^2 + \rho\, w_i^2) = \min_{c_o,c_1,c_2} (s_v^T\, s_v + \rho\, w_v^T\, w_v) \tag{26}$$

for any finite n. As before, the optimization problem in (26) can be solved using SOS programming approach discussed in Section 2. The lower and upper bounds on output variance can be computed by evaluating $s_v^T\, s_v$ and



(a) PI controller

(b) PID controller

Fig. 2. Trade-off between $\mathrm{Var}(y)$ and $\mathrm{Var}(\Delta\, u)$ for Case 1

$\|(1+\hat{g}\, c)^{-1}\, h\|_2^2$, respectively, using c_0, c_1 and c_2, which solve (26). The lower and upper bounds on the variance of input rate can be found similarly by evaluating $w_v^T\, w_v$ and $\|c\,(1+\hat{g}\, c)^{-1}\, h\|_2^2$, respectively.

Example 2 *For Case 1 in Table 1, we minimize the weighted sum of the variances of output and input rate for different values of ρ taken in the range* $0-100$*. The lower and upper bounds on the trade-off curve between* $\mathrm{Var}(y)$ *and* $\mathrm{Var}(\Delta\, u)$ *for PI and PID control are shown in Figure 2. The trade-off curves in Figure 2 confirm that the controller structure gives rise to additional limitations on achievable control performance. These limitations are more prominent for small values of ρ. When ρ is increased, the trade-off curves for different controllers approach each other. This is expected, as when $\rho \to \infty$, the optimal unrestricted and PI(D) controllers approach zero controllers implying small* $\mathrm{Var}(\Delta\, u)$*, but arbitrarily large* $\mathrm{Var}(y)$*. Similar trends are seen for other cases given in Table 1. Figure 2 also demonstrates that SOS programming provides tight lower bounds on the variances of output and input rate, as the lower and upper bounds are almost identical for different values of ρ.*

5 Conclusions

We propose the use of sums of squares (SOS) programming to find lower bound on achievable PID performance. Numerous examples taken from the literature demonstrate the tightness of the proposed bounds. Future research will focus on extending these results to characterize the fundamental limitation arising due to the decentralized structure of the controller. The main challenge in the extension is that for decentralized control, SOS programming often leads to ill-conditioned large-dimensional semidefinite programs, whose solution is prone to numerical errors and requires significant computational time.

Acknowledgements

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