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<td><strong>Author(s)</strong></td>
<td>Faust, Mathias; Chang, Chip Hong</td>
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Optimization of structural adders in fixed coefficient transposed direct form FIR filters

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Abstract—Over the last two decades, fixed coefficient FIR filters were generally optimized by minimizing the number of adders required to implement the multiplier block in the transposed direct form filter structure. In this paper, an optimization method for the structural adders in the transposed tapped delay line is proposed. Although additional registers are required, an optimal trade-off can be made such that the overall combinational logic is reduced. For a majority of taps, the delay through the structural adder is shortened except for the last tap. The one full adder delay increase for the last optimized tap is tolerable as it does not fall in the critical path in most cases. The criterion for which area reduction is possible is analytically derived and an area reduction of up to 4.5% for the structural adder block of three benchmark filters is estimated theoretically. The saving is more prominent as the number of taps grows. Actual synthesis results obtained by using the Full Adder (FA) count as the cost metric [6], [7]. The structural adder (SA), \(a_i\), \((i = N-2, N-3, \ldots, 0)\) is defined as the adder that adds the output of the register, \(p_i\) to the output of the coefficient multiplier, \(q_i\) in the tapped delay line. The structural adders have been omitted by most MCM algorithms, as it is not possible to share these adders. In general, the operands to the structural adders are binary numbers. This leads to a higher fanout for the Most Significant Bit (MSB) of the smaller input. For positive coefficients, the sign of the coefficient multiplier output follows the sign of the input value. As the bit width of the coefficient multiplier, \(q_i\), is always smaller than that of the partial sum, \(p_i\), a high load is imposed on the sign bit of the input.

In [8], a technique called MSB-Fix is used to avoid sign extension. It uses constant correction vectors as in the partial product addition of two’s complement multiplication. These constant correction vectors are grouped together and lumped at the last tap. Due to the delay of the partial sums in the TDF FIR filters, the first \(N-1\) outputs are incorrect as the cumulative error has to propagate to the last tap. In [8], the FIR filter in the all-digital modem has only \(N = 20\) taps. This system needs more than 20 clock cycles to start up, so the first 19 incorrect outputs are not critical. For generic FIR filters, in particular high order filters with large \(N\), the flushing of \(N-1\) incorrect output samples may not be acceptable. Therefore, an alternative approach to simplifying the structural adders is necessary.

For fixed coefficient FIR filters, the bit widths of the input and all coefficients are known. This enables the bit width of the coefficient multiplier to be determined from its dynamic range. As the partial sums are delayed before they are added with the coefficient multiplier outputs in the structural adders, the bit widths of the structural adders increase monotonically from the first structural adder towards the output. Careful analysis revealed that for most filters, the bit width of the adder increases only from coefficient \(N-1\) to about \(N/2\), after which the bit width stays relatively constant and increases by no more...
than two bits. As the bit width of the coefficient multiplier output reduces towards the last tap, longer sign extension is required for these structural adders. This paper proposes an addition scheme to reduce the bit widths of these structural adders so that the total combinational logic is reduced at the expense of some register overhead. To determine if the area reduction is able to offset the overheads of additional adders and registers, a lower bound for the difference between the adder bit width and the coefficient multiplier output bit width is established analytically.

II. PROPOSED STRUCTURAL ADDER OPTIMIZATION

As the proposed method optimizes the trade-off between the structural adders and registers, a ratio of these two component areas is required. A structural adder adds two two’s complement numbers using Carry Propagate Adder (CPA). The simplest CPA is a Ripple Carry Adder (RCA). Faster adders like carry look-ahead or carry select adder require more area than a RCA. For simplicity, the complexity of structural adder is assessed based on the number of FAs required to implement a RCA. A delay element is used to hold the output of the structural adder for one clock cycle and is typically implemented by delay Flip-Flop (FF). The authors of [9] suggest a ratio of $1:0.6 \sim 0.8$ for FA:FF. The TSMC 0.18$\mu$m standard cell library [10], which is used for our simulation, contains a variety of FFs with areas measured from 53.2 to 86.5$\mu$m$^2$ and a standard FA cell has an area of 69.9$\mu$m$^2$. These documented numbers and the simulation results showed an average area of 69.2$\mu$m$^2$ for FF and FA in typical FIR filters synthesized with this standard cell library. Therefore a FA:FF ratio of $1:1$ ($\rho = 1$) is adopted in this paper.

![Fig. 2. Bit width analysis for tapped-delay line signals of Filter 1 of [11].](image)

An area of several successive structural adders can be optimized by exploring the aggregate difference in the bit widths between the delayed signals and the outputs of the coefficient multipliers. Fig. 2 shows the analysis of Filter Example 1 of [11]. The bit width of the partial sum is the minimal bit width based on the conventional range estimation. It remains constant after $c_{60}$ whereas the coefficient multiplier output’s bit width reduces towards both ends. As the difference between the bit widths of the coefficient multiplier output and the partial sum increases towards the output tap, longer sign extension is required for these structural adders. Six or more bits are required for the sign extension for the structural adders, $a_{52}$ to $a_{60}$. The other two bit widths marked as reduced structural adder and lower bisection part in the legend are obtained from our proposed structural adder reduction scheme, which will be explained in what follows.

![Fig. 3. Example of adder size reduction for decimal number accumulation.](image)

The fundamental concept of our proposed method can be illustrated by an example in decimal. Let $\{610, -274, 2, 258\}$ be a set of coefficient multiplier outputs to be accumulated to a large partial sum 1234567 by the structural adders in a tapped delay line. A downwright approach is to add one number at a time from the set of smaller integers to the large integer. Alternatively, the integers in the set are summed and then added to the large integer. The latter accumulation scheme, when implemented in hardware, requires the large integer and the smaller integers to be stored at each tap. This incurs a large register overhead, which can be reduced if the large number is split into two smaller integers as shown in Fig. 3.

![Fig. 4. Binary example on the optimization of last three adders of Fig. 2.](image)

Fig. 4 shows the binary implementation of the proposed scheme on the last three coefficients of the filter example from Fig. 2. The reduction of adder lengths is observed to be several times more than the register and adder overheads it incurred. Furthermore, the delays through the structural adders, $a_2$ and $a_1$ have been reduced, while the delay through $a_0$ is increased by one FA delay. The slight increase in the delay through $a_0$ is not an issue as in most cases, there exists at least one tap ($i > 0$) for which delay($x \cdot c_i$) < delay($x \cdot c_0$). The FAs reduction for the structural adders can be offset by the increase in FF overhead. Therefore, information about the minimal difference between the addends of the structural adders is of interest.
With reference to Fig. 1, \( p_i \) and \( q_i \) are the inputs to the \( i \)th structural adder, \( a_i \), where \( q_i \) is the \( i \)th coefficient multiplier’s output and \( a_0 \) is the last adder that produces the output \( y \). Let \( n(z) \) be the bit width of an arbitrary signal \( z \) and let \([\bar{v}^-, \bar{v}^+]\) be the dynamic range of \( q \). The selected partial sum, \( p_i \), is bisected into two halves, i.e., \( p_i = p_i^+ \) \( p_i^- \) at bit position, \( h_i \) such that \( n(p_i^+) = h_i \) and \( n(p_i^-) = n(p_i) - h_i \). If \( p_i^u \) is added, after \( m \) delays, to the sign extended \( p_i^{u,m+1} \) where \( n(p_i^{u,m+1}) > h_i \), then the original structural adders, \( a_{i,j} \), for \( j = 1, 2, \ldots, m \). The minimal bit width \( n(a_{i,j}) \) of each reduced adder can be obtained by the following range estimation:

\[
n(a_{i,j}) = \left\lfloor \log_2 \left( \max \left( 2^{h_i} - 1 + \sum_{k=0}^{j-1} (v^+_{i-k}) - \sum_{k=0}^{j-1} (v^-_{i-k}) \right) \right) \right\rfloor + 1
\]

(1)

In (1), the value, \( 2^{h_i} - 1 \) is the maximum possible value of the lower bisected partial sum, \( p_i^- \). Since the first term provides the bit width of the magnitude from the dynamic range, the constant ‘1’ is added to account for the sign bit. As \( j \) increases, the values of \( n(a_{i,j}) \) grow monotonically as \( v^- \leq 0 \) and \( v^+ \geq 0 \). The total number of FAs saved by the \( m \) successive reduced structural adders starting from the \( i \)th tap is given as follows:

\[
\Delta_{i,m} = \sum_{k=1}^{m-1} \left[ (n(p_i) - n(a_{i,k}) - \rho) - (n(a_{i,m}) - h_i) \right] - (n(a_{i,m}) - h_i)
\]

(2)

where \( \rho \) is the FA:FF ratio. In (2), the term \( n(p_i) - n(a_{i,k}) \) denotes the adder cost reduction and the term \( n(a_{i,m}) - h_i \) represents the register overhead. The last term, \( n(a_{i,m}) - h_i \) refers to the overlapping bits for the addition of \( p_i^u \). As \( n(a_{i,m}) > h_i, \forall c \neq 0 \), it is apparent from (2) that there is no saving if \( m = 1 \).

Hypothetically, \( h_i \) can be any arbitrary value smaller than \( n(p_i) \). However, as \( n(a_{i,j}) \) is dependent on \( v^+ \) and \( v^- \), it can be optimally deduced. If \( h_i = \max \{ \left[ \log_2(v^+_{i-k}) \right] ; \left[ \log_2(-v^-_{i-k}) \right] \} \), the FA saving in the structural adders is reduced. If it is chosen to be smaller, the register overhead and the FA cost of the final addition are higher. As \( n(q_i) \) is not monotonically decreasing with \( i \), \( h_i \) needs to be selected by looking ahead into several coefficients. It was empirically determined that a look ahead of 5 coefficients is sufficient. This leads to

\[
h_i = \max \{ \left[ \log_2(v^+_{i-k}) \right] ; \left[ \log_2(-v^-_{i-k}) \right] \}, \forall k = i, i-1, \ldots, i-5
\]

(3)

Combining (1) and (3) it can be derived that \( n(a_{i,j}) \geq h_i + 2, \forall j > 0 \). Defining \( \delta_i = n(p_i) - h_i \), from (2) we have

\[
\Delta_{i,j} \leq (m-1) \cdot (\delta_i - 2 - 2\rho) - 2
\]

(4)

In order to not increase the area, the following lower bound can be derived from (4).

\[
\delta_i \geq 2 \cdot \frac{m(1 + \rho) - \rho}{m - 1}
\]

(5)

If \( m \to \infty, \delta_i \geq 2(1 + \rho) \). Usually, \( m \) is small and for TSMC 0.18um standard cell library, \( \rho \approx 1, \delta_i \geq 2(2m-1) \). For \( m = 2, \delta_i \geq 6 \) and for \( m = 3, \delta_i \geq 5 \).

The above formulae hold only if no zero valued coefficient is involved. Since no structural adder is required to add a zero coefficient, no saving can be gained from the structural adder but the overhead due to the overlapping bits remains. Experiment results showed that in many cases it is not beneficial to span the bisected partial sum addition process over zero coefficients. It was also observed that \( n(a_{i,j}) = h_i + 2 \) is valid only for \( j \leq 3 \). This observation suggests the use of \( n(p_i) - n(q_i) \geq 6 \) to select the partial sum for bisection. In Fig. 2, it can be seen that the first bisection is made at the partial sum of the tap where the partial sums of the tap and all successive taps have \( n(p_i) - n(q_i) \geq 6 \). \( h_i \) is always one bit below \( n(q_i) \), as the sign bit is not included in the range \([\bar{v}^-, \bar{v}^+]\) calculated in (3).

From (1), \( n(a_{i,j}) \) grows monotonically with increasing \( j \). This implies that the FF overhead is increasing as the FA reduction in the structural is declining. The gain diminishes with increasing \( m \) and can become negative. Furthermore, as \( n(q_i) \) decreases towards \( c_0 \), a smaller \( h_i \) can be chosen for better gain. This suggests that it is beneficial to make repeated bisections. The bisection can be repeated in two ways. If only area reduction is of interest, it is more optimal to merge \( p_i^u \) at a tap and then bisect the partial sum again on the next tap. This will increase the path delay at each merging structural adder. To avoid the delay penalty, the merging and bisecting can be performed simultaneously at the same tap. For this concurrent merge/bisect structural adder \( a_i \), the operand, \( p_i^u \) from the previous bisection is bisected again at \( h_i \) such that \( p_i^{u,2} = p_i^{u,1} + p_i^{h_i} \). \( p_i^{h_i} \) is sign extended and added to \( p_i^{u,1} \) while \( p_i^{h_i} \) is added to \( q_i \). The register overhead will be increased by two FAs, but since the additions are independent, the delay through the structural adder is also reduced as the operands are shortened due to the simultaneous merging and bisection. Experiment results showed that the delay reduction is more prominent than the cost of two FAs. Therefore, simultaneous merging and bisection is adopted. For the last tap, one FA delay increment due to the final merging is inevitable as shown in Fig. 4. In most cases, this is not a problem as \( c_0 \) does not fall in the critical path. In case the minimal delay is affected, the final merging could be moved to an earlier tap.

For the most optimal trade-off, it is suggested that (2) is calculated for every \( i > 2 \) for which \( n(p_i) - n(q_i) \geq 6 \) is valid. Every \( \{i, j\} \) pair will be checked to determine if a repeated bisection would lead to a better result than the best result obtained from the previous runs. This exhaustive search finds the most optimal cost reduction. The complexity of this search is \( O(n^2) \). On an Intel P4, 3.0 GHz PC with 2 GB RAM, the runtime is below 10 seconds for filters with \( N > 1000 \).

<table>
<thead>
<tr>
<th>Filter</th>
<th>( \omega_P ) [π]</th>
<th>( \omega_S ) [π]</th>
<th>( A_P ) [dB]</th>
<th>( A_S ) [dB]</th>
<th>Tap #</th>
<th>Word Length</th>
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<tr>
<td>A</td>
<td>0.042</td>
<td>0.14</td>
<td>0.2</td>
<td>-60</td>
<td>60</td>
<td>13, 8, 25</td>
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<tr>
<td>B</td>
<td>0.8</td>
<td>0.74</td>
<td>0.1</td>
<td>-80</td>
<td>121</td>
<td>14, 8, 25</td>
</tr>
<tr>
<td>C</td>
<td>0.2</td>
<td>0.2125</td>
<td>0.05</td>
<td>-50</td>
<td>441</td>
<td>14, 8, 26</td>
</tr>
</tbody>
</table>

TABLE I

Test Filter Data
TABLE II

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III. IMPLEMENTATION RESULTS

The proposed method was evaluated using Example 2
from [12] (Filter A), Example 1 from [11] (Filter B) and
Filter 3 from [5] (Filter C). The details are given in Table II.
All examples used signed digit to optimize the coefficients.
The estimated FA reduction of each filter is shown in the
second column of Table II. A CSD implementation and a
non-pipelined NRSCSE [3] optimized implementation of the
MCM block are used to demonstrate the effectiveness of our
structural adder optimization. The proposed structural adder
optimization scheme was combined with the optimized MCM
block and all designs were coded in VHDL and synthesized
by Synopsys Design Compiler (Ver. 2007.12) using TSMC
0.18µm standard cell library. The designs were optimized
under two different delay constraints, unconstrained (∞) and
fixed constraint (3ns). The synthesized area and the percentage
area reduction over the baseline CSD implementation were
shown in Table II. The compilation time was reported by
Design Compiler on Red Hat Linux (Ver. 4.1.2-13) running
on a Core2 2.66GHz workstation with 3.3GB RAM.

The area reduction over CSD implementation by [3] is small
due to the Boolean optimization made by Synopsys Design
Compiler. The combination of the proposed scheme and [3]
reduced the area by 2.96% to 13.13% and after discounting the
saving due to the MCM block optimization of [3], the actual
synthesis results still exceed the expected saving of 2.03%
2.63% and 4.58% from theoretical estimation for these filters.
The discrepancy between the estimated and simulation results
can be explained as follows. The proposed optimization trades
combinational area against registers and reduces the path de-
days in several taps. The reduced combinational logic requires
less area and the area reduction is more prominent under
tighter delay constraints. Moreover, the reduced path delays
create more slacks, which enable slower but smaller cells to
be used. As filter length increases, the area for the structural
adders dominates, up to 96% for Filter C. The time taken by
the synthesis tool to optimize the architecture generated by our
proposed method has also reduced significantly. The time for
unconstrained case was reduced by 7.6%, 11.5% and 29.1%
for Filters A, B and C, respectively and 23.2%, 26.6% and
45.2% for the case of fixed delay constrained optimization.

IV. CONCLUSION

This paper presents a new method to reduce the total area
of fixed coefficient transposed direct form FIR with a large
number of taps by minimizing the bit widths of the structural
adders. Sign extensions have been shortened and the delays
through the structural adders have been reduced at the expense
of some register overhead and a reduced size merged adder
for each bisection of a long partial sum. The condition for
which area reduction is possible and the optimal bisection
were analytically formulated. Theoretical estimate shows an
area reduction of up to 4.5% for the structural adder block
of the benchmark filters. The proposed method was combined
with a MCM optimization algorithm. An area reduction of
up to 13.13% (up to 1.33% by MCM optimization and up
to 11.8% by the proposed method) was obtained from the
synthesis result of long filter.

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