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<th>Title</th>
<th>Hybrid method of tolerance design (Published version)</th>
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<td>Author(s)</td>
<td>Foo, Say Wei; Lin, Yu</td>
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HYBRID METHOD OF TOLERANCE DESIGN

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ABSTRACT

A new method of tolerance design, which we call the Hybrid method, that accelerates the attainment of optimal design center, is proposed. The method combines geometry search with statistical process. At the start of the process, the region of acceptability is estimated using boundary search method. A new tolerance region that covers as much of the estimated region of acceptability is determined. The Centers-of-Gravity (CoG) method is then applied to find a new design center. The tolerance region is then progressively reduced to target size while the process of locating the optimal design centers continues. Compared to the popular CoG method, the speed of convergence of the Hybrid method is much faster while the additional computational load is insignificant.

1. INTRODUCTION

Design centring, the most important aspect of tolerance design, is concerned with finding the nominal design center that gives the maximum manufacturing yield for a given set of component tolerances and a given set of performance specifications. Two of the most popular methods for design centring are the Simplicial method pioneered by S.W.Director [1] and the Centers-of-Gravity method advocated by R.Spence [2].

In essence, the Simplicial method attempts to find a geometrical approximation to the region of acceptability. It makes extensive use of linear programming to determine the maximum size of some geometrical body such as a polytope or an ellipsoid that fits in the region of acceptability in the multi-dimensional space of design parameters. The center of this body is then taken to be the design center.

For the geometrical approach to give valid results the region of acceptability must be convex and the optimal design center must be located at the geometric center. These conditions may not always be met. The strength of the Simplicial method lies in its ability to determine the approximate physical size and location of the region of acceptability. However, the method suffers from the curse of dimensionality in that the computational load increases exponentially with the number of tolerated parameters. For this reason, the method is not popular as most practical problems involve a large number of tolerated parameters.

Another approach to design centring is to use statistical sampling. One of the most popular statistical approaches is the Centers-of-Gravity method. The method does not suffer from the curse of dimensionality, as the computational efforts are essentially dimensionally independent. No constraint on the shape of the region of acceptability is imposed. In addition, the confidence level of the accuracy of yield can be improved by using larger numbers of sampling points. Thus the approach has gained widespread acceptance. However, as this is basically an iterative local search method, the computational load is high, especially if many iterations are required to attain the optimal design center.

In this paper, a new method that inherits the advantages of both the Simplicial method and the Centers-of-Gravity method is proposed. Like the CoG method, no constraint on shape of region of acceptability need be imposed, confidence level in estimation of yield may be increased using larger numbers of sample points, and most important of all, convergence to optimal design is significantly sped up.

Detailed description of the Hybrid method is given in the next section. Two examples of the application of the method and comparison with the CoG method are presented in Section 3.

2. THE HYBRID METHOD

2.1 Notations

For a circuit of m variable components \( P = [p_1, p_2, \ldots, p_m] \) with tolerances \( T = \{ t_1, t_2, \ldots, t_m \} \), the target tolerance region \( R_T = \{ p \in \mathbb{P} | p_{i-} \leq p_i \leq p_{i+} \} \) for \( i = 1, 2, \ldots, m \) where \( p^0 = [p_{1-}, p_{1+}, \ldots, p_{m-}, p_{m+}] \) is the nominal design center. Let the performance of the circuit be represented by \( F(P) = [f_1(P), f_2(P), \ldots, f_n(P)] \), then the region of acceptability \( R_A \) is the set given by \( R_A = \{ p \in \mathbb{P} | f_j(P) - L_j \leq f_j(P) \leq U_j \} \) where \( f_j(P) \) is the performance of the circuit at the \( j^{th} \) point of interest and \( L_j \) and \( U_j \) are respectively the lower and upper specification limits of the circuit’s responses at this point. The feasible region \( R_F \), where the circuits meet the specifications, is the intersection of the tolerance
The tolerance region is progressively reduced, usually in three iterations, to the target size while the search process continues. When the tolerance region is reduced to the target size, the search process is then performed in the original search space with the target tolerance region. Searching continues until a suitable value of yield is obtained or until the number of iterations has exceeded a predetermined limit.

Note that too large an initial tolerance region $R_F$, where $R_T \supseteq R_A$, is not desirable as many points may then lie outside the region of intersection $R_T \cap R_A$ and this reduces the density of points in $R_A$. Insufficient sampling points in $R_A$, on the other hand, lead to a wrong prediction of the global optimal. The desirable initial tolerance size shall be one that is just big enough to cover the major part of $R_A$, i.e., $R_T \equiv R_A$.

### 3. EXAMPLES OF APPLICATION

Two examples of applications are described in the following. The first example involves only two tolerated parameters and hence the results can be easily envisioned. The second example involves a practical filter of some complexity. The example provides more convincing demonstration of the power of the proposed Hybrid method.

#### 3.1 Example 1.

The resistive potential divider is shown in Fig. 1.

![Fig. 1: The circuit and its constraints.](image)

The performance specifications of the circuit are given by the following two sets of constraints:

$$80 < R_1 + R_2 < 120$$  \hspace{2cm} (1)

$$4.5 < V_{out} < 5.5$$  \hspace{2cm} (2)

The two tolerated parameters are the two resistors, $R_1$ and $R_2$.

In the Fig. 2, the changes in the size and/or position of $R_F$ and the design centers for the first few iterations are shown.
3.2 Example 2.

The filter circuit commonly cited in literature on tolerance design is shown in Fig. 4(a). The performance criteria are that the relative attenuation values at a set of chosen frequencies must fall within specified limits.

The attenuation as a function of frequency $f$ is given by 

$$A(f) = 20 \log \left( \frac{|V_o(2\pi f)|}{|V_o(2\pi 990)|} \right).$$

The desirable performance in terms of relative attenuation at the various frequencies is depicted in Fig.4(b).

Although there are 11 components in the circuit, only the following four designable parameters denoted by $P^T = (C_1, C_3, C_4, C_5)$ are chosen as the tolerated parameters.

<table>
<thead>
<tr>
<th>experiment</th>
<th>$Y_0$</th>
<th>$Y_{C4}$</th>
<th>$Y_{\text{max}}$</th>
<th>$Y_{H3}$</th>
<th>$(Y_{H3} - Y_{C4})/Y_{C4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.54</td>
<td>0.93</td>
<td>0.98</td>
<td>0.98</td>
<td>5.4</td>
</tr>
<tr>
<td>2</td>
<td>0.33</td>
<td>0.79</td>
<td>0.93</td>
<td>0.96</td>
<td>21.5</td>
</tr>
<tr>
<td>3</td>
<td>0.01</td>
<td>0.44</td>
<td>0.68</td>
<td>0.96</td>
<td>118.2</td>
</tr>
<tr>
<td>average</td>
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<td>0.72</td>
<td>0.86</td>
<td>0.96</td>
<td>48.4</td>
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**Table 1:** Comparison of CoG Method and Hybrid Method

For purpose of comparison, the CoG method as well as the proposed Hybrid method are applied to this circuit using different initial nominal design values. 10 iterations were carried out for each of the different
starting points. The initial yield $y_0$, yield obtained at the 4th iteration $y_{C4}$, maximum yield obtained in 10 steps of CoG method $y_{C_{max}}$, 3rd step yield of the Hybrid method $y_{H3}$, and the improvement of $y_{H3}$ over $y_{C_4}$ are tabulated in Table 1. The numbers of extra circuit evaluations used to determine the approximate region of acceptability are tabulated in Table 2.

<table>
<thead>
<tr>
<th>experiment</th>
<th>extra evaluations to relocate the design center</th>
<th>total extra evaluations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>556</td>
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<tr>
<td>2</td>
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<td>430</td>
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<td>3</td>
<td>110</td>
<td>247</td>
</tr>
<tr>
<td>average</td>
<td>37</td>
<td>411</td>
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Table 2: Extra Evaluations of Hybrid Method

The yields achieved at the different iterations are plotted in Fig.5(a) for the CoG method and Fig.5(b) for the Hybrid method.

![Fig. 5: Comparison of CoG Method and the Hybrid Method.](image)

4. CONCLUSION

A novel method of design centring which we called the Hybrid method is proposed and described in this paper. The method combines boundary search of the region of acceptability with the Centers-of-Gravity approach. The Hybrid method inherits the advantages of both geometrical approach and the statistical approach while compensates for their intrinsic disadvantages to some extent.

The Hybrid method does not impose any special condition on the shape and size of the region of acceptability. Compared with the CoG method, the proposed method is computationally more efficient: the computational efforts to reach the optimal design are substantially reduced. Its performance is also less dependent on initial design point.

The Hybrid method proposed is robust, simple to understand and simple to use.

ACKNOWLEDGMENT

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REFERENCES


