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<td><strong>Author(s)</strong></td>
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Optimization of a compact falling-droplet absorber for cooling power generation

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Abstract
Refrigeration has become a necessary component for comfort living. Absorption refrigeration is a valid option for waste-heat-to-cool conversion. Coupling this technology with cheap heat energy sources is an interesting prospect, however downsizing of this type or chiller for small environments has been proven difficult, especially regarding the absorber. Large interface area between the two operating fluids returns higher absorption rates, but lack of control on the fluid distribution results in an inefficient use of the space available. This study proposes a space-efficient design based on finned-plate technology coupled with a droplet flow regime. Manufacturing through 3D printing technique is used to study the effect of fins shape. Droplet behaviour is firstly studied with an analytical model based on the variational approach. Experimental results were obtained using a high speed camera employed to validate the analytical results and obtain qualitative and quantitative data to complete the analysis. The results show that the analytical model reproduces with sufficient accuracy the droplet dynamics in some regions. The rhomboidal geometry with 120° angle proved able to produce the smallest droplets without allowing merging of more droplets, ensuring the maintenance of droplet flow. Disturbances in the droplet profiles were observed, caused by the pin-droplet interaction. Further study is required to refine the model (to account for these disturbances) and obtain a more precise prediction of the droplet sizes.

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Keywords: Refrigeration; Absorption; Absorber; Droplet Flow; Fluid Dynamics

1. Introduction

The next years will see a great increase in demand for cooling, which might reach a value ten times higher than the current one by 2050 according to Isaac and Van Vuuren (2009).

Heat-driven cooling systems might help reduce the potential stress on electricity usage, especially if the driving force can be provided by cheap solar energy (Kim and Infante Ferreira, 2008) or waste heat (Ammar et al., 2012). In this field many options are available, but absorption refrigeration is one of the main technologies for effective low-grade heat-to-cool conversion (Little and Garimella, 2011), thanks to its direct heat-to-cool conversion (Cola et al., 2016). However, implementation of absorption refrigeration in fields such as domestic and automotive air-conditioning is hindered by the large footprint of this chiller. A few attempts have been made at developing a small scale chiller, such as the device of Determan and Garimella (2012). The main obstacle to face in this regard is the absorber downsizing (Killion and Garimella, 2001). Extensive research has been focusing on how to improve its performance and consequently reduce its size. Raisul Islam et al. (2003) used film inverting techniques, while flat fins by Mortazavi et al. (2015) and a herringbone structure by Bigham et al (2013) are notable examples of film perturbation strategies that can improve the solution mixing and cooling, and consequently the absorption process.

The use of an adiabatic absorber can allow for more diverse strategies. The efficacy of droplet and column flow for the solution has been assessed by Li et al. (2015) while the use of flat fan sheets of solution by use of a vee-jet nozzle has been studied by Palacios et al. (2009). The use of an adiabatic absorber does increase the contact area between refrigerant and solution, but this comes at the cost of reduced solution mixing.

The authors here propose a solution to solve the internal mixing issue while keeping the design flexibility of an adiabatic absorber. The absorber consists of a pin-finned plate coupled with a droplet flow regime (see Figure 1), which is shown to return a better absorption mass flux than a film flow regime, according to Ben Hafsia (2015).
This study deals with the shape analysis of the plate structure. In particular, shape of the pins is investigated from the point of view of droplet formation phenomena, in order to find the configuration that increases the contact area of the solution the most, by reducing the droplet size and by preventing droplet coalescence on the pins. In Figure 1 a summary of the cases investigated is given.

2. Theory and modelling

To ensure that the droplet flow regime is maintained, droplet formation at the pins bottom is studied. Specifically, it needs to be ensured that droplets are not merging; this translates to the requirement that the droplet formed at the pins be smaller than the one calculated using equation 1, where d is the diameter of the orifices generating the droplet flow, $d_m$ the droplet diameter.

$$d_m/d = (3\pi/\sqrt{2})^{1/3} (1 + 30h)^{1/6}$$

An analytical model is used, based on the variational approach previously employed by Pitts (1974) and Babu (1987). The model considers the balance between the surface tension force, the potential gravitational energy and the interfacial energy of the droplet (Figure 2). The droplet profile can then be calculated by finding the minimum of the droplet energy at constant volume. This translates into the Euler-Lagrange equation, which can be reformulated into the following system of differential equations:

$$dX/dS = \cos \phi$$

$$dZ/dS = \sin \phi$$

$$d\theta/dS + \sin \phi/X = 2/B - Z$$

Where X and Z are the geometrical coordinates, S is the arc length and B the curvature radius at the droplet axes, after non-dimensionalization by multiplication of the respective dimensional variables by the factor C:

$$C = \sqrt{(\rho_d - \rho_L)g/\sigma}$$

Where $\rho_d$ and $\rho_L$ are the droplet and the surrounding fluid density respectively, and $\sigma$ is the surface tension. The system of equations is then solved using the following boundary conditions and assuming fully wetted condition of the pins.
\[ X = 0 \] at \( Z = 0 \) \hspace{1cm} (5)

\[ \sin \theta / X = 1 / B \] at \( Z = 0 \) \hspace{1cm} (6)

\[ dX/dZ = \cot(90 - \beta) \] at \( Z = Z_0 \) \hspace{1cm} (7)

Here \( \beta \) is the angle between the vertical axis of symmetry and the tangent to the pin at the point of contact between the droplet and the pin. For the circular and elliptical cases, this angle varies depending on the point of contact, while for the rectangular and rhomboidal cases this value is constant.

The fourth order Runge-Kutta method is used in Matlab to solve the system iteratively for different values of the curvature radius \( B \). Moreover, for the circular and elliptical cases the angle \( \beta \) is varied as well. The equations are integrated until \( X = X_0 \), where \( X_0 \) is taken to be half the pin width for the rhomboidal and rectangular cases, while for the circular and elliptical cases it’s varied according to the value of \( \beta \) used for each iteration.

The droplet volume is calculated as volume of revolution, solving the following equation along with equations 2-3:

\[ \frac{dV}{dS} = \pi X^2 \sin \theta \] \hspace{1cm} (8)

Then, the volume occupied by the pin is subtracted from the total. This is calculated by multiplying the frontal area of the pin by \( X_0 \), keeping the assumption of rotational symmetry. Finally, the maximum volume calculated among the different values of \( B \) and \( \beta \) (where applicable), is taken for the verification of the droplet merging risk.

3. Experimental setup and procedure

Experiments have been carried out to test the validity of the analytical model previously described. Water droplets are manually generated with a syringe with a 2mm orifice diameter. A high speed camera (Photron SA5) and an Argon light source are used to record the water droplet formation at the pins bottom. A 768x768 resolution is used, with at least 2000 fps to capture the droplet dynamics. Captured images are processed with imageJ and coordinates of the droplet profile are exported for each test case. Of these coordinates, few points (those closer to the bottom of the droplet) are used in another model (Matlab) to calculate the corresponding experimental radius of curvature. If three points only are used, the radius is univocally calculated. If more than three points are used, a least square method is used to find the best fit circle that can approximate the data used. The calculated experimental radius is then exported back in the system of ODE described in Section 2 and used as input. With this value the model returns the theoretical droplet profile having the same radius of curvature of the experimental one, making the comparison and validation possible.

4. Results

The results are now presented. The calculation of the experimental radius of curvature is influenced by the
amount of experimental points chosen for the calculation. To assess the actual effect of changing the number of points, a sensitivity analysis has been performed. Five different ranges have been chosen; their length is determined by the “x” coordinate of the profile, set at five different values from 0.2mm to 1mm. All points falling in each range are then considered for the calculation. Each of these iteration returns a radius of curvature, which is then used to run the model described in Section 2.

The case of the circular pin is presented in Figure 3. For each pin three images are chosen, to represent three stages of the droplet formation, before it detaches from the pin. In the first row of figures, the curves of the sensitivity analysis are presented and compared with the experimental curve (black dashed line). Then, below each case, the experimental droplet shape is shown and compared with the theoretical profile (red dashed line in the third row of Figure 3).
The profiles match the experimental data with higher accuracy close to the droplet bottom, i.e. close to the origin. Farther from the origin instead, different ranges may result in considerably different profiles as it can be observed for example in the “Circular-end” case “c” of Figure 3. Nevertheless, analysis done by the authors on different pin shapes (data not reported here) shows that in many instances the analytical profiles tend to be less affected by the choice of the range length, behaving then in a similar way, as it can be observed for the “Circular-mid” case “b”.

Beside the level of accuracy in the experimental profile curve extraction, observation of the overlapped profiles shows that the main cause of the discrepancy between the analytical and experimental curves can be explained by the finite dimension of the pins. The analytical model assumes droplets forming on an infinite surface, or a surface large enough to be approximated as infinite. Experiments instead show that this condition is not always met. In cases such as the circular pin of Figure 3, the droplet surrounds completely the pin, and this creates a distortion in the profile that the current model cannot compensate for. Table 1 summarizes the results of the sensitivity analysis by reporting the error of the various curves with respect to the experimental data. The error is calculated on the X variable, for same values of the Z variable, and finally the average error is reported.

Table 1: Average errors between the experimental curves and the best fit curves derived from the sensitivity analysis

<table>
<thead>
<tr>
<th>Pin type</th>
<th>Pin</th>
<th>Error (%)</th>
<th>Range</th>
<th>Phase</th>
<th>Pin type</th>
<th>Phase</th>
<th>Error (%)</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular</td>
<td>Start</td>
<td>14</td>
<td>“1 mm”</td>
<td>Rhomb 60°</td>
<td>Start</td>
<td>16.3</td>
<td>“0.2 mm”</td>
<td></td>
</tr>
<tr>
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<td>Middle</td>
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<td>“1 mm”</td>
<td>Rhomb 60°</td>
<td>Middle</td>
<td>14.5</td>
<td>“0.2 mm”</td>
<td></td>
</tr>
<tr>
<td>Circular</td>
<td>End</td>
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<td>“0.6 mm”</td>
<td>Rhomb 60°</td>
<td>End</td>
<td>28.7</td>
<td>“1 mm”</td>
<td></td>
</tr>
<tr>
<td>Elliptic -1x1.25</td>
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<td>6.4</td>
<td>“0.4 mm”</td>
<td>Rhomb 90°</td>
<td>Start</td>
<td>4.6</td>
<td>“0.4 mm”</td>
<td></td>
</tr>
<tr>
<td>Elliptic -1x1.25</td>
<td>Middle</td>
<td>31.4</td>
<td>“0.2 mm”</td>
<td>Rhomb 90°</td>
<td>Middle</td>
<td>28.2</td>
<td>“0.8 mm”</td>
<td></td>
</tr>
<tr>
<td>Elliptic -1x1.25</td>
<td>End</td>
<td>26.5</td>
<td>“0.4 mm”</td>
<td>Rhomb 90°</td>
<td>End</td>
<td>27.4</td>
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</tr>
<tr>
<td>Elliptic -1x1.5</td>
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<td>“0.2 mm”</td>
<td>Rhomb 120°</td>
<td>Start</td>
<td>17.3</td>
<td>“0.8 mm”</td>
<td></td>
</tr>
<tr>
<td>Elliptic -1x1.5</td>
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<td>28.9</td>
<td>“0.8 mm”</td>
<td>Rhomb 120°</td>
<td>Middle</td>
<td>18.9</td>
<td>“1 mm”</td>
<td></td>
</tr>
<tr>
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<td>24.3</td>
<td>“1 mm”</td>
<td>Rhomb 120°</td>
<td>End</td>
<td>10.4</td>
<td>“0.6 mm”</td>
<td></td>
</tr>
<tr>
<td>Rectangular</td>
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<td>8.1</td>
<td>“0.2 mm”</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rectangular</td>
<td>Middle</td>
<td>8.7</td>
<td>“0.6 mm”</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Rectangular</td>
<td>End</td>
<td>15.7</td>
<td>“0.4 mm”</td>
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</tbody>
</table>

The table shows how the errors fall between 4% and 35%, the rectangular and the 120° rhomboidal case returning, on average, a smaller error and with a smaller variation among the phases investigated. The fact that these two cases refer to pins with a flat and quasi-flat bottom surface might explain the similar behaviour. The sharp angle at the pins sides might also be the cause for the higher accuracy measured. As previously explained, droplet distortion caused by the presence of water in the upper half of the pins is one of the main cause of the low accuracy of the model, far from the droplet base. In the case of the rectangular and 120° rhomboidal pins, these sharp angles might create a good separation between the upper and lower part of the pin, reducing the distortion effect during droplet formation.

4.1 Design criteria for the LiBr-H2O absorber

After validating the models for the distribution and the pin-finned plate, the model was solved iteratively not for a specific radius of curvature B, but for a range of values, using the LiBr-H2O solution as working fluid. In this way it is possible to explore all the possible droplet profiles obtainable and calculate the corresponding volume. Once the different volumes are calculated, the diameter of largest droplet calculated is used for comparison. Figure 4 shows the result of the comparison. The 120° rhomboidal pin is the one that returned the smallest droplet volume. This results in the largest surface area per unit of fluid and thus the most efficient absorption. Following this, the two elliptical pins returned a similar performance, while the other two rhomboidal, rectangular and circular pins all returned a higher volume. A 2 mm orifice could ensure the maintenance of the droplet flow regime of the LiBr solution. In particular, a 1 mm orifice is the best choice for the chosen 120° rhomboidal pin, as it can produce smaller droplets without the risk of coalescence. The other pins instead could be theoretically able to collect a larger volume and thus create coalescence. Further experiments with different pin sizes would be needed to further confirm the model validity for different designs.
The presented study showed how the variational approach is capable of predicting the droplet size forming at the pins with good accuracy in the region close to the droplet bottom, and when the assumption of droplet forming on an infinite or quasi-infinite surface is verified during the experiment. This condition is verified when the fluid is able to collect entirely in the bottom part of the pin. However, when residual fluid is present in the upper part of the circular pin, it caused a distortion in the profile and hindered the model accuracy. Further study will be required to evaluate the effect of these distortions and thus refine the model. In this study, the 120° rhomboidal and the rectangular pins were the most effective in isolating the bottom part and thus reduce the distortion effect of fluid on the top side; this resulted in a better accuracy of the model and a higher accuracy consistency between the different phases investigated. Looking at the droplet size instead, the 120° rhomboidal case pin returned the lowest maximum droplet size and is the best option for the plate absorber manufacturing. Coupled with a 1 mm orifice, for the droplet flow generation, this geometry can ensure a flow of small droplets of diameter smaller than 2 mm, with no risk of droplet merging. The reduced droplet size increases the contact surface area and should thus augment the absorption rate of the absorber. Further study will focus on testing the heat and mass transfer performance of this design.

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**References**


