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Phase retrieval for high-speed 3D measurement using binary patterns 
with projector defocusing

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ABSTRACT

Recent digital technology allows binary patterns to be projected with a very high speed, which shows great potential for high-speed 3D measurement. However, how to retrieve an accurate phase with an even faster speed is still challenging. In this paper, an accurate and efficient phase retrieval technique is presented, which combines a Hilbert three-step phase-shifting algorithm with a ternary Gray code-based phase unwrapping method. The Hilbert three-step algorithm uses three squared binary patterns, which can calculate an accurate phase even under a slight defocusing level. The ternary Gray code-based method uses four binary patterns, which can unwrap a phase with a large number of fringe periods. Both simulations and experiments have validated its accuracy and efficiency.

Keywords: 3D measurement, projector defocusing, phase-shifting, ternary Gray code

1. INTRODUCTION

3D measurement using fringe projection technique has been widely used due to its high-resolution, non-contact, and flexibility, etc.$^{1,5}$. This technique often uses a phase-shifting algorithm to calculate the phase, for which several phase-shifting sinusoidal patterns are projected.$^1$ However, the projection speed of sinusoidal patterns is often less than 120 frames per second, which cannot be used for high speed 3D measurement such as face profiling in real-time.$^1$ Recent digital technology allows binary patterns to be projected with thousands of frames per second.$^3$ When the projector is manually defocused, the binary patterns become sinusoidal.$^4$ Because binary patterns contain many high-order harmonics, the calculated phase always contains phase error. A high defocusing level is required to reduce the phase error, which results in a lower fringe contrast, and consequently, a smaller dynamic range of 3D measurement.$^5$ The phase error can be reduced by using a large number of binary patterns.$^4$ Therefore, how to calculate an accurate phase by using a small number of binary patterns is a challenge. In addition, the calculated phase is wrapped in the range of $(-\pi, \pi]$ and contains $2\pi$ phase jumps.$^5$. Phase unwrapping is necessary to unwrap the calculated phase, and a Gray code-based phase unwrapping method is commonly used due to its robustness.$^6$. However, the traditional Gray code-based method also requires a large number of binary patterns.$^6$ Therefore, how to unwrap the calculated phase by reducing binary patterns is another challenge. To solve the above mentioned two challenges, an accurate and efficient phase retrieval technique is introduced by using fewer binary patterns, which combines a Hilbert three-step phase-shifting algorithm with a ternary Gray code-based phase unwrapping method. The phase retrieval technique is presented in Section 2. Experiments are provided in Section 3. Section 4 concludes this paper.

2. PRINCIPLE OF THE PHASE RETRIEVAL TECHNIQUE

This section is organized as follows. Section 2.1 adapts the Hilbert three-step phase-shifting algorithm to calculate the phase, and Section 2.2 introduces the ternary Gray-code based phase unwrapping method to unwrap the calculated phase.

2.1 The Hilbert three-step phase-shifting algorithm

The projector defocusing can be approximated by a 2D Gaussian filter, whose size is truncated to $\lceil 6\sigma \rceil \times \lceil 6\sigma \rceil$ (where $\lceil \cdot \rceil$ is the ceiling function), making this filter sufficiently close to the ideal Gaussian function.$^4$. The symbol $\sigma$ denotes the standard deviation of the Gaussian filter, which describes the projector defocusing level. To calculate an accurate
phase, one approach is to design binary patterns as close as possible to ideal sinusoidal patterns after Gaussian filtering, which can be mathematically described by

$$\min_{\phi} \| I - G \odot B \|_F,$$

where $\| \cdot \|_F$, $G$, and $\odot$ denote Frobenius norm, Gaussian kernel, and convolution operation, respectively. The sinusoidal pattern and the binary pattern are denoted by $J$ and $B$, respectively. Recently, pulse width modulation\(^5\), dithering\(^7\), and optimized\(^8\) techniques are used to designed binary patterns, respectively. Generally, to obtain ideal sinusoidal patterns, a proper defocusing level should be selected\(^4\). However, it is difficult to select such a proper defocusing level.

Another flexible approach is to use squared binary patterns combined with a phase-shifting algorithm with an odd step of $N$, which is insensitive to high-order harmonics lower than $(2N-1)^{\text{th}}$\(^4\). When the fringe period of squared binary patterns is selected as $T$, i.e., in one period of the squared binary pattern, $T/2$ pixels are white and the other $T/2$ pixels are black\(^4\). Because the phase change $2\pi$ in one fringe period, three squared binary patterns are designed with their respective pattern shifts of $-T/3$, $0$, and $T/3$ to generate three sinusoidal patterns with phase shifts of $-2\pi/3$, $0$, and $2\pi/3$, respectively. When the projector is manually defocused, three phase-shifting sinusoidal patterns are generated, which can be described by $I_1^c$, $I_2^c$ and $I_3^c$, respectively. The three-step phase-shifting algorithm calculates the phase by

$$\varphi = \arctan\left[\sqrt{3}(I_1^c - I_3^c)/(2I_2^c - I_1^c - I_3^c)\right].$$

When the projector is slightly defocused, we seldom observe high-order harmonics larger than the $10^{\text{th}}$-order. In this condition, the phase error of the three-step phase-shifting algorithm is described by

$$\Delta \varphi_3 = \arctan\left[\frac{(B_3 - B_1) \sin(6\varphi)}{B_1 + (B_2 + B_3) \cos(6\varphi)}\right],$$

where $B_k$ is the amplitude of the $k^{\text{th}}$-order harmonics. To reduce the phase error, a Hilbert three-step phase-shifting algorithm has been proposed\(^4\). The Hilbert transform (i.e., HT) is applied to introduce a phase shift of $\pi/2$ to all the three sinusoidal patterns\(^9\). Then, a Hilbert phase is calculated by using Eq. (2) again. Because phase errors of the two phases have opposite distributional tendency, the phase error is reduced when the two phases are averaged in the final phase, whose phase error can be described by

$$\Delta \varphi_{3\text{HC}} = \frac{1}{2} \arctan\left[\frac{B_2}{B_1} \sin(6\varphi)\right],$$

which shows that the phase error leading by the $5^{\text{th}}$-order harmonic is removed. As background knowledge\(^4\), squared binary patterns using a high fringe frequency only require a slight defocusing level. However, the fringe frequency cannot be selected too high to satisfy the sampling-rate of the camera\(^4\). Generally, the fringe period is selected around 20 pixels, and thus we select 18 pixels to satisfy the pattern-shift requirement of three-step phase-shifting algorithm.

A simulation is provided to demonstrate the performance of the Hilbert three-step phase-shifting algorithm. By using different values of the standard deviation, the RMS phase errors of the three-step and Hilbert three-step algorithms are provided in Table 1. When the standard deviation is selected as $\sigma = 0.83$, the phase error distributions of the three-step and Hilbert three-step phase-shifting algorithms are shown in Fig. 1. The blue and red lines plot the phase error distributions of the three-step and the Hilbert three-step, respectively. The Hilbert three-step performs consistently better than the three-step. In the real measurement, the system random RMS phase error is about 0.02 rad\(^4\). The Hilbert three-step can obtain an accurate phase if $\sigma \geq 0.83$. That is to say, the Hilbert three-step works well even under a slight defocusing level. It should be noted that the Hilbert transform always leads to a weak edge effect\(^4\), which can be neglected for object without a sharp height change in real measurement.
2.2 The ternary Gray code-based phase unwrapping method

To unwrap a phase with \( f \) fringe periods, the Gray code-based phase unwrapping method requires \( \lceil \log_2 f \rceil \) binary patterns, because it only uses black and white patterns. Recently, a ternary Gray code-based phase unwrapping method has been proposed\(^1\), which only requires \( \lceil \log_3 f \rceil \) binary patterns, because it uses black, gray and white patterns. The gray pattern is created by using the narrowest squared binary patterns when the projector is defocused\(^1\). The black, gray and white patterns can be segmented into three states of “0”, “1” and “2”, respectively\(^1\). The ternary Gray code-based method is illustrated in Fig. 2. The states of the two ternary patterns are denoted by \( S_1 \) and \( S_2 \), respectively. In each fringe period, the combination of states in \( S_1 \) and \( S_2 \) is unique, which can determine a unique phase order of \( K \), and then the calculated phase of \( \varphi \) can be unwrapped by\(^1\)

\[
\Phi = \varphi + K \times 2\pi,
\]

where \( \Phi \) denotes the unwrapped phase. Therefore, \( m \) binary patterns can determine \( 3^m \) phase orders for the phase unwrapping. By using the method in Ref. 3, the ternary Gray code-based method is tested by also selecting \( T = 18 \) pixels. When the standard deviation is selected in the range of \([0.5, 1.83]\), the simulated black, gray and white patterns can be correctly segmented, and the ternary Gray code-based method performs consistently error-free.

In a real measurement, the captured patterns are affected by object reflectivity and measurement environment. If \( m \) binary patterns are used, each of the captured pattern can be denoted by \( I_k^h, k = 1, 2, \cdots, m \), respectively. Before doing the segmentation, these captured patterns should be normalized by\(^3\)

\[
I_k^{nh} = \frac{(I_k^h - I_{\min})}{(I_{\max} - I_{\min})},
\]

where \( I_{\min} \) and \( I_{\max} \) denote minimum and maximum intensity, respectively, which can be easily calculated from phase-shifting sinusoidal fringe patterns\(^5\). Because of the discrete sampling of the camera, there will be normalization error around \( 2\pi \) phase jump pixels, which leads to unwrapping errors with multiples of \( 2\pi \). To remove these unwrapping errors, a simple median filtering is introduced by\(^1\)

\[
\Phi_M = medilt2(\Phi, s_x \times s_y),
\]

where \( \Phi_M \) represents the unwrapped phase after the median filtering; \( medilt2 \) represents the median filter operator;
\( s_i \times s_j \) denotes the median filter size. To make the filtering result congruent, a congruent operation is introduced by

\[
\Phi_c = \phi + 2\pi \times \text{Round}\left(\frac{\Phi_{1:4} - \phi}{2\pi}\right),
\]

(8)

where \( \text{Round}[\cdot] \) is the rounding function, and \( \Phi_c \) denotes the congruent phase we desired.

![Fig. 2 Schematic diagram of the ternary Gray code-based phase unwrapping method.](image)

3. EXPERIMENTS

The developed system includes a TI DLP Discovery 4100 projector with a resolution of 1140×912 and a Basler CMOS acA2000 camera with a resolution of 800×600. The designed fringe patterns include \( f = 50.7 \) (i.e., 912/18) periods due to the projector’s resolution. The traditional and ternary Gray code-based methods use \( \lceil \log_2 50.7 \rceil = 6 \) and \( \lceil \log_3 50.7 \rceil = 4 \) binary patterns, respectively.

To evaluate the Hilbert three-step phase-shifting algorithm, a white flat board is measured, which is placed in front of the projector with the distance of 75 cm. The projector is under a slight defocusing level, and the captured phase-shifting patterns are shown in Figs. 3(a)-3(c), respectively. These phase-shifting patterns still look like binary ones due to the slight defocusing level. Three of the four ternary Gray code patterns are shown in Figs. 3(d)-3(f), respectively. The three-step and Hilbert three-step calculated phases are shown in Fig. 3(g)-3(h), respectively. Figure 3(i) shows the actual phase, which is calculated from eighteen-step phase-shifting algorithm. By subtracting the above two calculated phases from the ideal phase, the phase errors of the three-step and Hilbert three-step are obtained. The blue and red lines in Fig. 4 plot the phase error distributions of the three-step and the Hilbert three-step, respectively. The RMS phase errors of the three-step and Hilbert three-step are 0.0623 rad and 0.0218 rad, respectively. The three-step generates obvious periodic phase errors, while the Hilbert three-step generates much less obvious periodic phase errors.

By using the ternary Gray code-based method, the above three calculated phases can be correctly unwrapped, which are shown in Figs. 3(j)-3(l), respectively. To further evaluate the ternary Gray code-based method, a computer fan as shown in Fig. 5(a) is placed on the white flat board and then measured. One of the phase-shifting patterns and one of the ternary Gray code-based patterns are shown in Figs. 5(b)-5(c), respectively. The Hilbert three-step calculated phase and the ternary method unwrapped phase are shown in Figs. 5(d)-5(e), respectively. Although this computer fan contains complex surface, the ternary Gray code-based method perform robustly, and the unwrapped phase does not contain phase unwrapping errors. For clarity, by subtracting the unwrapped phase of the computer fan from that of the white flat board, the computer fan modulated phase is obtained and shown in Fig. 5(f), which does not contain phase errors from phase unwrapping.
Fig. 3 Experimental results of a white flat board.

Fig. 4 Phase error distributions in the experiment.

Fig. 5 Experimental results of a computer fan.
4. CONCLUSION

This paper presents a phase retrieval technique by using an even fewer number of binary patterns, which combines a Hilbert three-step phase-shifting algorithm and a ternary Gray code-based phase unwrapping method. This phase retrieval technique can be used for 3D measurement with high-speed requirement.

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