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Tuning the dispersion of effective surface plasmon polaritons with multilayer systems

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Abstract: Recently, effective surface plasmon polaritons (ESPPs) induced by structural dispersion in bounded waveguides were theoretically demonstrated and experimentally verified. Despite the theoretical and experimental efforts, whether ESPPs can mimic real SPPs in every aspect still remains an open question. In this work, we go one step further to study the hybridization of ESPPs in multilayer systems. We consider transverse electric (TE) modes in a conventional rectangular waveguide and a parallel-plate waveguide (PPW) and derive analytically the dispersion relations and asymptotic frequencies of the corresponding ESPPs modes in sandwiched structures consisting of alternating dielectrics of different permittivities. Our results show that the ESPPs can be categorized into odd and even parities (owing to the ‘plasmon’ hybridization) in a similar way as natural SPPs supported by the insulator/metal/insulator (IMI) and metal/insulator/metal (MIM) heterostructures in the optical regime. The similarities and differences between ESPPs and their optical counterparts are also discussed in details, which may provide valuable guidance for future application of ESPPs at the microwave and terahertz frequencies.

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References and links

1. Introduction

Plasmonics offers a possible route to confine and manipulate electromagnetic waves at subwavelength scales [1–9]. As a result, light-matter interactions at metal-dielectric interfaces or in small metallic nanostructures can lead to huge near-field enhancement in the optical regime. Two ingredients of plasmonics, surface plasmon polaritons (SPPs) and localized surface plasmons (LSPs) have been developed throughout the 20th century, holding great promise for applications in miniaturization of photonic circuits, near-field optics, surface-enhanced spectroscopy, plasmonic antennas and photovoltaics, etc [1]. However, SPPs and LSPs only occur at metal/dielectric interface at optical frequencies and large dissipative losses impede the development of metal-based plasmonic devices at optical frequencies. In 2004, the concept of spoof surface plasmons at low frequencies (far infrared, terahertz or microwave) was proposed by Pendry to mimic real SPPs by engineering metal surfaces with periodic subwavelength grooves or holes [10]. Later on, in 2012, Garcia-Vidal et al. demonstrate that spoof localized surface plasmons (spoof-LSPs) can be supported by periodically textured perfect electric conducting particles [11]. A series of theoretical and experimental efforts have been made to realize low frequency plasmonics as well as its applications ever since [12–38]. However, all these work were in the same framework of spoof surface plasmons. In 2016, Engheta et al. provided another route to realize a variety of plasmonic phenomena by exploiting structural dispersion of TE modes in bounded waveguides filled with materials of positive permittivity only [39]. Then, Li et al. gave a theoretical insight and experimental verification of this structural dispersion induced ESPPs in a rectangular waveguide and substrate integrated waveguide [40]. Later on, the transmission of ESPPs at a single interface in a parallel-plate waveguide was experimentally verified by Prudencio and associates [41].

Despite the theoretical and experimental attempts [39–41], whether ESPPs can mimic real SPPs in every aspect still remains an open question. So in this work, we go one step further to study the hybridization of ESPPs in multilayer systems. Specifically, we investigate the dispersion and transmission characteristics of ESPPs in multilayer systems in a conventional rectangular waveguide and a parallel-plate waveguide by engineering the TE modes. The dispersion relations of the odd and even ESPPs modes in two sandwiched structures consisting of alternating dielectrics of different relative permittivities are studied in detail. We find that these two sandwiched structures resemble nearly the same characteristics of the IMI and MIM heterostructures in the optical regime. Long-ranging ESPPs can be realized by decreasing the thickness of the air layer of the effective IMI structure. And negative group velocity can also be achieved by tuning the geometrical dimensions of each layer both for the effective IMI and MIM structures. In addition, the odd and even ELSPs modes in effective IMI/MIM structures are found to perfectly emulate the ultra-subwavelength confinement of SPPs in real IMI/MIM systems, which cannot be easily realized by conventional spoof LSPs [11,18,19,26] that a periodic array of grooves is needed with a depth on the order of one quarter of wavelength.

2. Theory

First, we consider a conventional rectangular waveguide with the cross section dimensions \(a\times b\) in Fig. 1, which is filled with three layer isotropic and homogeneous dielectrics. For simplicity, it is assumed in this work that the dielectrics in Layer II \((-b/2<y<-s)\) and Layer III \((s<y<b/2)\) are the same \(\varepsilon_2=\varepsilon_3\) and \(\mu_2=\mu_3=1\) and we only focus on two specific cases when \(\varepsilon_2=\varepsilon_3>\varepsilon_1\) and \(\varepsilon_2=\varepsilon_3<\varepsilon_1\), which are two triple-layer systems acting as effective IMI and effective MIM structures. As we know [40], the effective relative permittivity
\( \varepsilon_e = \varepsilon_r - (m/2a)^2 \lambda_0^2 \) (\( \lambda_0 \) is the operating wavelength) of the dielectric filling in the rectangular waveguide corresponding to \( TE_{ml0} \) mode can be tuned to be either positive, zero or negative by changing the operating frequency, lateral dimension of the waveguide and filling materials [39]. Then, the effective wave number \( k_e \) would be \( k_e = k_0 \sqrt{\varepsilon_e} \). Thus, it is anticipated that effective surface confined modes can be supported at Interface I (interface between Layer I and Layer II) and Interface II (interface between Layer I and Layer III) when \( \text{Re}(\varepsilon_{e1}) \cdot \text{Re}(\varepsilon_{e3}) < 0 \) and \( \text{Re}(\varepsilon_{e2}) \cdot \text{Re}(\varepsilon_{e3}) < 0 \), in which \( \varepsilon_{ei} = \varepsilon_r - (m/2a)^2 \lambda_0^2, i = 1, 2, 3 \).

In order to derive the dispersion relations of the coupled ESPPs modes, we start by setting the magnetic vector potential for the effective coupled modes in Layer III in the form of

\[
\vec{A}_3 = y \psi_3, \quad \psi_3 = e^{-j\beta z} \left( B_3 e^{k_{3y}y} + C_3 e^{-k_{3y}y} \right)
\]

with no variations in the x direction, in which \( B_3 \) and \( C_3 \) are the amplitudes of the decaying fields from Interface II to Layer III and the reflected fields bouncing back from the upper wall. \( k_{3y} = \beta^2 - k_{30}^2 \) is the wave number in the y direction with \( k_{30} = k_0 \sqrt{\varepsilon_{e3}} \) being the wave number in the effective dielectric of Layer III. Thus, from Maxwell's equations we can obtain all components of the electric and magnetic fields in Layer III as follows.

\[
H_x^3 = j \frac{\beta}{\mu_0} e^{-j\beta z} \left( B_3 e^{k_{3y}y} + C_3 e^{-k_{3y}y} \right)
\]

\[
E_y^3 = -j \frac{1}{\omega \mu_0 \varepsilon_0 \varepsilon_{e3}} \left( k_3^2 + k_{3y}^2 \right) e^{-j\beta z} \left( B_3 e^{k_{3y}y} + C_3 e^{-k_{3y}y} \right)
\]

\[
E_z^3 = -\frac{\beta k_{3y}}{\omega \mu_0 \varepsilon_0 \varepsilon_{e3}} e^{-j\beta z} \left( -B_3 e^{k_{3y}y} + C_3 e^{-k_{3y}y} \right)
\]

Similarly, we set the magnetic vector potentials in Layers I and II in the form of

\[
\vec{A}_i = y \psi_i, \quad \psi_i = e^{-j\beta z} \left( B_i e^{k_{iy}y} + C_i e^{-k_{iy}y} \right), \quad i = 1, 2,
\]

in which \( B_i \) and \( C_i \) are the amplitudes of
the decaying fields from **Interface II** to **Interface I** and reflective fields from **Interface I** to **Interface II**, $B_2$ and $C_2$ are the amplitudes of the decaying fields from **Interface I** to **Layer II** and the reflected fields from the lower wall. $k_i^2 = \beta^2 - k_w^2$, $i = 1, 2$ are the wave numbers in the $y$-direction with $k_w = k_n \sqrt{\varepsilon_n}$ being the wave numbers in the effective dielectrics of **Layers I and II**. So, the electric and magnetic fields in **Layers I and II** are derived as follows.

\[
H_i^i = \frac{j \beta}{\mu_0} e^{i \beta \gamma} \left( B_i e^{k_i y} + C_i e^{-k_i y} \right)
\]
\[
E_i^i = -j \frac{1}{\omega \mu_0 \varepsilon_0 e_i} \left( k_i^2 + k_w^2 \right) e^{-i \beta \gamma} \left( B_i e^{k_i y} + C_i e^{-k_i y} \right)
\]
\[
E_i^i = -\frac{\beta k_w}{\omega \mu_0 \varepsilon_0 e_i} e^{-i \beta \gamma} \left( B_i e^{k_i y} - C_i e^{-k_i y} \right), \quad i = 1, 2
\]

By matching the following boundary conditions

\[
y = s, H_s^{III} = H_s^{I}; H_s^{III} = H_s^{I}; E_s^{III} = E_s^{I}
\]
\[
y = -s, H_s^{III} = H_s^{I}; H_s^{III} = H_s^{I}; E_s^{III} = E_s^{I}
\]
\[
y = b / 2, E_s^{III} = 0; y = -b / 2, E_s^{III} = 0
\]
\[
y = s, \varepsilon_s E_s^{III} = \varepsilon_s E_s^{I}, y = -s, \varepsilon_s E_s^{III} = \varepsilon_s E_s^{I}
\]

we can get the dispersion relation for possible surface confined modes as

\[
e^{-4k_i y} = \left( \frac{k_{i1} + \frac{k_{2w}}{k_{i2}}} {\varepsilon_{i1}} \frac{k_{i1} + \frac{k_{3w}}{k_{i3}}} {\varepsilon_{i3}} \tan h \left( k_{2w}, t \right) \right) \left( \frac{k_{i1} - \frac{k_{2w}}{k_{i2}}} {\varepsilon_{i1}} \frac{k_{i1} - \frac{k_{3w}}{k_{i3}}} {\varepsilon_{i3}} \tan h \left( k_{2w}, t \right) \right)
\]

(1)

with $k_i^2 = \beta^2 - k_w^2 + \left( \frac{m \pi}{a} \right)^2$, $i = 1, 2, 3; m = 1, 2, 3, \ldots$

As stated above, **Layer II** and **Layer III** has the same permittivity $\varepsilon_{s1} = \varepsilon_{s3} (\varepsilon_{s2} = \varepsilon_{s3})$ and thickness, and thus $k_{s2} = k_{s3}$. In this case, the dispersion relation (1) can be split into a pair of equations, namely

\[
\tan h \left( k_{s1}, s \right) = -\frac{k_{2s} \varepsilon_{s1}} {k_{s2} \varepsilon_{s2}} \tan h \left( k_{2s}, t \right),
\]

(2)

\[
\tan h \left( k_{s1}, s \right) = -\frac{k_{1s} \varepsilon_{s2}} {k_{2s} \varepsilon_{s1}} \coth \left( k_{2s}, t \right).
\]

(3)

It can be observed that Eq. (2) describes modes of odd vector parity ($E_y$ is odd, $H_x$ and $E_x$ are even functions), while Eq. (3) describes modes of even vector parity ($E_y$ is even, $H_x$ and $E_x$ are odd functions). If we consider an extreme case when the thickness of the two
claddings becomes infinite that \( t \to \infty \) (the rectangular waveguide becomes a parallel-plate waveguide with the separation \( a \) between the two plates), the two dispersion relations denoted by Eqs. (2) and (3) for the odd and even modes lead to

\[
\tan h(\kappa_{1}s) = -\frac{k_{2}^{2}e_{\varepsilon_{1}}}{k_{1}^{2}e_{\varepsilon_{2}}},
\]

(4)

\[
\tan h(\kappa_{1}s) = -\frac{k_{1}^{2}e_{\varepsilon_{2}}}{k_{2}^{2}e_{\varepsilon_{1}}},
\]

(5)

which are actually in the same form as those for the odd and even modes supported in sandwich structures in the optical regime [42] with only the replacement of \( \varepsilon_{i} \to \varepsilon_{\eta} \) \((i = 1, 2)\).

3. Analysis and discussions

The dispersion relations [Eqs. (2) and (3)] and [Eqs. (4) and (5)] can now be applied to the effective IMI (\( \varepsilon_{\varepsilon_{2}} = \varepsilon_{\gamma_{2}} > \varepsilon_{\varepsilon_{1}} \)) and effective MIM (\( \varepsilon_{\varepsilon_{2}} = \varepsilon_{\gamma_{3}} < \varepsilon_{\varepsilon_{1}} \)) structures to investigate the properties of the coupled ESPPs modes in these two systems. In the following calculations and simulations, only \( TE_{1} \) mode is considered in a parallel-plate waveguide with the separation \( a = 22.86\,\text{mm} \) and \( TE_{1\eta} \) mode \((m = 1)\) in an X-band rectangular waveguide \((a = 22.86\,\text{mm} \text{ and } b = 10.16\,\text{mm})\). The purple region is filled with a dielectric of relative permittivity \( \varepsilon_{\varepsilon_{2}} = 4 \) and the blue region is filled with air of \( \varepsilon_{\varepsilon_{1}} = 1 \). In the same manner as our previous work [40], we place a series of thin metallic wires with radius \( r = a / 200 \) and period \( d = a / 40 \) along the entire interface between the blue and purple regions. These thin metallic wires contribute to the suppression of TM modes and accumulation of electric charges to sustain the normal components of the electric field to face opposite to each other at the two interfaces. We remark that increasing the period \( d \) (such as \( a / 10 \) or \( a / 5 \)) could affect the cutoff frequencies of the modes. However, the trend of the dispersion curves would not change. Thus, for better agreement with our analytical dispersion results, \( d \) is chosen as \( a / 40 \) in this work.

Fig. 2. Dispersion relations of odd and even ESPPs modes in the effective IMI structure in a parallel-plate waveguide.
We first consider an effective **IMI** structure in a parallel-plate waveguide with \( \varepsilon_{s2} = \varepsilon_{s3} = 4; \varepsilon_{s1} = 1 \) and calculate the dispersion curves for the odd and even modes using the analytical results in Eqs. (4) and (5) compared with the corresponding simulated dispersion curves using the commercial software CST STUDIO in Fig. 2. It can be observed that the simulations agree quite well with the analytical results. It is also quite interesting to note that the frequency of the odd modes are higher than the respective frequencies for a single interface ESPPs and the even modes. When the thickness of the air layer (Layer 1) decreases, the odd modes also evolve into the TE\(_1\) mode supported in the dielectric filled parallel-plate waveguide and the confinement of the coupled ESPPs becomes weak. This implies that the propagation length of the ESPPs would drastically increased with negligible loss in the air, which is in analogous to the long-ranging SPPs in optical frequencies supported by IMI structures with decreasing metal film thickness [42]. The even ESPPs modes exhibit the opposite behavior—their confinement increases with decreasing air layer thickness, resulting in a reduction in propagation length. When the air layer thickness increases, the odd and even modes become increasingly decoupled and converge to the ESPPs at a single interface shown as the red solid line in Fig. 2. Different from the dispersion curves of real SPPs starting from zero frequency, all the dispersion curves of the odd and even modes start from the cutoff frequency of TE\(_1\) mode in the parallel-plate waveguide.

Accordingly, we can also study the effective **MIM** structure in a parallel-plate waveguide with \( \varepsilon_{s2} = \varepsilon_{s3} = 1; \varepsilon_{s1} = 4 \) and calculate the dispersion curves for the odd and even modes using the analytical results in Eqs. (4) and (5) compared with the corresponding simulated results in Fig. 3. Excellent agreement further demonstrate the validity of the above theory. It is also quite interesting to observe that the frequency of the even modes are higher than the respective frequencies for a single interface ESPPs and the odd modes. The even modes can exhibit negative group velocity with decreasing thickness \( 2s \) of the dielectric layer. The odd ESPPs modes, however, being the fundamental modes of the system, only have positive group velocity no matter how thick Layer I is. Fig. 4 further shows the dispersion relations of the fundamental odd mode for the effective MIM structure with the thickness of Layer I \( 2s = b \) and the dielectric in Layer I taken as lossy. We can observe that \( \text{Re}[\beta] \) does not go to infinity as the surface plasmon frequency is approached, but folds back, which is analogous to the real SPPs in silver/air/silver heterostructure [42]. With the increase of the dielectric loss, the dispersion curve folds back at a smaller value of \( \text{Re}[\beta] \).
For comparison, we also plot the simulated and analytical dispersion relations [Eqs. (2) and (3)] of the odd and even ESPPs modes in the effective IMI and MIM structures in a rectangular waveguide shown in Figs. 5 and 6, respectively. Due to the imaging effect of the upper and lower walls of the rectangular waveguide, the dispersion relations show a bit difference, especially for the effective MIM structure shown in Fig. 6, in which the starting frequency of the even mode does not change no matter how thick the dielectric is. However, all these dispersion relations explicitly show the hybridization of the ESPPs modes in bounded waveguides and give us a route to design functional devices based on ESPPs by engineering the dispersion relations.
Then, we show in Figs. 7 and 8 how to excite coupled ESPPs modes in the effective IMI and MIM structures under wave port excitations in a standard X-band rectangular waveguide with the cross section dimension $a \times b = 22.86 \times 10.16 \text{mm}^2$ and length $L = 10a$.

For the effective IMI structure in Fig. 7(a), the middle region is of length $l_3 = 6a$ (with the air thickness $2s = 0.2b$) and the two side regions with length $l_1 = l_2 = a$ are mode conversion ones with continuous variation of the air layer thickness from 0 to $2s$. Considering that the wave port excitations ($TE_{10}$ mode) is of odd mode form with respect to the symmetric plane at $y = 0$, the expected ESPPs are also odd mode with spectrum ranging from 3.28GHz to 4.62GHz according to the above theory. Simulations in CST STUDIO also demonstrate a passband between 3.2GHz and 4.62GHz in the S parameters spectrum from 2 to 6GHz shown in Fig. 7(e). At an arbitrary frequency point $f_0 = 4.5GHz$, we calculate the effective relative permittivities of these three dielectrics as $\varepsilon_{e1} = -1.126$ and $\varepsilon_{e2} = \varepsilon_{e3} = 1.874$ and show in Figs. 7(b) to 7(d) the electric lines of force and $E_y$ distributions in the $yz(x = 0)$ and $xz(y = b/2)$ planes. We can observe perfect conversion between the $TE_{10}$ mode and the odd ESPPs mode. According to the dispersion relations in Fig. 5, the odd ESPPs mode exhibits both positive and negative group velocity when Layer II is thin enough. And with the increase of the working frequency $f_0$, the wavelength of the ESPPs is dramatically shortened and the field confinement significantly enhances.

For the effective MIM structure in Fig. 8(a), the dielectric thickness is set as $2s = 0.5b$ in the middle region and the two side regions are mode conversion ones with continuously decreasing dielectric thickness from $b$ down to $2s$. Due to the odd mode excitations($TE_{10}$), the expected ESPPs are also odd mode with spectrum ranging from 3.28GHz to 4.15GHz, theoretically. Excellent agreement can also be obtained by comparison with the simulated passband between 3.28GHz and 4.11GHz shown in Fig. 8(e). Figs. 8(b)-8(d) show the electric lines of force and $E_y$ distributions in the $yz(x = 0)$ and $xz(y = b/2)$ planes at 3.9GHz.
Fig. 7. Simulation of the odd ESPPs mode in effective IMI structure in a standard X-band rectangular waveguide, in which $l_1 = 6a$ and $l_1 = l_2 = l_4 = l_5 = a$. (a) Structure of the interface (b) Distributions of electric lines of force on the yz plane (c) Distributions of Ey on the yz plane (d) Distributions of Ey on the xz plane (e) Odd ESPPs mode spectrum.
Fig. 8. Simulation of the odd ESPPs mode in effective MIM structure in a standard X-band rectangular waveguide, in which $l_2 = 6a$ and $l_3 = l_4 = l_5 = a$ (a) Structure of the interface (b) Distributions of electric force of lines on the yz plane (c) Distributions of $E_y$ on the yz plane (d) Distributions of $E_y$ on the xz plane (e) Odd ESPPs mode spectrum.

In both effective IMI/MIM structures, fields are concentrated in the dielectrics, which is analogous to the fields concentrating in the air in the IMI/MIM structures at optical frequencies. Different from real SPPs, the cutoff and asymptotic frequencies of the ESPPs in multilayer systems in bounded waveguides can be flexibly controlled by the lateral dimension of the waveguide, dielectric parameters and mode number. Although we have limited our
discussion of coupled ESPPs in three-layer structures in the parallel-plate waveguide and the 
rectangular waveguide to the fundamental bound modes of the system, it should be noted that 
the family of modes supported by this system is much richer than described above. The 
coupling between ESPPs at the two core/cladding interfaces changes dramatically when the 
permittivities of the sub- and superstrates are different.

4. Summary

In summary, the hybridization of the ESPPs in multilayer systems are studied in this work. 
Odd and even ESPPs modes in the effective IMI and MIM structures are investigated in detail 
by engineering the TE modes in a rectangular waveguide and a parallel-plate waveguide. We 
derive and analyze the dispersion relations and asymptotic frequencies of the odd and even 
ESPPs modes in these two sandwiched structure systems. Simulations demonstrate that the 
odd and even ESPPs modes in effective IMI and MIM structures resemble nearly the same 
characteristics of the odd and even SPPs in IMI and MIM heterostructures in the optical 
regime. Our results also show that different from SPPs in optical frequencies, the cutoff 
frequency and asymptotic frequency of the ESPPs in bounded waveguides can be flexibly 
tuned by the waveguide dimension, dielectric parameters and mode numbers. This work 
进一步 demonstrate that ESPPs mimic real SPPs in an ultra-subwavelength confinement 
nature and can find potential applications in compact devices and circuits in the microwave 
and terahertz frequencies.

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