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<th>Joint pricing and power allocation for multibeam satellite systems with dynamic game model</th>
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Abstract—Multibeam satellite systems (MSS) enable transmission flexibility and spatial diversity while efficiently reusing the scarce spectrum resource. However, as spectrum reuses tend to introduce co-channel interference, MSS need to address power allocation and interference management carefully. In this paper, we tackle the joint interference pricing and power allocation problems of MSS by formulating the underlying resource allocation problem as a dynamic game model—the Stackelberg model. In our proposed scheme, a fresh satellite user will be charged according to its interference on the satellite system. MSS can dynamically adjust the interference price in order to make a trade-off between inter-cell interference and operating profit. Meanwhile, for the satellite user, an equilibrium power allocation should be ascertained in response to the MSS’s pricing. A novel market-based solution is proposed for interference management in MSS by introducing an elastic price mechanism. The Nash equilibrium for interference pricing and its iterative convergence for power allocation have further been proven. Numerical results are provided to evaluate the impact of different prices on the utility functions of both MSS and satellite users.

Index Terms—Multibeam satellite systems (MSS), satellite communications, power allocation, interference management

I. INTRODUCTION

With the wide applications of various satellite systems, higher demand for bandwidth efficiency and transmission capacity are expected to be fulfilled. Multibeam antenna technology, which can help enhancing frequency reuse and increasing communication capacity, is a significant part of satellite communication systems that has attracted growing attention of the satellite communication community [1]-[3].

The most obvious characteristics of multibeam technique is its capability of transferring a single wide beam into dozens, or even hundreds, of beams so as to increase the coverage gain for satellite antennae. With the application of multiple antennae, multibeam technique can allocate the same spectrum resource to serve more users, i.e., allowing it to carry larger capacity without increase in bandwidth. With the continuous development and application of multibeam technique, it will be equipped by more satellite mobile communication systems especially on Ka-band. However, in the face of spatial diversity as well as band reuse of multibeam satellite systems (MSS), interference among beams will affect system performance to a large extend [4][5]. Besides, since it is difficult for satellite systems to keep perfect orthogonality during frequency reuse, inter-cell interference seems inevitable especially for adjacent cells [6].

In general, each spot beam in MSS has different traffic demand as well as channel condition depending on the service requirement and location of the users [7][8]. Meanwhile, each beam intends to compete with others for wireless resources such as bandwidth and power to achieve satisfactory communication. Thus, in order to manage the issue of inter-cell interference and to enhance MSS performance, various techniques are explored by researchers with the objective of achieving efficient wireless resource allocation.

Unlike static resource allocation in satellite system, dynamic allocation needs to take into account a number of deciding factors including individual communication requirement, user priority, transmit diversity as well as service type in real time. Mainly working in time division mode (TDM) with elastic network schemes, existing satellite systems manage their wireless resources by rationally allocating time slot, transmit power as well as spectrum band to address these concerns. Different TDM slots of the same carrier can transmit information in various rates while depending on accurate synchronization technology to guarantee its practicability [9]. The allocation of time-slot resource takes an important position in MF-TDMA systems due to its major impact on the resource efficiency and user’s QoS [10]. For another, resource constraints in transmission power of satellite systems have become a serious obstacle for the networking of satellite communications, air-space-ground integration and high-speed capacity. The optimization objective of power control in MSS needs to match the differential demands of traffic, rate or throughput for different beams [11]. Various efforts in the literature to improve both energy efficiency and spectrum efficiency have been paid [12][13]. Besides, as the allocated capacity of each beam should be changed adaptively according to the time-variant traffic distribution over MSS, more bandwidths can be focused on cells with hot traffic [14][15]. Dynamic spectrum combined with power allocation can make MSS more feasible in complex network circumstance. Furthermore, the allocation of spectrum band in satellite systems is always performed by ways of spectrum sharing or dynamic spectrum access where
cognitive satellite communications sharing satellite spectrum with terrestrial networks have attracted plenty of attention in recent years [16][17].

Compared to the study of resource allocation on terrestrial networks, works on satellite systems, especially on MSS, have not been investigated well. As resource allocation in wireless networks always needs to balance the benefits of different participants, the effort tends to be deemed like a zero-sum game, though market-driven mechanism consisted of auction-based or pricing-based approach has long been studied in terrestrial networks. It is widely accepted that market-based method can more efficiently redeploy the scarce resource, balance the demands and even attract potential participants. However, to our knowledge, few related work is available for the satellite systems. Only several pricing-based methods for resource allocation in satellite communication networks have been proposed before [18][19]. In [18], a congestion pricing scheme is designed to drive the buffer queue length to an appropriate reference queue length. In [19], a novel MAC protocol is proposed based on some pricing strategy that aims to allocate network resources efficiently according to users' demand.

In this paper, we investigate the problem of interference management in MSS by introducing a concept namely interference pricing. Compared with single beam satellite, in order to enhance spectrum efficiency, MSS easily incurs co-channel interference thus resulting in deterioration in system capacity. In the proposed scenario, when a new terrestrial satellite user joins in the MSS network, it can be foreseen that adjacent cells sharing the same band will suffer additional interference accordingly. With the intention of properly controlling inter-cell interference, we try to apply market-based solution to adjust new user’s power and manage inter-cell interference. When new satellite users access the multibeam network with larger transmit power, they should bear relatively higher cost. The satellite system can achieve higher revenue and better control of inter-cell interference through this pricing scheme, which raises the charge to adaptive level when inter-cell interference is too high. To be specific, we adopt Stackelberg game model to formulate the pricing problem in this paper. The Stackelberg game, which is a strategic game consisting of a leader and several followers competing with one another on certain resources, has been widely investigated and applied in terrestrial networks to solve dynamic game difficulties [20]-[23]. In particular, [20] proposed a negotiation-based throughput maximization algorithm which adjusts the operating channel and transmit power among access points from a game-theoretic perspective. [21] investigated a price-based resource allocation strategy for two-tier femtocell networks where a central macrocell is underlaid with distributed femtocells over same band. [22] proposed a novel cell ON/OFF scheduling algorithm based on the Stackelberg game to optimize packet throughput performance with a tradeoff in energy consumption. [23] designed a joint pricing and power allocation scheme for cognitive radio networks by using Stackelberg game.

For MSS, due to the provision of spectrum reuse in its dozens of cells, when a new satellite user joins the MSS, there will be an increase in interference in the system, especially between adjacent cells sharing the same frequency band. We find that the economic-perspective method can well model the inherent relationship between managing MSS inter-cell interference and allocating user’s power resource. To the best of our knowledge, no prior literature addresses the issue of interference management in MSS by using market-driven solution. The special fading model of the satellite channel and the oblique projection in MSS should be paid more attention to.

The contributions of this paper can be highlighted as follows.

- A novel resource allocation algorithm is proposed to address the interference management in MSS by introducing a market-based mechanism which plays an effective role to combat satellite networks’ interference and achieve higher spectrum efficiency.
- We adopt a dynamic game model—Stackelberg model to describe the relationship between the satellite system’s profit, interference pricing and user’s power allocation. A satellite user can observe the satellite system’s pricing and then decides its power allocation in order to maximize cost-efficiency.
- Asymptotic analysis on the changes in pricing and player’s profit is provided to evaluate the performance of the proposed method. This paper also proves the pure Nash equilibrium of the pricing algorithm and its convergence characteristics for the power allocation solution.

The rest of this paper is organized as follows. A system model of MSS network is provided in Section II. Then, a pricing algorithm based on dynamic game model is proposed in Section III. In Section IV, numerical results are provided to testify the performance of our proposed scheme. We conclude this paper in Section V.

II. SYSTEM MODEL

A high-throughput MSS is designed to comprise of N beams to serve a specified area, in which every single section is covered by a different satellite beam. In this paper, we consider a geostationary satellite that employs a transparent architecture operating in the Ka-band. Since the scenario involves flexible power allocation and interference management, it is assumed that the satellite payload is equipped with the necessary modules, such as multiport amplifiers, flexible traveling wave tube amplifiers. In the course of spectrum reuse, we assume that a four-color spectrum reuse pattern is employed which is a simple and practical scheme to avoid coverage conflict and improve spectrum efficiency as shown in Fig. 1.

Generally, when investigating the resource allocation and interference management in multibeam satellite, the earth surface can be considered as a plane, on which each cell is approximated by an orthographic projection for the satellite beam. Thus, a cone has been formed by a cell and the corresponding beam. We should take into account during the design phase of satellite networks that the signal strength in the border of a beam is weaker than that in the center. Hence, in this work, we suppose that most of the cells in the proposed system model as shown in Fig. 1, are not working under the
mode of orthographic projection, but the oblique projection which makes the bottom of the cone similar to an ellipse. Moreover, the cell’s shape is changing with the angle between the central line and the bottom.

In terrestrial communication systems, the strength of inter-cell interference mainly depends on the distance from the adjacent cells of first tier. If the orthogonality between the terrestrial cells is perfect, less interference will be invoked by the other cells. However, in mobile satellite communication system, the satellite antenna acts as the role of spatial filter. Generally, the angular selectivity of beams is hard to be ideal in practice, and the interference strength is also affected by the angle between the selected user’s position and the central line of the corresponding beam, as shown in Fig. 2.

As shown in Fig. 2, when taking into account the oblique projection, the subastral point does not match the cell’s center well. In this case, the angle \( \theta \) describing the deviation angle between user \( (U,c) \) and cell center \( o \) can be expressed as

\[
\theta = \arccos \left( \sqrt{(d_o^p)^2 + (d_{U,c}^p)^2 - 2R^2(1 - \cos(d_{U,c}^o/R))} \right)
\]

where \( d_o^p \) denotes the distance between cell center \( o \) and the satellite as shown in Fig. 2. \( d_{U,c}^p \) denotes the distance between user \( (U,c) \) and the subastral point, \( d_{U,c}^o \) denotes the distance between user \( (U,c) \) and cell center \( o \), and \( R \) means the earth radius.

In the GEO satellite communication system proposed in this paper, we adopt the communication mode of channelized TDM. In general, for channelized TDM systems, a user in any single cell will suffer from interference caused users working in other cells which share the same band. As for the number of interference users, it depends on the network pattern and whether the cell is fully loaded. As shown in Fig. 1, user \( (U,c) \) denotes the user causing interference to user \( (M,n) \). Then, for the uplink channel, the carrier power can be obtained as

\[
P_{\text{up}} = \frac{P_{\text{up}}Mn g_{\text{Mn}}(\alpha_{\text{Mn}})G_M(\theta_{\text{Mn}}^M)}{(4\pi d_{\text{Mn}}/\lambda)^2 f_{\text{Mn}}(\alpha_{\text{Mn}})}
\]

where \( p_{\text{up}}Mn \) denotes the transmit power of satellite terminal \( (M,n) \), \( \alpha_{\text{Mn}} \) denotes the elevation angle from user \( (M,n) \) to the satellite system, \( g_{\text{Mn}}(\alpha_{\text{Mn}}) \) denotes the antenna gain

of terminal user \( (M,n) \) at direction \( \alpha_{\text{Mn}} \). \( \theta_{\text{Mn}}^M \) is the derivation angle from user \( (M,n) \) to the central line of cell \( M \). \( G_M(\theta_{\text{Mn}}^M) \) is the satellite antenna gain of cell \( M \) at direction \( \theta_{\text{Mn}}^M \). \( d_{\text{Mn}} \) is the straight-line distance between user \( (M,n) \) and the satellite system. \( \lambda \) denotes the wavelength, and \( f_{\text{Mn}}(\alpha_{\text{Mn}}) \) denotes the channel fading of user \( (M,n) \) at direction \( \alpha_{\text{Mn}} \). Thus, the interference among the terrestrial cells can be given as

\[
I = \frac{1}{k} \sum_{U=1}^{k} \frac{p_{\text{up}}Uc g_{\text{Uc}}(\alpha_{\text{Uc}})G_U(\theta_{\text{Uc}}^U)}{(4\pi d_{\text{Uc}}/\lambda)^2 f_{\text{Uc}}(\alpha_{\text{Uc}})} p_{\text{up}}Uc p_{\text{up}}Uc^M
\]

where \( k \) denotes the number of the cells sharing the same frequency with cell \( M \), \( p_{\text{up}}Uc \) denotes the transmit power of satellite terminal \( (U,c) \), \( G_U(\theta_{\text{Uc}}^U) \) is the satellite antenna gain of cell \( U \) at direction \( \theta_{\text{Uc}}^U \). \( d_{\text{Uc}} \) is the straight-line distance between user \( (U,c) \) and the satellite system, \( \lambda \) denotes the wavelength, and \( f_{\text{Uc}}(\alpha_{\text{Uc}}) \) denotes the channel fading of user \( (U,c) \) at direction \( \alpha_{\text{Uc}} \). \( \rho_{\text{Uc}}^M \) denotes the active factor of user \( (U,c) \) which is related to the user’s service type. \( \rho_{\text{Uc}}^M \) is the polarization isolation factor between cell \( M \) and \( N \). Then, the uplink SINR can be expressed as

\[
SINR_{\text{U},c} = \frac{P_{\text{up}}Mn g_{\text{Mn}}(\alpha_{\text{Mn}})G_M(\theta_{\text{Mn}}^M)}{(4\pi d_{\text{Mn}}/\lambda)^2 f_{\text{Mn}}(\alpha_{\text{Mn}}) + N_0(\alpha_{\text{Mn}})B_{\text{Mn}}}
\]

III. DYNAMIC GAME AND NASH EQUILIBRIUM

The power allocation and interference pricing problem over MSS can be modeled as a dynamic game, in which each player (satellite system or satellite user) tries to maximize its payoff function. A kind of strategic games known as the Stackelberg game which matches our model well is employed in this case.
By deriving the Nash equilibrium, an algorithm for joint power allocation and interference pricing is proposed and the relevant characteristic analysis is also provided.

A. Cost Function of The Model

The Stackelberg game model is a dynamic strategic model where the players involved in the process are categorized into two groups (the leader players and the follower players). The leader moves first and then the follower moves sequentially. Therefore, the follower chooses a strategy to optimize its objective function in response to the leader's action. On the other hand, the leader can also pursue its benefit maximization by predicting the optimal response of the follower.

In this work, we formulate the optimization problem of resource allocation in the process of spectrum reuse in MSS as a non-cooperative, two-stage Stackelberg game where the satellite system acts as leader and new satellite users as followers.

Stage I: The satellite system tends to receive more profits from its satellite relay service by rationally deciding interference price \( \pi \) and scheduling transmit power \( p_n \) for the original satellite user \((M, n)\) suffered co-channel interference from a new user \((U, c)\) as shown in Fig. 1. In this scenario, due to the coexistence of time-division and frequency-division transmission mode, we consider the original satellite user \((M, n)\) who suffers the co-channel interference from a new satellite user \((U, c)\). And we will give an analysis on the scenario where the co-channel interference is raised by multiple satellite users. Thus, in this case, the satellite system’s utility function \( U_s(p_c, \pi, p_n, B_c) \) can be expressed as follows

\[
\max_{p_n \geq 0, \pi \geq 0} \quad U_s = \pi p_c f_{cs} + \varepsilon B_c - \kappa X_{loss}
\]

\[
\text{subject to} \quad \gamma_n = \frac{p_n f_{ns} + N_0(\alpha_c) B_n}{p_c f_{cs} + N_0(\alpha_c) B_n} \geq \gamma_{tar}
\]

where \( \pi p_c f_{cs} \) denotes the charge paid by new satellite user \( c \) due to its interference on the satellite system. \( p_c \) denotes the transmit power of the new satellite user, \( f_{cs} \) denotes the path loss from the satellite transmitter \( c \) to the satellite system which can be expressed as \( f_{cs} = g_{Uc}(\alpha_c) G_{Uc}(\theta_U) \lambda^2 \mu_{Uc} p_U^M / 16 \pi^2 d_{Uc}^2 f_{Uc}(\alpha_U) \). Furthermore, \( X_{loss} \) is the increasing power consumed by the original satellite user \( n \) when the new satellite user has joined in the system, thus \( X_{loss} = p_n - p_n' \), where \( p_n' \) is the power required by the user under no co-channel interference, which can be achieved from the equation \( p_n f_{ns} / N_0(\alpha_c) B_n = \gamma_{tar} \). Thus, we can obtain \( X_{loss} = \frac{\gamma_{tar} p_c f_{cs}}{f_{cs} p_n} \) when QoS is merely attained at the given threshold for original user \( n \). In this profit function equation, \( \pi, \kappa \) and \( \varepsilon \) denote the monetary coefficient which transfers the system income into monetary profit. Besides, \( B_c \) is the bandwidth purchased by the satellite user \( c \).

Stage II: Having detected the interference price and power allocation of the satellite system, new satellite user \( c \) can select appropriate transmit power \( p_c \) to maximize its utility function \( U_c(p_c, \pi, p_n, B_c) \), which can be expressed as follows

\[
\max_{p_c \geq 0, \pi \geq 0} \quad U_c = \omega \log_2(1 + \frac{p_c f_{cs}}{p_n f_{ns} + N_0(\alpha_c) B_c}) - \varepsilon B_c - \pi p_c f_{cs}
\]

\[
\text{subject to} \quad \gamma_c = \frac{p_c f_{cs}}{p_n f_{ns} + N_0(\alpha_c) B_n} \geq \gamma_{tar}
\]

where \( \omega \) is the monetary coefficient and \( \log_2(\cdot) \) is the transmission capacity obtained by the new satellite user. The explanations about the parameters \( \varepsilon, \pi, p_n, p_c \) are given before. After paying for the cost of system interference and transmission bandwidth, the new user can achieve the benefit in terms of transmission capacity in given spectrum band.

B. Derivation of the Nash Equilibrium

Having detected the interference pricing and power allocation of the satellite system, the Nash equilibrium of the satellite user’s utility function can be firstly derived. Based on the necessary conditions for Nash equilibrium and system’s decision on power allocation, we have

\[
\frac{\partial U_c}{\partial p_c} = 0 = \frac{\omega}{\ln 2} \cdot \frac{f_{cs}}{p_n f_{ns} + N_0(\alpha_c) B_n} - \pi f_{cs}
\]

Solving equation (7), we can obtain

\[
p_c = \frac{\omega / (\pi \ln 2) - N_0(\alpha_c) B_c - p_n f_{ns}}{f_{cs}}
\]

Combining (8) and the SINR requirement of the satellite system transmission as \( \frac{p_n f_{ns}}{p_c f_{cs} + N_0(\alpha_c) B_c} = \gamma_{tar} \), the power allocation \( p_n \) can be expressed as

\[
p_n(\pi) = \frac{\omega / (\pi \ln 2) - N_0(\alpha_c) B_c - N_0(\alpha_c) B_n}{f_{ns}(1 + 1/\gamma_{tar})}
\]

Back to (8), substituting \( p_n \) by (9), the power allocation for the new satellite user can be achieved as

\[
p_c(\pi) = \frac{\omega / (\pi \ln 2) - N_0(\alpha_c) B_c - N_0(\alpha_c) B_n}{f_{ns}(1 + 1/\gamma_{tar})}
\]

Due to \( X_{loss} = \frac{\gamma_{tar} p_c f_{cs}}{f_{cs} p_n} \), substituting \( p_c \) in (5), then the utility function for the satellite system can be derived as follows

\[
U_s(\pi) = (\pi - \frac{\kappa_{max} f_{cs}}{f_{ns}})[\frac{\omega}{\pi \ln 2} - N_0(\alpha_c) B_c - \frac{\omega / (\pi \ln 2) - N_0(\alpha_c) B_c + N_0(\alpha_c) B_n}{1 + 1/\gamma_{tar}}] + \varepsilon B_c
\]

Since the utility function \( U_s \) is concave with the parameter \( \pi \), taking its derivation, we can obtain the optimal pricing for the interference pricing as

\[
\pi^* = \sqrt{\frac{\omega \kappa_{max} f_{cs}}{f_{ns} \ln 2 [\gamma_{tar} N_0(\alpha_c) B_n + N_0(\alpha_c) B_c]}}
\]

Substituting \( \pi \) in (9) and (10), the equilibrium power allocation \( p_c(\pi^*) \) and \( p_n(\pi^*) \) can be obtained respectively. Furthermore,
the optimal profit for the satellite system can be achieved by putting \( \pi^* \) into (11), then we have

\[
U_s^*(\pi) = \frac{\omega N\gamma}{f_{ns}} \ln 2\gamma \ln N + N_0(\alpha) B_n + N_0(\alpha) B_c 
\times \left( \frac{\omega}{f_{ns} \ln 2\gamma \ln N + N_0(\alpha) B_n} - N_0(\alpha) B_c - \frac{\omega}{f_{ns} \ln 2\gamma \ln N + N_0(\alpha) B_n} \ln 2 \right) 
\times \left( 1 + 1/\gamma \right) 
+ \varepsilon B_c - \frac{\kappa \gamma}{f_{ns}} B_{cs}
\]

(13)

Then, the optimal profit for the new satellite user in this satellite communication can also be obtained by putting \( \pi^*, p_c, p_n^* \) into (6) which can be expressed as

\[
U_s^*(\pi) = \omega \log_2 (1 + \frac{\omega}{\pi^* \ln 2} - N_0(\alpha) B_c + N_0(\alpha) B_n) 
\times \left( \frac{\omega}{\pi^* \ln 2} - N_0(\alpha) B_c + N_0(\alpha) B_n \right)
\]

(14)

Therefore, a game algorithm for power allocation and interference pricing has been proposed to maximize the profit of both the satellite system and satellite user.

C. Existence of The Nash Equilibrium

**Proposition 1:** There exists a Nash equilibrium for satellite system’s utility, and the equilibrium can be unique when condition (C1): \( \frac{\omega}{\pi^* \ln 2} > N_0(\alpha) B_c + \frac{\omega}{\pi^* \ln 2} - N_0(\alpha) B_c + N_0(\alpha) B_n \)

holds.

**Proof:** Based on Kakutani’s Fixed Point Theorem [24], a equilibrium point, or a fixed point exists, in given response \( U_s \) and variable \( \pi \), is a convex and nonempty with \( \pi \), and \( U_s : \Sigma \rightarrow \Sigma \) is a compact convex subset. (2) Closed mappings exist for \( U(\pi) \).

Define \( S_1(p_n, p_c, \pi) \) is the strategy combination for \( U(\pi) \), thus \( \Sigma_1 \) is a simple in dimension \( |\Sigma_1 - 1| \). Besides, since \( U(\pi) \) is linear and continuous whose maximal value can be fixed in its compact subset, we can obtain condition (1) should be met.

Then, condition (2) can be proven by the method of the proof by contradiction. If condition (2) cannot be met, there exist \( \varepsilon > 0 \) and \( \pi^*_i \), letting \( U_i(\pi^*_i, \pi, \pi, \ldots, \pi) > U(\bar{\pi}_i, \pi, \pi, \ldots, \pi) + 3\varepsilon \). Because \( U_i \) is continuous and \( (\pi^*, \bar{\pi}) \rightarrow (\pi, \bar{\pi}) \), giving enough large \( \varepsilon \), there is

\[
U_i(\pi^*_i, \pi^*_i) > U_i(\bar{\pi}_i, \pi, \pi, \ldots, \pi) - \varepsilon > U_i(\bar{\pi}_i, \pi, \pi, \ldots, \pi) + 2\varepsilon > U_i(\bar{\pi}_i, \pi, \pi, \ldots, \pi) + \varepsilon
\]

(15)

For \( \pi^*_i \), \( \pi^* \) is strictly prior to \( \bar{\pi}_i \) which is contrary to \( \bar{\pi}_i \in U_i(\pi^*_i) \), so condition (2) can be satisfied. Same principle fits to \( p_c \) and \( U_c \). Besides, based on (10), in order to ensure \( p_c(\pi) > 0 \) and \( p_n(\pi) > 0 \), we have \( \frac{\omega}{\pi \ln 2} > C_1 N_0(\alpha) B_c + \frac{\omega}{\pi \ln 2} - N_0(\alpha) B_c + N_0(\alpha) B_n \). According to [25], we can conclude the equilibrium is unique.

D. Asymptotic Behavior Analysis

We can achieve from the deductions above that the satellite system’s equilibrium profit \( U_s \) and satellite user’s equilibrium utility \( U_c \) as well as the optimal equilibrium price \( \pi^* \) are all affected by the parameters’ selection including \( \omega, \kappa \) and \( \gamma \).

Thus, we give the asymptotic analysis for these parameters on the effects of optimal utilities and prices.

**Remark 1:** Fixing \( \kappa \) and \( \gamma \), when satellite user’s monetary coefficient \( \omega \rightarrow 0 \), satellite system’s optimal price \( \pi^* \rightarrow 0 \). On the other hand, original satellite user’s optimal power \( p_n(\pi) \rightarrow N_0(\alpha_c) B_n / f_{ns}(1+1/\gamma) \), and the new satellite user’s power \( p_c = \frac{N_0(\alpha_c) B_c}{f_{cs}(1+1/\gamma)} - N_0(\alpha_c) B_c \). The changes of optimal power allocation and optimal spectrum pricing with various monetary coefficients are shown in Fig. 3. In the following simulation tests, we consider a MSS network with one centralized satellite and a number of beams as shown in Fig. 1. In this case, satellite users work in the mode of channelized TDM where the interior interference and the interference caused by other satellite networks are ignored. Suppose the MSS network has 16 beams, and each beam’s radius is 200km. Ten satellite users randomly locate in each cell. We consider the interference in each beam to be mainly due to spectrum reuse in the MSS network. In this subsection, we investigate the situation where an original satellite user suffers from the co-channel interference from a new satellite user located in the nearest cell. In the following subsection, we will further give the utility functions for the MSS network in which one original satellite user and a group of new satellite users work in the same channel.

In this situation, we can also obtain \( U_s^*(\pi) \rightarrow N_0(\alpha_c) B_n + \frac{\omega}{\pi \ln 2} - N_0(\alpha_c) B_c + N_0(\alpha_c) B_n \)

(16)

And \( U_c^*(\pi) \rightarrow -\varepsilon B_c \leq 0 \). The effects of optimal utility functions are shown in Fig. 4. In the tests, we set \( \kappa = 10, \kappa = 0.4, N_0(\alpha_c) = 0.004, N_0(\alpha_c) = 0.006, f_{ns} = 0.2, f_{cs} = 0.15, B_c = 1, B_n = 2 \) and \( \omega \in [0, 30] \).

Therefore, we can obtain from Fig. 3 that the optimal pricing and power allocation are all decreasing when monetary coefficient, \( \omega \) → 0. We can also achieve similar conclusions as deduced from the above. Same change rule appears in Fig. 4 where profits for both the satellite system and new satellite user are damaged when monetary coefficient decreases. If the satellite user cannot expect to receive enough benefit from leasing the spectrum of the satellite system, it can be imaged that the user will not be eager to purchase more band for usage hence leads to decline in profit at the end.

**Remark 2:** Derived from condition (C1), we define \( \lambda = \pi \ln 2(N_0(\alpha_c) B_n + \frac{\omega}{\pi \ln 2} - N_0(\alpha_c) B_c + N_0(\alpha_c) B_n) \). Given fixed \( \gamma \) and \( \kappa \), when \( \omega \rightarrow \lambda \), equilibrium price

\[
\pi^* \rightarrow \frac{\kappa \gamma}{f_{ns}(1+1/\gamma)} B_c + \frac{\omega}{\pi \ln 2} - N_0(\alpha_c) B_c + N_0(\alpha_c) B_n
\]

(17)

And \( p_n \rightarrow N_0(\alpha_c) B_c + \frac{\omega}{\pi \ln 2} - N_0(\alpha_c) B_c + N_0(\alpha_c) B_n \), \( p_c \rightarrow 0 \). In this situation, we can obtain \( U_s^* \rightarrow \varepsilon B_c \) and \( U_c^* \rightarrow -\varepsilon B_c \), thus the
benefit received by satellite systems equals to the cost paid by the satellite user which makes the trading to become a zero-sum game.

Remark 3: Given $\kappa$ and $\omega$, when $\gamma^{tar} \to \infty$, which means the QoS requirement of the satellite users is decreasing, then equilibrium price $\pi^* \to \infty$, optimal power allocation $p_n^* \to \frac{N_0(\kappa)}{B_{tar}} - \frac{N_0(\kappa)}{B_n}$, $p_c^* \to \frac{-N_0(\kappa)}{B_n}$. Besides, the equilibrium profits for both the satellite system and the satellite user become to be $U_n^* \to 0$, $U_c^* \to 0$. Fig. 5 shows the change tendency of power allocation, equilibrium pricing and utilities with increasing QoS threshold.

E. Iterative Algorithm

In practice, we can also identify the equilibrium power allocation $p_n^*$ and $p_c^*$ by iterative algorithm. For generality, we assume both the original satellite user and new satellite user have sufficient power capacity $p_{n, max}$ and $p_{c, max}$, then the iterative algorithm can be performed as the following steps.

Iterative Power algorithm (IPA): Step 1. Initialization: initialize original satellite power $p_{n, (0)} = 0$ and $p_{c, (0)} = 0$; Step 2: begin iteration, (i) original satellite user updates its power by $p_n = \min\{\frac{p_{c, (0)}}{f_{tar} - N_0(\kappa)B_n}, p_{n, max}\}$, and the new satellite user updates $p_c = \left\{ \begin{array}{ll} \frac{f_{tar} - N_0(\kappa)B_n - p_{n, (0)}f_{tar}}{f_{tar}}, & \gamma^{tar} \geq 1 \\ \omega/(\kappa \ln 2) - N_0(\kappa)B_n - p_{n, (0)}, & \gamma^{tar} < 1 \end{array} \right.$; Repeat (i) and (ii) over again until the algorithm converges. END

Proposition 2: Under Condition (C1), the IPA can be convergent if $\gamma^{tar} \geq 1$. Furthermore, the Nash equilibrium for power allocation is unique.

Proof: To testify the iterative algorithm proposed above convergent, we need to rewrite the algorithm in matrix form. Thus, we have

$$[p_n; p_c] = W^{\tau}(\Phi)[\tilde{K}(\Phi)[p_n, p_c] + b^{\tau}(p_n, \Phi), \forall \tau \in \Sigma \ (16)$$

where $\tilde{K}(\Phi) = I - K(\Phi)$. $\Sigma$ is the complete set of vector space. $\tau$ denotes a specific form of the space mapping which means the thresholds of the new satellite user’s power. Here, matrix $W^{\tau}(\Phi)$ is a diagonal matrix, $b^{\tau}(p_n, \Phi)$ is a $(|\Phi| + 1) \times 1$ vector. According to [26], if there is $\|\tilde{K}(\Phi)\|_{\infty} \leq 1$, where $\|\cdot\|_{\infty}$ is the matrix norm of $\tilde{K}(\Phi)$ in this situation, then the iterative algorithm in (16) will converge to a fixed and unique point. Based on the procedure of IPA, we achieve $K(\Phi) = \begin{pmatrix} 0 & -p_{n, (0)}f_{tar}/f_{tar} \\ -p_{n, (0)}f_{tar}/f_{tar} & \omega/(\kappa \ln 2) - N_0(\kappa)B_n \end{pmatrix}$ and $b^{\tau} = \begin{pmatrix} -N_0(\kappa)B_n \\ \omega/(\kappa \ln 2) - N_0(\kappa)B_n \end{pmatrix}$. Then, we can obtain that $\gamma^{tar} \geq 1$ can make the condition $\|\tilde{K}(\Phi)\|_{\infty} \leq 1$ holds which will guarantee the equilibrium unique.

F. Single original satellite user and multiple new satellite users

We further consider the satellite network in which one original satellite user and a group of new satellite users $Q = \{2, 3, \ldots, N + 1\}$ coexist. $N$ is the user number. We also
assume that condition (C1) is met for every single user in \( Q \). In this case, the original user may refer to the most important user whom the satellite system mainly needs to supply service for. When multiple satellite users access the spectrum band at the same time, the system requires to calculate additional cost and proper power allocation. Then, the new Stackelberg game model can be expressed as follows.

**Stage I:** The satellite system decides its interference price \( \pi \), the original satellite user’s transmit power \( p_n \) based on the set \( Q \) of multiple new satellite users to optimize its profit by addressing the following expression

\[
\max_{p_n \geq 0, \pi \geq 0} U_s = \pi \sum_{c \in Q} p_c f_{cs} + \varepsilon \sum_{c \in Q} B_c - \kappa X_{loss}
\]

subject to \( \gamma_n \geq \gamma_{tar} \)  \hspace{1cm} (17)

**Stage II:** In response to satellite system’s action, each satellite user \( c \in Q \) schedules its power level by solving the following problem

\[
\max_{p_c \geq 0, \pi \geq 0} U_c = \omega \log_2(1 + \frac{p_c f_{cs}}{p_n f_{ns} + \sum_{i \in Q(c)} p_i f_{is} + N_0(\alpha_c)B_c}) - \varepsilon B_c - \pi p_c f_{cs}
\]

subject to \( \gamma_c \geq \frac{p_c f_{cs}}{p_n f_{ns} + \sum_{i \in Q(c)} p_i f_{is} + N_0(\alpha_c)B_c} \geq \gamma_{tar} \)  \hspace{1cm} (18)

We can obtain that each satellite user involved in the multiple users scenario will suffer from interference caused by the original user and other new satellite users. The mathematical deductions of equilibrium price, optimal power allocation as well as iterative convergence are similar to the case of single new user scenario as mentioned above, thus we ignore the related process. In this case, it can also be proven that there exists a unique Nash equilibrium for the extended Stackelberg game model.

**IV. NUMERICAL RESULTS**

In this section, we further evaluate the performances of the proposed utility functions and transmission power allocation with changing interference price. As shown in Fig. 6, we give the performance of utility functions of the satellite system and new satellite user denoted by \( U_s \) and \( U_c \), with increasing interference pricing \( \pi \). In this test, we fix the relevant parameters as \( \gamma_{tar} = 10, \kappa = 0.4, N_0(\alpha_c) = 0.004, N_0(\alpha_n) = 0.006, f_{ns} = 0.2, f_{cs} = 0.15 \) and \( B_c = 1, B_n = 2 \). The equations of utility functions are expressed by (5) and (6). We can obtain from the figure that the utility function of the satellite system is not linear and has a maximum at the point of equilibrium pricing. Thus, if the interference price is too high, it will hamper consumer demand thus effectively cannot increase the satellite system’s profit. A proper interference pricing should be carefully designed to maximize the system’s benefit. On the other hand, it is apparent that the benefit of new satellite user will be damaged with increasing interference pricing as the satellite user needs to bear the cost. Similar conclusions can also be obtained from (5) and (6). Besides, as shown in the figure, when the monetary coefficient decreases, benefits for both the satellite system and user are degrading accordingly. High coefficient means more benefit for the game follower with same bandwidth, which will attract satellite users to participate in the dynamic trading in depth.

Then, we give the performances of equilibrium power allocation with different interference prices as shown in Fig. 7. From the figure, we can obtain that power allocation for both the original and new satellite users are decreasing with upgrading interference pricing. It is easy to understand that the satellite user’s transmit power should be restricted since the cost is just accounted for by the interference caused by the user. On the other hand, when the transmit power of the satellite user decreases which will lead to a decrease in inter-cell interference as a result, thus the original satellite user does not need a high transmit power to combat outside interference. Besides, when \( f_i \) become low which means the path loss is relatively severe, then a higher power allocation is required.
As shown in Fig. 7, the transmit powers for satellite users increase with degrading $f_i$.

We further testify the optimal pricing $\pi$ which has been given in (12). From the equation, we can obtain that optimal pricing is affected by several parameters, such as $f_n, \omega, \gamma_{tar}$. As shown in Fig. 8, we give the performance of optimal pricing with changing $\omega$ and $f_n$. First of all, we can achieve from the figure that an increasing optimal pricing is attained when the monetary coefficient $\omega$ is growing. It can be concluded that the satellite system can properly enhance its interference pricing when more benefits are expected for the satellite consumer. Moreover, when the path loss increases as shown in this figure which means a higher transmit power for the satellite user is essential, thus the satellite system can raise the interference price to balance the inter-cell interference and improve its profit.

At last, we give the utility performances for both the satellite system and satellite users under optimal interference pricing as shown in Fig. 9. The utility functions have been expressed in (13) and (14). It can be concluded from Fig. 8 that a higher optimal utility can be attained with increasing $\omega$ and decreasing $\gamma_{tar}$. In fact, it is a win-win cooperation between the satellite system and satellite users. When satellite users can achieve an ideal benefit from the leased spectrum, a higher price is more acceptable. As a result, the profit of the satellite system can also be improved. From another perspective, if the satellite user is eager to lease the communication band which implies a desired performance is expected for the user, then we can predict that it is an opportunity for the satellite system to reach the deal with an ideal price. A proper pricing will smooth the band trading and benefit the satellite system in the end.

V. CONCLUSION

In this paper, we have proposed a novel algorithm of power allocation and interference pricing for MSS in which the problem of inter-cell interference has been carefully considered in the context of spectrum reuse. We have described the dynamic game issue by using the Stackelberg model for satellite users. As shown in Fig. 7, the transmit powers for satellite users increase with degrading $f_i$. We further testify the optimal pricing $\pi$ which has been given in (12). From the equation, we can obtain that optimal pricing is affected by several parameters, such as $f_n, \omega, \gamma_{tar}$. As shown in Fig. 8, we give the performance of optimal pricing with changing $\omega$ and $f_n$. First of all, we can achieve from the figure that an increasing optimal pricing is attained when the monetary coefficient $\omega$ is growing. It can be concluded that the satellite system can properly enhance its interference pricing when more benefits are expected for the satellite consumer. Moreover, when the path loss increases as shown in this figure which means a higher transmit power for the satellite user is essential, thus the satellite system can raise the interference price to balance the inter-cell interference and improve its profit.

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wherein the satellite system acted as a leader and satellite users as followers. The satellite system intends to increase system profit and balance inter-cell interference. Meanwhile, satellite users observe the action of the system and properly allocate their transmit power to maximize their utility function. In MSS, the problem of inter-cell interference is required to be addressed, and we introduce the concept of market-based proposal to control interference. A complicated resource allocation problem has been properly modeled and solved by fixing the Nash equilibrium. Numerical results were further provided to evaluate the performance of our proposed pricing on the system’s benefits. In the following research, we will investigate the pricing diversity of different satellite bands in order to present the problem of resource allocation in MSS in a more detailed manner.

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