

This document is downloaded from DR-NTU, Nanyang Technological University Library, Singapore.

Title	Hybrid method of tolerance design( Published version )
Author(s)	Foo, Say Wei; Lin, Yu
Citation	Foo, S. W., & Lin, Y. (1999). Hybrid method of tolerance design. IEEE International Conference on Electronics, Circuits and Systems (pp. 557-560) Singapore: National University of Singapore.
Date	1999
URL	<a href="http://hdl.handle.net/10220/4634">http://hdl.handle.net/10220/4634</a>
Rights	<p>© IEEE. Personal use of this material is permitted. However, permission to reprint/republish this material for advertising or promotional purposes or for creating new collective works for resale or redistribution to servers or lists, or to reuse any copyrighted component of this work in other works must be obtained from the IEEE. This material is presented to ensure timely dissemination of scholarly and technical work. Copyright and all rights therein are retained by authors or by other copyright holders. All persons copying this information are expected to adhere to the terms and constraints invoked by each author's copyright. In most cases, these works may not be reposted without the explicit permission of the copyright holder. This material is presented to ensure timely dissemination of scholarly and technical work. Copyright and all rights therein are retained by authors or by other copyright holders. All persons copying this information are expected to adhere to the terms and constraints invoked by each author's copyright. In most cases, these works may not be reposted without the explicit permission of the copyright holder.</p> <p><a href="http://www.ieee.org/portal/site">http://www.ieee.org/portal/site</a></p>

# HYBRID METHOD OF TOLERANCE DESIGN

Say Wei FOO<sup>1</sup> and Yu LIN<sup>2</sup>  
Department of Electrical Engineering  
National University of Singapore  
10 Kent Ridge Crescent  
Singapore 119260

<sup>1</sup>elefoosw@nus.edu.sg, <sup>2</sup>yulin@eeserver.ee.nus.edu.sg

## ABSTRACT

A new method of tolerance design, which we call the Hybrid method that accelerates the attainment of optimal design center, is proposed. The method combines geometry search with statistical process. At the start of the process, the region of acceptability is estimated using boundary search method. A new tolerance region that covers as much of the estimated region of acceptability is determined. The Centers-of-Gravity (CoG) method is then applied to find a new design center. The tolerance region is then progressively reduced to target size while the process of locating the optimal design centers continues. Compared to the popular CoG method, the speed of convergence of the Hybrid method is much faster while the additional computational load is insignificant.

## 1. INTRODUCTION

Design centring, the most important aspect of tolerance design, is concerned with finding the nominal design center that gives the maximum manufacturing yield for a given set of component tolerances and a given set of performance specifications. Two of the most popular methods for design centring are the Simplicial method pioneered by S.W.Director [1] and the Centers-of-Gravity method advocated by R.Spence [2].

In essence, the Simplicial method attempts to find a geometrical approximation to the region of acceptability. It makes extensive use of linear programming to determine the maximum size of some geometrical body such as a polytope or an ellipsoid that fits in the region of acceptability in the multi-dimensional space of design parameters. The center of this body is then taken to be the design center.

For the geometrical approach to give valid results the region of acceptability must be convex and the optimal design center must be located at the geometric center. These conditions may not always be met. The strength of the Simplicial method lies in its ability to determine the approximate physical size and location of the region of acceptability. However, the method suffers from the curse of dimensionality in that the computational load increases exponentially with the number of tolerated parameters. For this reason, the

method is not popular as most practical problems involve a large number of tolerated parameters.

Another approach to design centring is to use statistical sampling. One of the most popular statistical approaches is the Centers-of-Gravity method. The method does not suffer from the curse of dimensionality, as the computational efforts are essentially dimensionally independent. No constraint on the shape of the region of acceptability is imposed. In addition, the confidence level of the accuracy of yield can be improved by using larger numbers of sampling points. Thus the approach has gained widespread acceptance. However, as this is basically an iterative local search method, the computational load is high, especially if many iterations are required to attain the optimal design center.

In this paper, a new method that inherits the advantages of both the Simplicial method and the Centers-of-Gravity method is proposed. Like the CoG method, no constraint on shape of region of acceptability need be imposed, confidence level in estimation of yield may be increased using larger numbers of sample points, and most important of all, convergence to optimal design is significantly sped up.

Detailed description of the Hybrid method is given in the next section. Two examples of the application of the method and comparison with the CoG method are presented in Section 3.

## 2. THE HYBRID METHOD

### 2.1 Notations

For a circuit of  $m$  variable components  $\mathbf{P} = \{p_1, p_2, \dots, p_m\}$  with tolerances  $\mathbf{T} = \{t_1, t_2, \dots, t_m\}$ , the target tolerance region  $\mathbf{R}_T$ , is the set given by  $\mathbf{R}_T = \{\mathbf{P} \mid p_i^0 - t_i \leq p_i \leq p_i^0 + t_i; i = 1, 2, \dots, m\}$  where  $\mathbf{P}^0 = \{p_1^0, p_2^0, \dots, p_m^0\}$  is the nominal design center. Let the performance of the circuit be represented by  $\mathbf{F}(\mathbf{P}) = \{f_1(\mathbf{P}), f_2(\mathbf{P}), \dots, f_n(\mathbf{P})\}$ , then the region of acceptability  $\mathbf{R}_A$ , is the set given by  $\mathbf{R}_A = \{\mathbf{P} \mid L_j \leq f_j(\mathbf{P}) \leq U_j; j = 1, 2, \dots, n\}$ , where  $f_j(\mathbf{P})$  is the performance of the circuit at the  $j^{\text{th}}$  point of interest and  $L_j$  and  $U_j$  are respectively the lower and the upper specification limits of the circuit's responses at this point. The feasible region  $\mathbf{R}_F$ , where the circuits meet the specifications, is the intersection of the tolerance

region and region of acceptability. Mathematically,  $\mathbf{R}_F = \mathbf{R}_A \cap \mathbf{R}_T$ .

An estimate of the manufacturing yield  $Y$ , is given by the following  $m$ -fold integral

$$Y = \int \dots \int_{R_F} \phi(p_1, \dots, p_m) dp_1 \dots dp_m.$$

where  $\phi(p_1, \dots, p_m)$  is the joint *pdf* of the values of the  $m$  variable components.

Monte Carlo method is commonly used to estimate this yield. For the Monte Carlo method, component values are randomly generated according to their probability distributions. The performances of circuits using the sets of component values so generated are then assessed either using formulae or through computer simulation.

A sufficiently large number of circuits must be evaluated to obtain statistically meaningful results. Hence a large number of sets of component values are generated. The manufacturing yield is then estimated as the percentage of circuits that pass the performance specifications. Mathematically, the yield  $Y$  is given by

$$Y = \text{number of pass circuits} / \text{number of total circuits tested}.$$

## 2.2 The Algorithm

It is well known that most search-methods, including the CoG method, may end up at a local optimum instead of the global optimum if the initial point is not properly chosen. The proposed Hybrid method aims to smoothen the search space by taking a global view before zooming in specific area to search for the optimal point. In addition to targeting at the global optimum, the method is also effective in reducing the number of iterations required for the searching process.

The Hybrid method starts with geometric search to give a crude estimate of the region of acceptability. A region large enough to just cover the estimated region of acceptability is then determined and used as an initial tolerance region. This initial tolerance region is in most practical cases, larger than the target tolerance region. With this larger tolerance region, a solution close to the most probable global optimum can be identified and zoomed in. This also ensures that  $\mathbf{R}_T \cap \mathbf{R}_A = \mathbf{R}_A$  and hence reduces the possibility that to get stuck at a local optima. Search methods such as the Centers-of-Gravity method is then applied to search for this global optimal design center.

The size of the initial tolerance region is then adjusted towards the target size at the end of the first iteration. CoG method is again used to determine an improved design center.

The tolerance region is progressively reduced, usually in three iterations, to the target size while the search process continues. When the tolerance region is reduced to the target size, the search process is then performed in the original search space with the target tolerance region. Searching continues until a suitable value of yield is obtained or until the number of iterations has exceeded a predetermined limit

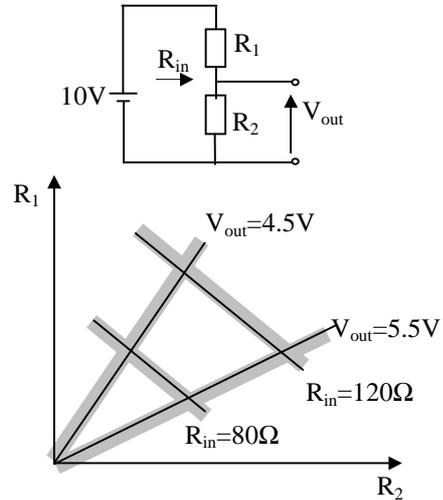
Note that too large an initial tolerance region  $\mathbf{R}_T$ , where  $\mathbf{R}_T \supseteq \mathbf{R}_A$ , is not desirable as many points may then lie outside the region of intersection  $\mathbf{R}_T \cap \mathbf{R}_A$  and this reduces the density of points in  $\mathbf{R}_A$ . Insufficient sampling points in  $\mathbf{R}_A$ , on the other hand, lead to a wrong prediction of the global optimal. The desirable initial tolerance size shall be one that is just big enough to cover the major part of  $\mathbf{R}_A$ , i.e.,  $\mathbf{R}_T \cong \mathbf{R}_A$ .

## 3. EXAMPLES OF APPLICATION

Two examples of applications are described in the following. The first example involves only two toleranced parameters and hence the results can be easily envisioned. The second example involves a practical filter of some complexity. The example provides more convincing demonstration of the power of the proposed Hybrid method.

### 3.1 Example 1.

The resistive potential divider is shown in Fig. 1,



**Fig. 1:** The circuit and its constraints.

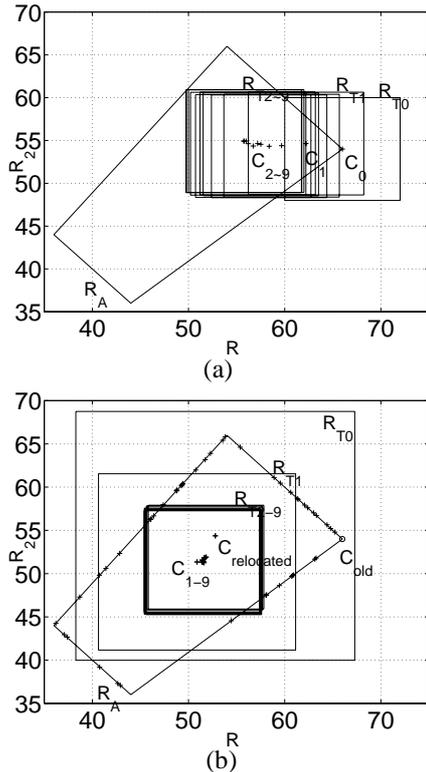
The performance specifications of the circuit are given by the following two sets of constraints:

$$80 < R_1 + R_2 < 120 \quad (1)$$

$$4.5 < V_{out} < 5.5 \quad (2)$$

The two toleranced parameters are the two resistors,  $R_1$  and  $R_2$ .

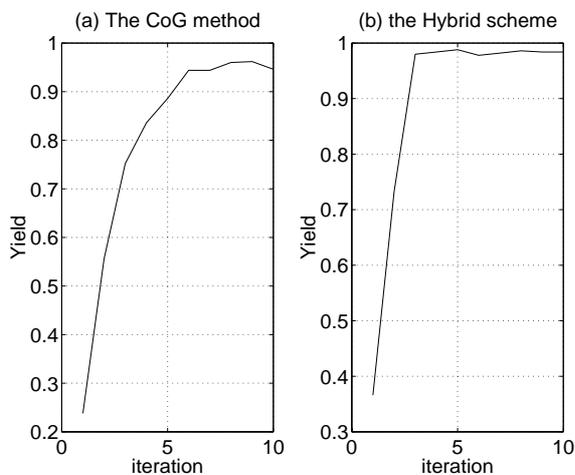
In the Fig. 2, the changes in the size and/or position of  $\mathbf{R}_T$  and the design centers for the first few iterations are shown.



**Fig.2:** Graphical Illustration of the Search Process of (a) the CoG Method and (b) the Hybrid Method.

Comparing Fig.2(a) and Fig.2(b), it can be seen that by first doing a boundary search, the Hybrid method is able to rapidly capture the major part of  $R_A$  and zoom in to the optimal design centre. The CoG method, on the other hand, moves towards the optimal centre slowly by trial and error.

The yields achieved at the different iterations using the Hybrid method and the CoG method are also plotted in Fig.3. It is apparent that the Hybrid method is able to reach the optimal yield in a much shortest time.



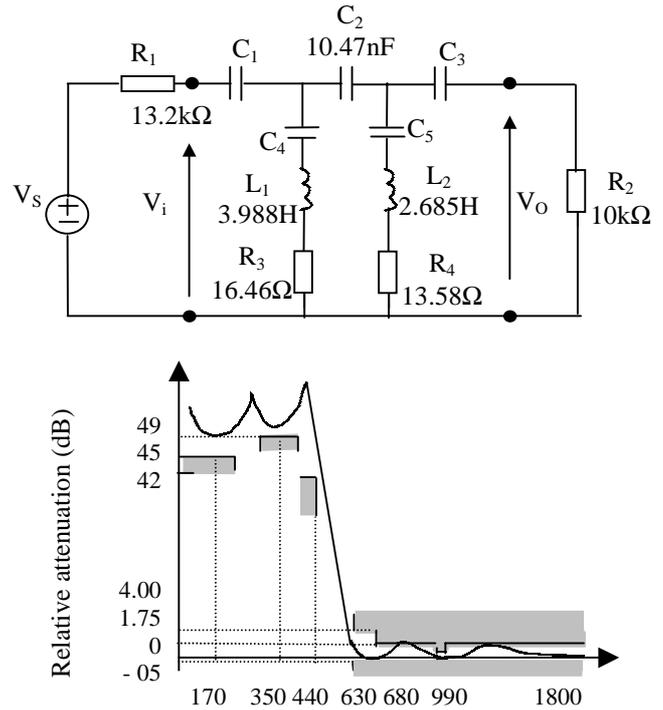
**Fig. 3:** Comparison of CoG Method and the Hybrid Method.

### 3.2 Example 2.

The filter circuit commonly cited in literature on tolerance design is shown in Fig.4(a). The performance criteria are that the relative attenuation values at a set of chosen frequencies must fall within specified limits.

The attenuation as a function of frequency  $f$  is given by  $A(f) = 20 \log ( |V_o(2\pi 990) / V_o(2\pi f)| )$ .

The desirable performance in terms of relative attenuation at the various frequencies is depicted in Fig.4(b).



**Fig. 4:** (a) The filter circuit (b) Desired specifications.

Although there are 11 components in the circuit, only the following four designable parameters denoted by  $P^T = ( C_1, C_3, C_4, C_5 )$  are chosen as the tolerated parameters.

experiment	CoG			Hybrid	Improvement %
	$y_0$	$y_{C4}$	$y_{Cmax}$		
1	0.54	0.93	0.98	0.98	5.4
2	0.33	0.79	0.93	0.96	21.5
3	0.01	0.44	0.68	0.96	118.2
average	0.29	0.72	0.86	0.96	48.4

**Table 1:** Comparison of CoG Method and Hybrid Method

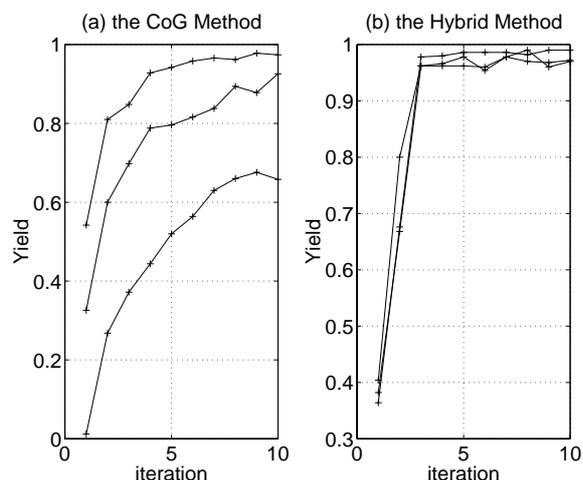
For purpose of comparison, the CoG method as well as the proposed Hybrid method are applied to this circuit using different initial nominal design values. 10 iterations were carried out for each of the different

starting points. The initial yield  $y_0$ , yield obtained at the 4<sup>th</sup> iteration  $y_{C4}$ , maximum yield obtained in 10 steps of CoG method  $y_{Cmax}$ , 3<sup>rd</sup> step yield of the Hybrid method  $y_{H3}$ , and the improvement of  $y_{H3}$  over  $y_{C4}$  are tabulated in Table 1. The numbers of extra circuit evaluations used to determine the approximate region of acceptability are tabulated in Table 2.

experiment	extra evaluations in		total extra evaluations
	relocate the design center	boundary search	
1	0	556	556
2	1	430	431
3	110	137	247
average	37	374	411

**Table 2:** Extra Evaluations of Hybrid Method

The yields achieved at the different iterations are plotted in Fig.5(a) for the CoG method and Fig.5(b) for the Hybrid method.



**Fig. 5:** Comparison of CoG Method and the Hybrid Method.

As in Example 1, it can be observed that the Hybrid method is able to attain the optimal design point much faster than the CoG method. From the lowest one of the three curves in Fig.5(a), it can also be concluded that if the initial design point is not properly chosen, the CoG method may not be able to reach the optimal point at all. It is also apparent that the performance of the Hybrid method is more or less independent of the initial design point and optimal yield is achieved in three to four iterations.

From the data recorded in Table 2, the additional computational effort required of the Hybrid method over the CoG method is approximately equal to the computations required for one iteration of the CoG search process. This additional computational effort is required to provide a crude estimate of the region of acceptability at the start of the process.

## 4. CONCLUSION

A novel method of design centring which we called the Hybrid method is proposed and described in this paper. The method combines boundary search of the region of acceptability with the Centers-of-Gravity approach. The Hybrid method inherits the advantages of both geometrical approach and the statistical approach while compensates for their intrinsic disadvantages to some extent.

The Hybrid method does not impose any special condition on the shape and size of the region of acceptability. Compared with the CoG method, the proposed method is computationally more efficient: the computational efforts to reach the optimal design are substantially reduced. Its performance is also less dependent on initial design point.

The Hybrid method proposed is robust, simple to understand and simple to use.

## ACKNOWLEDGMENT

The authors would like to thank the National University of Singapore for providing the University Research Grant to support the research presented in this paper.

## REFERENCES

- [1] Stephen W. Director, Gary D. Hachtel and Luis Manuel Vidigal, "Computationally Efficient Yield Estimation Procedures Based on Simplicial Approximation," *IEEE Transactions on Circuits and Systems*, Vol. CAS-25, No.3, March 1978, pp.121-130.
- [2] Robert Spence, Randeep Singh Sooin, Tolerance design of electronic circuits, *Addison-Wesley Publishing Company*, 1988.
- [3] B. V. Batalov, Y. N. Belyakow, and F. A. Kurmaev, "Some methods for statistical optimization of integrated microcircuits with statistical relations among the parameters of the components", *Soviet Microelectronics (USA)*, Vol. 7, No. 14, 1978, pp. 228-238.
- [4] K. Singhal, and J. F. Pintel, "Statistical design centring and tolerancing using parametric sampling," *IEEE Transactions Circuit and Systems*, Vol. CAS-28, No. 7, 1981, pp.692-702.
- [5] M. L. Stein, "An efficient method of sampling for statistical circuit design." *IEEE Transactions Computer-Aided Design*, Vol. CAD-5, No. 1, 1986, pp.23-29.
- [6] Yu LIN, and Say Wei FOO, "Fast search algorithm for tolerance design." *IEE Proceedings on Circuits, Devices and Systems*, Vol.145, No.1, February, 1998, pp.19-23.